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Telemanipulation with Time Delays

Abstract

In this paper we survey the development of the wave variable concept and examine wave-based teleoperation. We study the behavior of force reflecting systems under unknown but constant transmission delays, ranging from periods less than the human reaction time to several seconds. Passive transmission procedures guarantee system stability, but wave reflections and spurious dynamics may interfere with normal operation. Using wave variables for the analysis and implementation, and making appropriate design choices, a system with consistent and predictable behavior is constructed. This design methodology aims to create a virtual tool which accounts for the implicit limitations imposed by the delay. These developments also form the basis for extensions to wave-based prediction and application to variable delays, such as those inherent to Internet-based telemanipulation.

KEY WORDS—teleoperation, force reflection, time delay, wave variables, passivity, impedence matching, virtual tool

1. Introduction

Telemanipulation defines the idea of a user interacting with and manipulating a remote environment (Sheridan 1992) and has led to applications ranging from space-based robotics to telesurgery. Of particular interest are telerobotic systems, in which the user commands the motion of a remote slave robot via a local master joystick. Bilateral systems also feed forces back through this joystick to provide the operator with better information and a more complete interaction, thereby improving his or her ability to perform complex tasks (Massimino and Sheridan 1994).

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Passivity and impedance models have become a popular method for examining and ensuring the stability of bilateral teleoperators, as an environment model is not required (Hannaford 1989). In addition, transparency is desired to accurately render the environment to the user. The dual objectives of stability and transparency have led to numerous controller architectures that transmit different combinations of position and force signals (Lawrence 1993). These architectures have also been extended to admittance-type manipulators (Hashtrudi-Zaad and Salcudean 2001). Fundamental limits for force feedback gains have been explored and shown to depend on the relative manipulator inertias (Daniel and McAree 1998).

However, by their very definition, teleoperation systems frequently experience significant time delays in communication between the local and remote sites, which necessarily limit the user's performance (Ferrell 1965; Lane et al. 2000). Furthermore, the combination of force feedback with even small time delays creates stability problems (Sheridan 1993), which has led to alternative control approaches. For example, sensory substitution allows the independent display of force information via other channels, such as auditory or visual (Massimino 1995). Predictive displays utilize information about the remote manipulator and environment to display an expected response, most commonly as a graphic overlay (Bejczy, Kim, and Venema 1990). Teleprogramming issues only high-level commands to the remote site and assumes appropriate local autonomy (Funda, Lindsay, and Paul 1992; Sayers et al. 1998).

Nevertheless, to provide an immersive and natural experience to the user, much effort has been invested into extending direct force feedback methods to time delayed systems (Hashtrudi-Zaad and Salcudean 2002; Niculescu, Taoutaou, and Lozano 2003). In particular, Lawrence (1993) uses impedance models to describe the stability and transparency of various teleoperation schemes accounting for transmission delays. Leung, Francis, and Apkarian (1995) design a controller based on μ-synthesis, while Kim, Hannaford, and Bejczy (1992) propose and investigate shared compliant control. Eusebi and Melchiorri (1998) further introduce criteria for stability, both independent of and dependent on time delay.

Stability independent of the transmission delay can be achieved by scattering approaches, which naturally preserve passivity (Anderson and Spong 1989; Stramigioli et al. 2002). A reformulation of these ideas led to the introduction of wave variables (Niemeyer and Slotine 1991, 1997a), which provide a framework for designing and analyzing force-reflecting teleoperators. Transmission of wave variables also leads to efficient implementations and ensures stability without knowledge of the time delay. Prediction, generally in the form of Smith predictors (Smith 1957), can be combined with wave-based systems to reduce the effects of the delay (Arioui, Kheddar, and Mammar 2002; Ganjefar, Momeni, and Janabi-Sharifi 2002).

Considerable attention has also been devoted to Internet-based teleoperation, in which the communications delay is variable (Fiorini and Oboe 1997; Oboe and Fiorini 1998). For a supervisory control architecture, a time-forward observer and appropriate delay model are developed by Brady and Tarn (2001). For direct force feedback, wave-variable based approaches have been used extensively (Kikuchi, Takeo, and Kosuge 1998; Yokokohji, Imaida, and Yoshikawa 1999; Yokokohji, Tsujioka, and Yoshikawa 2002), in particular guaranteeing passivity via energy-conserving filters (Niemeyer and Slotine 1998, 2001) or a suitable time-variable gain (Lozano, Chopra, and Spong 2002). They have been further extended to include estimation of the delay (Benedetti, Franchini, and Fiorini 2001), prediction of the delay (Mirfakhrai and Payandeh 2002), as well as Smith predictors in combination with energy regulation (Munir and Book 2002, 2003).

In this paper, we survey and explore the development of the wave variable concept applied to time-delayed teleoperation, under the assumption of an unknown but constant delay (Niemeyer 1996). Delays from less than the human reaction time to several seconds are considered. The dynamics of a wave-based system are analyzed in the context of wave reflections, which may interfere with normal operation and unnecessarily decrease performance. Using an impedance matched design, position feedback, and optional wave filtering, a system with consistent and predictable behavior is constructed. It becomes transparent to the user if the delay remains below the human reaction time.

The described process centers on the creation of a virtual tool. This design goal accounts for the limitations of delayed feedback and automatically reduces performance for longer delays. An on-line tuning parameter trades off the properties of the system as best suited for individual tasks.

After discussing the general goal of virtual tools and the concept of wave variables in Sections 2 and 3, we analyze the wave-based communications and basic teleoperator setup in Sections 4 and 5. In Section 6 we introduce an additional position feedback loop, while in Section 7 we examine the behavior of the entire impedance matched system. It is characterized by basic parameters that display the effects of the time delay and the on-line tuning parameter. Brief summarizing remarks are offered in Section 8.

2. Virtual Tool Design Goal

The introduction of time delays in the communications imposes fundamental limitations on the achievable performance of a telerobot, regardless of the individual technique. In particular, no command can be transmitted and acted on in less time than the one way delay T. Furthermore, the reaction to an unknown disturbance or an environment contact cannot take effect in less time than the total round trip delay $2T$. The closed-loop bandwidth is effectively limited by the same time constant.

As such, it is all the more important to present the operator with a simple and predictable system. Any unexpected behavior will lead to distractions and further complicate the situation. Instead, the operator should be free to concentrate on the task at hand and not worry about the details of the telerobots.

To account for the limitations as well as these objectives, we propose the goal of a virtual tool. This approach aims to modify the entire dynamics of the teleoperator system into a simple and well-understood form, where the dynamics include both the master and slave robots, as well as their controllers and the communications. It contrasts with the notion of telepresence, which attempts to completely hide all dynamics and is not achievable for delayed systems.

Such a virtual tool requires both inertial and compliance characteristics. The inertial property is necessary because the slave robot cannot accelerate or decelerate to execute a motion command until after the delay time T. In effect, it resists a change of its motion for at least the duration of the delay. This limitation is captured as inertia in the virtual tool.

Similarly, the master cannot display a contact force until after the delay, at which time it may no longer be at the same location as the slave. This limitation is embodied in the compliance of the virtual tool.

The magnitude of both the inertial and compliance properties is directly proportional to the time delay T. A small delay requires only low inertia and low compliance, leaving the user with a light, stiff, high fidelity tool. A larger delay forces a heavier and/or softer virtual tool, which will slow down the user consistent with the delay limitations. Note that this adjustment of the virtual tool to the delay occurs automatically if the system is based on wave variables, so performance is automatically reduced to appropriate levels.

Indeed, as we see in Section 4.3, the “natural frequency” of the wave-based communications can be defined as the square root of stiffness over inertia and equals the inverse of the delay $1/T$. This captures the limiting bandwidth of the system.

Fortunately, a tradeoff between the inertia and compliance of the virtual tool remains open for tuning. Thus, a system with large delay may be selected as light and soft for ease of motion, or as heavy and stiff for good contact resolution. Indeed the tuning parameter, the wave impedance b, is accessible on-line, so the user may adjust this characteristic to the current task (see Section 7.2).

These concepts surface throughout the following developments, as they recognize the fundamental limitations of time-delayed teleoperation. Our objective is thus centered around making the system dynamics fit the virtual tool as best as possible to create a simple behavior. For example, Section 7 introduces impedance matching and effectively adds dissipation to the virtual tool to avoid an "undamped resonance" at the natural frequency.

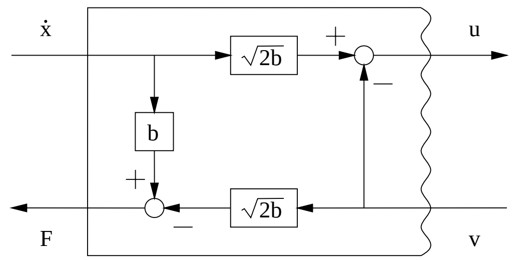


Fig. 1. The wave transformation encodes a velocity command $\dot{\mathbf{x}}$ into the forward moving wave $\mathbf{u}$ and simultaneously generates a force feedback signal $\mathbf{F}$ from the returning wave $\mathbf{v}$, coupling both operations. The wavy line signifies the wave variable port.

3. Wave Variables

Wave variables present a modification or extension to the theory of passivity which creates robustness to arbitrary time delays. They are also closely related to the scattering and small gain theories (Desoer and Vidyasagar 1975). Based only on the concepts of power and energy, they are applicable to nonlinear systems and can handle unknown models and large uncertainties. As such, they are well suited for interaction with real physical environments.

To achieve their goals, wave variables provide an alternative information encoding scheme to the standard power variables. The required algebraic transformations are simple and preserve all information. Meanwhile, the inherent symmetry avoids the classic effort/flow and admittance/impedance distinctions and duality.

3.1. Definition

A complementary pair of wave variable $(\mathbf{u}, \mathbf{v})$ is defined based on a complementary pair of standard power variables $(\dot{\mathbf{x}}, \mathbf{F})$ by the following transformation or encoding

$$\mathbf{u} = \frac{b\dot{\mathbf{x}} + \mathbf{F}}{\sqrt{2b}} \qquad \mathbf{v} = \frac{b\dot{\mathbf{x}} - \mathbf{F}}{\sqrt{2b}} \tag{1}$$

where the force $\mathbf{F}$ and velocity $\dot{\mathbf{x}}$ variables may be replaced by any other effort and flow pair. We select $\mathbf{u}$ to denote the forward or right moving wave, while $\mathbf{v}$ denotes the backward or left moving wave. The characteristic wave impedance b is a positive constant or a symmetric positive definite matrix and assumes the role of a tuning parameter, trading off speed of motion against level of forces as we will see below.

The transformation is bijective, so that it is always unique and invertible. No information is lost or gained in either representation. In particular, the power variables can also be computed via

$$\dot{\mathbf{x}} = \frac{1}{\sqrt{2b}}(\mathbf{u} + \mathbf{v}) \qquad \mathbf{F} = \sqrt{\frac{b}{2}}(\mathbf{u} - \mathbf{v}). \tag{2}$$

In practice, the wave transformations provide an interface between systems described in power and wave variables. For example, as depicted in Figure 1, encoding a velocity command $\dot{\mathbf{x}}$ into the forward moving wave $\mathbf{u}$ (transmitted from the master to the slave) uses knowledge of the returning wave $\mathbf{v}$ via

$$\mathbf{u} = -\mathbf{v} + \sqrt{2b}\,\dot{\mathbf{x}}, \tag{3}$$

but also generates a force feedback command to the master as

$$\mathbf{F} = b\dot{\mathbf{x}} - \sqrt{2b}\,\mathbf{v}. \tag{4}$$

All other combinations, sending a force command forward or sending a velocity or force command back, are quickly obtained by manipulating eq. (1).

3.2. Power Flow

The wave variable definition (1) is supported by the redefinition of power flow as

$$P = \dot{\mathbf{x}}^{\mathrm{T}}\mathbf{F} = \tfrac{1}{2}\mathbf{u}^{\mathrm{T}}\mathbf{u} - \tfrac{1}{2}\mathbf{v}^{\mathrm{T}}\mathbf{v}. \tag{5}$$

We use $\frac{1}{2}\mathbf{u}^{\mathrm{T}}\mathbf{u}$ to specify the power flowing along a main direction leading to a positive value for P. In contrast, $\frac{1}{2}\mathbf{v}^{\mathrm{T}}\mathbf{v}$ gives the power flowing against the main direction and introduces a negative element into P.

This highlights that a wave variable contains not only the information encoded by its definition, but also the power needed to execute the associated action. A velocity command carries the power to generate motion and create kinetic energy, and a force command contains the power to compress an environment and create potential energy.

In particular, notice the apparent damper of magnitude b within the wave transformation in Figure 1. When attempting to create a velocity command, the damper will retain the power necessary to compose the wave signal and then implicitly transmit the energy with the wave to the remote location. If a returning wave is present it may return energy and negate the effect of the apparent damping, allowing the operator to feel no resistance. However, until a wave response arrives, the transformation provides a local dissipative feedback path, as we will discuss in Section 5.2.

3.3. Passivity

The original motivation for introducing wave variables is their effect on the condition for passivity. In the power domain, passivity is tested by

$$\int_0^t P_{input}\,\mathrm{d}\tau = \int_0^t \dot{\mathbf{x}}^{\mathrm{T}}\mathbf{F}\,\mathrm{d}\tau \geq -E_{store}(0) \quad \forall t \geq 0 \qquad (6)$$

where $P_{input}(t)$ is the power flowing to the right (into the system) and $E_{store}(0)$ denotes the initial stored energy of that system (Slotine and Li 1991). This condition states that more energy must be introduced into a passive system than can be removed from it, subject to the initial stored energy.

In the wave domain, as depicted in Figure 2, the condition becomes

$$\int_0^t \tfrac{1}{2}\mathbf{u}_{out}^{\mathrm{T}}\mathbf{u}_{out}\,\mathrm{d}\tau \leq \int_0^t \tfrac{1}{2}\mathbf{u}_{in}^{\mathrm{T}}\mathbf{u}_{in}\,\mathrm{d}\tau + E_{store}(0) \quad \forall t \geq 0. \qquad (7)$$

Not surprisingly, a system is passive if the energy in the outgoing wave $\mathbf{u}_{out}$ is limited to the energy provided by the incoming wave $\mathbf{u}_{in}$ plus the initial stored energy.

Most importantly, a time delay becomes a passive system in the wave domain. Indeed, if

$$\mathbf{u}_{out}(t) = \mathbf{u}_{in}(t - T) \qquad (8)$$

the power in the input wave is temporarily stored for the duration of the delay:

$$E_{store}(t) = \int_{t-T}^t \tfrac{1}{2}\mathbf{u}_{in}^{\mathrm{T}}\mathbf{u}_{in}\,\mathrm{d}\tau. \qquad (9)$$

The power dissipation is zero, making the delay lossless.

Systems expressed in wave variables thus become completely robust to delays of any amount or phase lags of any level. Furthermore, a concatenation of passive wave-based elements also remains passive. Nevertheless, the addition of delays will affect the system dynamics, as we see below.

3.4. Properties and Interpretation

Unfortunately, wave variables cannot be physically measured and are often less familiar than velocity and force data. Indeed their units ($\sqrt{\text{Watt}}$) are quite unusual. Nevertheless, wave variables have a useful meaning in themselves. It has even been suggested that wave variables may occur in biological systems, particularly humans, to control motor functions (Massaquoi and Slotine 1996). To further their understanding, let us make some brief observations.

Symmetry. Wave variables are symmetric in that both outgoing and returning waves are interpreted in the same fashion. From eq. (1) they are distinguished only by a change of sign in the force $\mathbf{F}$, which determines whether they travel along or against the main power flow direction. No distinction is made between command and feedback signals.

Hybrid Encoding. This symmetry also removes the distinction between force versus velocity, effort versus flow, and admittances versus impedances. While the example in Figure 1 encodes a velocity command into the wave variable, the wave itself does not distinguish between velocity or force, and the recipient cannot detect the original form. Instead, any element may interpret an incoming wave as best suited to its current needs. So a robot in contact with the environment will use an incoming wave command to generate a force, whereas a robot in free space will use the same wave command to generate a motion.

Effectively, the same wave system may function as an impedance or an admittance as needed. This property gives wave-based systems greater flexibility in handling unknown environments by mimicking position controllers, force controllers, and general impedance controllers as needed.

Move or Push Commands. Wave commands determine the strength and direction of an action, but not its form. They are therefore best described as a “shove” or more generally as a “move or push” instruction. The sign determines the direction, as forward or backward. Also, each command contains just enough energy for its own execution; this energy may be converted to either potential or kinetic energy as needed.

Wave Impedance. The wave impedance b presents a tuning parameter which can trade off the speed of motion and levels of force. Consider a system described in the wave domain and assume a given wave variable $\mathbf{u}$. Increasing the wave impedance will place a larger weight on the velocity $\dot{\mathbf{x}}$ as compared to the force $\mathbf{F}$. So the same value of $\mathbf{u}$ leads to smaller values for $\dot{\mathbf{x}}$ and larger values for $\mathbf{F}$. This also makes the system appear more damped. In contrast, when the wave impedance is decreased, force levels are lower, motion is easier and the system appears less damped.

This tuning mechanism is automatically built into every wave system and we will use it to adjust the system behavior to the required task in Section 7.2.

3.5. Wave Responses

An alternative way to understand wave signals is to examine their responses to simple systems. To satisfy passivity, a wave response is limited in magnitude to the original wave

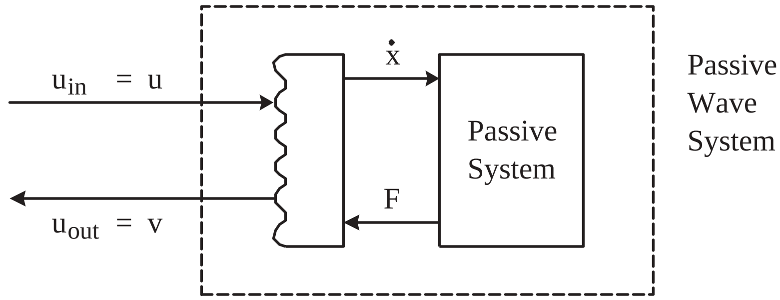


Fig. 2. A system is passive in the wave domain if the output wave $\mathbf{u}_{out}$ returns no more energy than the input wave $\mathbf{u}_{in}$ carried into a system plus any initial stored energy.

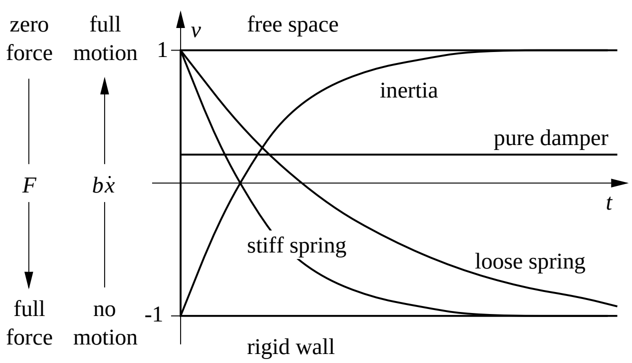


Fig. 3. Step responses of individual elements in the wave domain. The magnitude is bounded by ± 1, while the sign determines the type of action: motion or force.

command, subject to stored energy. Indeed the relative magnitude provides a measure of power losses or temporary energy storage.

The relative sign of the response determines the type of action taken. Actions dominated by motion ($|b\dot{\mathbf{x}}| > |\mathbf{F}|$) return waves of the same sign, while force dominated actions ($|\mathbf{F}| > |b\dot{\mathbf{x}}|$) return waves of opposite sign. Effectively, the wave response either "moves with" or "pushes against" the command.

Figure 3 shows the wave responses of classic elements to step wave commands. Rigid walls and free space neither dissipate nor store energy, so the return wave will be ± 1. For springs and inertias, again no power is dissipated but the storage of potential or kinetic energy temporarily reduces the step response, which exponentially converges to ± 1 in steady state. Only for dampers is power lost, and even in steady state the wave response remains low.

Examine, for example, the response of a spring in more detail. Initially, the force levels are very low so that the spring compresses easily. The action contains mostly motion and provides a positive response like free space. However, as the potential energy builds up, the spring forces slow down further motion and the response becomes negative. In steady state, the spring feels like a rigid wall simply reflecting the applied force without any movement.

For LTI systems, wave responses may also be examined in the Laplace domain:

$$H(s) = \frac{\mathbf{V}(s)}{\mathbf{U}(s)}. \tag{10}$$

In particular, we plot the wave frequency response $H(j\omega)$. Its magnitude represents the power gain at each frequency ω, which must remain below unity to satisfy passivity, so that the graph lies entirely within the unit circle of the $H(s)$ plane.

Various regions of the $H(s)$ plane correspond to different behaviors, as sketched in Figure 4. The location on the real axis describes the relative dominance of forces versus motions, positive being motion dominated and negative locations exhibiting higher force values. In the extremes, $(+1)$ defines free space with no forces and (-1) rigid interactions with no motion. On the imaginary axis, we find capacitive versus inertial behavior, corresponding to phase lead or lag. The distance to the origin signifies the dissipation; lower magnitudes show more losses, while higher magnitudes provide better efficiency, and unit magnitudes are lossless.

These data may also be obtained and plotted experimentally, providing a different perspective than classic Bode plots.

4. Wave Communications

Traditional force reflecting teleoperators transmit force and position/velocity commands between the master and slave sites. Untreated, they experience stability problems when the transmission involves even small time delays. These problems have been linked to delay-induced power generation in the communications, which violates passivity arguments (Anderson and Spong 1989, 1992; Spong 1993; Anderson 1995).

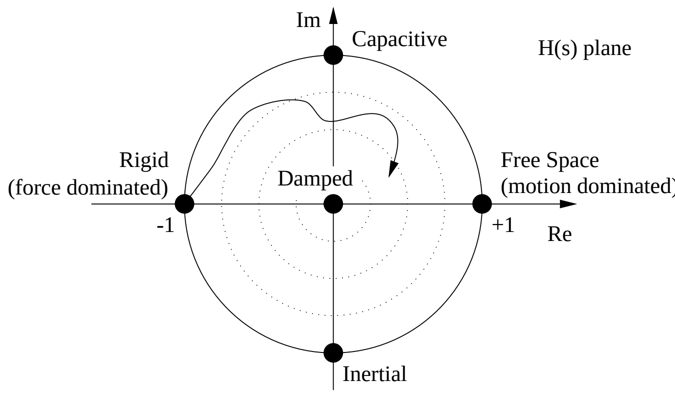


Fig. 4. The graph of the wave frequency response $H(j\omega)$ remains within the unit circle of the $H(s)$ plane. Various regions are responsible for different behaviors.

On the other hand, the passivity of wave variables is by their very construction robust to time delays. This suggests performing the communications directly in the wave domain, as shown in Figure 5. We denote the master variables at the local site with subscript "m" and the slave variables at the remote location with "s". Note that the delays in the forward and reverse path need not be the same, although the following derivations make this assumption to simplify notation. However, for this analysis to be applicable both delays should remain constant and add up to $2T$.

4.1. Definition

At both sites the velocity $\dot{\mathbf{x}}$ and force $\mathbf{F}$ are encoded into wave variables via an appropriate transformation. Only the wave variables themselves are then transmitted.

The equations governing the transmission are

$$\mathbf{u}_s(t) = \mathbf{u}_m(t-T) \tag{11}$$

$$\mathbf{v}_m(t) = \mathbf{v}_s(t-T) \tag{12}$$

while, based on the transformation, the input waves are computed as

$$\mathbf{u}_m(t) = \frac{b\dot{\mathbf{x}}_m(t) + \mathbf{F}_m(t)}{\sqrt{2b}} \tag{13}$$

$$\mathbf{v}_s(t) = \frac{b\dot{\mathbf{x}}_s(t) - \mathbf{F}_s(t)}{\sqrt{2b}}. \tag{14}$$

Notice that this does not specify or restrict whether force or velocity are considered the input or output at either side. Indeed any combination is possible and we later use this flexibility to design alternative configurations. In essence, the communications element acts as both impedance and admittance, depending on the surrounding elements. For now, we allow both possible output equations on both sides. The master thus commands either

$$\dot{\mathbf{x}}_m(t) = \sqrt{\frac{2}{b}}\,\mathbf{v}_m(t) + \frac{1}{b}\,\mathbf{F}_m(t) \tag{15}$$

or

$$\mathbf{F}_m(t) = b\dot{\mathbf{x}}_m(t) - \sqrt{2b}\,\mathbf{v}_m(t) \tag{16}$$

while the slave determines either

$$\dot{\mathbf{x}}_s(t) = \sqrt{\frac{2}{b}}\,\mathbf{u}_s(t) - \frac{1}{b}\,\mathbf{F}_s(t) \tag{17}$$

or

$$\mathbf{F}_s(t) = -b\dot{\mathbf{x}}_s(t) + \sqrt{2b}\,\mathbf{u}_s(t). \tag{18}$$

When the delay time T reduces to zero, these definitions provide a simple identity between master and slave. That is, the communications element becomes completely transparent.

Also note that while we propose to transmit only wave signals and continue to make this assumption throughout our developments, one could imagine transmitting the velocity and force information separately and executing all of the above transformations at a single site. This is a pure implementation issue which does not affect the analysis, so that we describe only the preferred method of communicating wave data.

4.2. Passivity

To verify passivity, we examine the overall power input P_{in} entering the communications:

$$P_{in} = \dot{\mathbf{x}}_m^{\mathrm{T}}\mathbf{F}_m - \dot{\mathbf{x}}_s^{\mathrm{T}}\mathbf{F}_s. \tag{19}$$

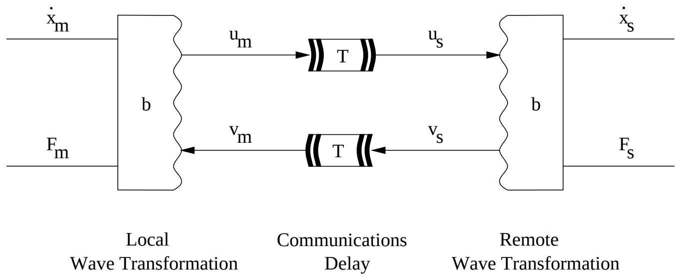


Fig. 5. The wave-based communications transform both local and remote information into wave variables before transmission to the other side.

The minus sign appears because power is considered positive while flowing in the main direction from left to right. So positive power enters the left side and exits the right side.

Substituting the wave transformation equations, we also compute this power input as

$$P_{in} = \tfrac{1}{2}\mathbf{u}_m^{\mathrm{T}}\mathbf{u}_m - \tfrac{1}{2}\mathbf{v}_m^{\mathrm{T}}\mathbf{v}_m - \tfrac{1}{2}\mathbf{u}_s^{\mathrm{T}}\mathbf{u}_s + \tfrac{1}{2}\mathbf{v}_s^{\mathrm{T}}\mathbf{v}_s \tag{20}$$

where all variables are measured at the current time t.

Substituting eqs. (11) and (12) and integrating, we find that all input power is stored according to

$$E_{store}(t) = \int_0^t P_{in}\,\mathrm{d}\tau = \int_{t-T}^t \tfrac{1}{2}\mathbf{u}_m^{\mathrm{T}}\mathbf{u}_m + \tfrac{1}{2}\mathbf{v}_s^{\mathrm{T}}\mathbf{v}_s\,\mathrm{d}\tau \geq 0 \tag{21}$$

assuming zero initial conditions. The wave energy is thus temporarily stored while the waves are in transit, making the communications not only passive, but also lossless. This is independent of the selected output equations (15)–(18) and the actual delay time T. Furthermore, it does not require knowledge of the delay nor equal delays in forward and reverse paths.

4.3. Spring and Inertial Characteristics

We have noted that the communications element may play the role of either an impedance or an admittance by outputting and commanding either a force or velocity respectively. It takes characteristics from both alternatives.

In particular, it will exhibit a spring-like deflection under constant force inputs

$$\mathbf{F}_m = \mathbf{F}_s = K_{comm}\,(\mathbf{x}_m - \mathbf{x}_s) \tag{22}$$

while maintaining momentum like an inertia in free space:

$$\dot{\mathbf{x}}_m = \dot{\mathbf{x}}_s = \frac{1}{M_{comm}}\int_0^t \mathbf{F}_m - \mathbf{F}_s\,\mathrm{d}\tau. \tag{23}$$

To determine the stiffness, we examine the deflection $\Delta\mathbf{x}(t)$. From eqs. (2), (11), and (12) we find that

$$\Delta\mathbf{x}(t) = \mathbf{x}_m(t) - \mathbf{x}_s(t) = \frac{1}{\sqrt{2b}}\int_{t-T}^t \mathbf{u}_m(\tau) - \mathbf{v}_s(\tau)\,\mathrm{d}\tau. \tag{24}$$

In steady state, if both sides have reached a constant position, we find that the forces must be equal on both sides

$$\mathbf{F}_m = \mathbf{F}_s = \sqrt{2b}\,\mathbf{u} = -\sqrt{2b}\,\mathbf{v}. \tag{25}$$

We can furthermore combine these equations and see that the forces are also proportional to the deflection with the stiffness

$$K_{comm} = \frac{b}{T}. \tag{26}$$

This is a steady-state result as the delay obviously prevents the immediate feedback of a true spring. Also notice that the value depends on both the wave impedance b and the actual delay time T. In particular, for small delays the stiffness becomes very large and the communications appear much like a rigid connection.

Should the element see no steady-state forces but be free to move in space, then it will conserve its momentum. Following similar steps we find that the equivalent inertia is given by

$$M_{comm} = bT. \tag{27}$$

Again we see the dependence on the wave impedance b and the delay T. In this case the mass becomes zero for small delays.

Together these two parameters define the “natural frequency” of the communications as

$$\omega_{comm} = \sqrt{\frac{K_{comm}}{M_{comm}}} = \frac{1}{T}. \tag{28}$$

Indeed one can imagine the wave communications as a chain of infinitesimal lumped parameter spring/masses, which add to the above parameters and define a wave speed that will transmit the wave signals to the remote site in the delay time T.

We can also use the wave impedance as a tuning mechanism to trade off the respective behaviors, selecting between a stiff and heavy configuration and a light but flexible arrangement. We use this later to control the feel of the teleoperator. Meanwhile, the system will exhibit each characteristic when it is placed in an appropriate context, showing stiffness under contact and inertia in motion.

5. Simple Wave Teleoperator

We now incorporate such a wave transmission into the teleoperator system. In particular, a wave transformation is added to both sides, so that all delays are isolated and incorporated passively.

5.1. Basic Layout

In general, both master and slave sides may have a separate controller. In particular, both sides may control either position and velocity or force. We will use this flexibility later in Section 7, but for now we assume that the slave follows a desired motion and is placed under PD control. Meanwhile, the master applies the desired force directly to the joystick. As such, this layout is closest to the traditional system with position feedforward and force feedback. A block diagram of the transfer functions for a single degree of freedom system is depicted in Figure 6.

The slave PD controller uses constant symmetric positive definite matrices for both the velocity gain B and position gain K to force the slave to track the desired velocity $\dot{\mathbf{x}}_{sd}$. The necessary force is

$$\mathbf{F}_s = -B(\dot{\mathbf{x}}_s - \dot{\mathbf{x}}_{sd}) - K(\mathbf{x}_s - \mathbf{x}_{sd}). \tag{29}$$

The desired velocity is decoded from the wave transformation as

$$\dot{\mathbf{x}}_{sd} = \frac{\sqrt{2b}\,\mathbf{u}_s - \mathbf{F}_s}{b} \tag{30}$$

and the return wave is computed by

$$\mathbf{v}_s = \frac{b\dot{\mathbf{x}}_{sd} - \mathbf{F}_s}{\sqrt{2b}} = \mathbf{u}_s - \sqrt{\frac{2}{b}}\,\mathbf{F}_s. \tag{31}$$

Notice that the combination of PD controller (29) and wave transformation (30) creates an algebraic loop. The slave force depends on the desired velocity, which in turn depends on the current force. This loop is quickly solved by writing

$$\dot{\mathbf{x}}_{sd} = \frac{\sqrt{2b}\,\mathbf{u}_s + B\dot{\mathbf{x}}_s + K(\mathbf{x}_s - \mathbf{x}_{sd})}{B + b}. \tag{32}$$

Also, the desired position must be computed from this velocity via

$$\mathbf{x}_{sd} = \int_0^t \dot{\mathbf{x}}_{sd}(\tau)\,\mathrm{d}\tau. \tag{33}$$

In this basic form, numerical integration errors may cause a slow drift between the actual master and slave positions. This issue arises because the wave variable transmission does not explicitly encode position information. It is addressed in detail and fixed in Section 6.

On the master side, the desired force is applied to the joystick without additional changes. The value is determined by the local transformation via eq. (16) as

$$\mathbf{F}_m = b\dot{\mathbf{x}}_m - \sqrt{2b}\,\mathbf{v}_m. \tag{34}$$

Finally, the right moving wave is computed from

$$\mathbf{u}_m = \frac{b\dot{\mathbf{x}}_m + \mathbf{F}_m}{\sqrt{2b}} = \sqrt{2b}\,\dot{\mathbf{x}}_m - \mathbf{v}_m. \tag{35}$$

We should also point out that all the elements are passive and so this system is stable, regardless of the delay T. Indeed the value need not even be known, as it is not used anywhere in the controller. For small delays, the system reverts to a simple PD connection between master and slave robots.

5.2. Feedback Paths

This simple teleoperator layout contains multiple internal loops created by the wave transformations. Remember that each wave transformation contains an apparent damping element. So we find three distinct paths that may carry signals back to the joystick and the operator (Figure 6).

First, for every motion of the master manipulator, there is an immediate feedback in the form of damping created in the wave transformation. This energy is not dissipated, but rather used to construct the wave signal and transmitted to the remote location. Nevertheless, this feedback appears to the operator like a simple damper. Notice it is clearly visible in eq. (34). Also, it is the only feedback that does not travel through the delay.

The second path contains and is based on wave reflections at both transformations. When the right moving wave $\mathbf{u}_s$ reaches its destination, part of the signal may return with the left moving wave $\mathbf{v}_s$ back towards the operator, as we see in eq. (31). When the returning wave $\mathbf{v}_m$ arrives at the local transformation, it may again be reflected into the forward path as $\mathbf{u}_m$, as governed by eq. (35). Not only do wave reflections contain little useful information, but the cyclic layout may allow them to last for several cycles before dying out. They easily create unexpected disturbances and distractions and can cause substantial performance problems. The experimental results in Figure 7 display repeated wave reflections lasting five complete cycles before dying out.

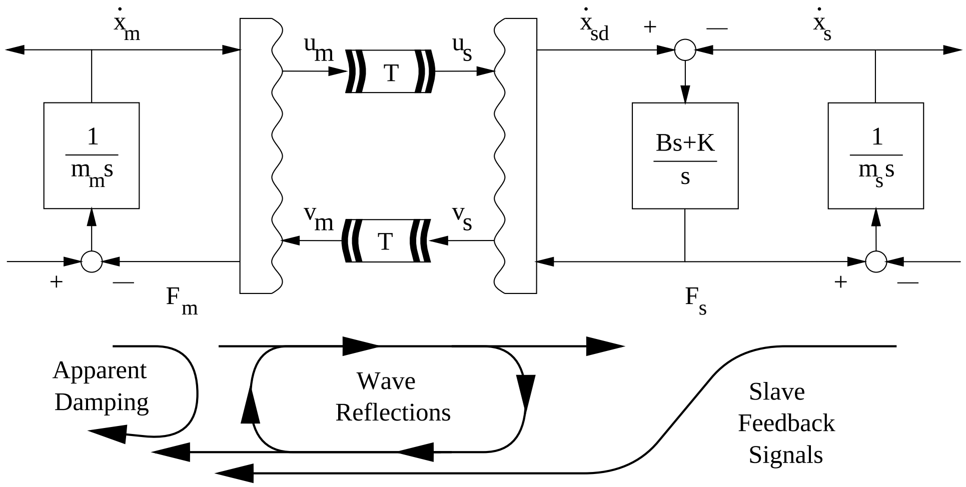


Fig. 6. The basic wave teleoperator places the slave robot under PD control and applies forces directly to the master. The wave transformations both encode information for transmission and decode the incoming waves to compute the desired values. The resulting structure provides three separate paths through which signals may travel to the operator.

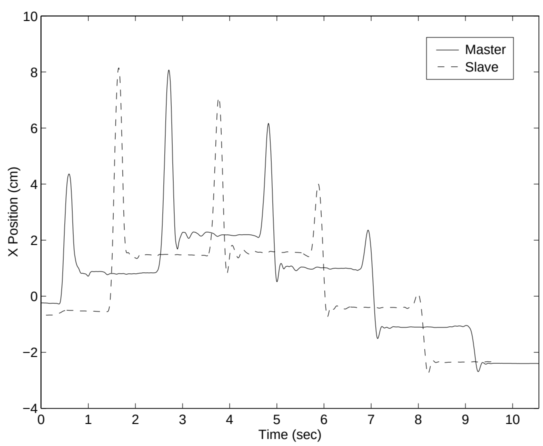


Fig. 7. This experiment is initiated by a brief input or disturbance, after which both master and slave robots are untouched. The graph of master and slave position shows clear evidence of wave reflections that appear at multiples of the delay time of $T = 1.0$.

Based on the discussion of Section 4.3, the wave reflections can also be interpreted as a resonance of the wave communications at its “natural frequency”. The communication itself is furthermore lossless, so the oscillations continue until the energy is dissipated by either the master or slave subsystems.

The third and final path moves signals from the remote robot via the PD controller back to the operator. This is the main feedback path and provides the information needed to complete the tasks. Whether motion occurred or contact was made is described by the slave position and velocity and appears in the tracking error. The slave force encodes this information and provides a close representation of the actual contact forces. Its feedback to the operator is the main goal of the system.

Of the three feedback components, the last contains the important data and must be protected. The first creates appropriate damping and is easily tolerable if not even beneficial, but the second, being the wave reflections, can cause significant problems and overshadow the others. It is this feedback path that we need to eliminate, for example by impedance matching the controllers, as demonstrated in Section 7. Alternatively, we may wish to add wave filters to reduce the effect of the reflections, as described in Section 7.4.

6. Position Feedback

We have noted that the only data being transmitted between the two locations consist of wave signals that implicitly contain velocity and force information. So far we have not seen any direct position feedback nor an explicit guarantee of positional convergence between master and slave. Position information only appeared via integration of the velocity command in eq. (33).

This is consistent with the passivity approach, which cannot guarantee the power necessary to move either robot to any position. Imagine moving the master to a new location differing from the slave location and holding it there firmly. Also imagine the slave in contact and prevented from moving. Forcing the position of the master back to its original position and hence back to the slave position may require more energy than is available. Thus the system cannot guarantee such convergence in all cases.

The lack of position feedback is also consistent with the notion that a wave-based system will change roles to adapt to the current task. If contact is made, the system will act as a force controller which generally does not consider position convergence.

In practice, most applications require position feedback and seek a guarantee of tracking between master and slave that is robust to numerical errors and based on actual position measurements. Two solutions to this problem have been developed. First, we may transmit the wave integrals in addition to the wave signals. These integrals contain position information and can be constructed directly from position measurements. Similarly, they can be decoded to provide position commands.

The second solution adds a corrective term to the actual wave command in response to any observed drift between the two robots. This strategy establishes absolute position feedback and handles any initial position offsets as well. It generates smooth behavior within the limitations imposed by the passivity requirements.

6.1. Theoretical Position Tracking

The wave-based teleoperator layout proposed in the previous section will force the master and slave robot velocities to track each other. In theory, this also forces the robot positions to track each other, reducing the steady-state position error to zero. However, this relies on the numerical integration of the desired slave velocity into a desired position (33), so that in practice the master and slave may drift apart.

First consider ideal conditions without numerical errors. The master and the desired slave positions can be computed as

$$\mathbf{x}_m(t) = \frac{1}{\sqrt{2b}} \int_0^t \mathbf{u}_m(\tau) + \mathbf{v}_m(\tau)\,\mathrm{d}\tau \tag{36}$$

$$\mathbf{x}_{sd}(t) = \frac{1}{\sqrt{2b}} \int_0^t \mathbf{u}_s(\tau) + \mathbf{v}_s(\tau)\,\mathrm{d}\tau. \tag{37}$$

Their difference forms the position error across the wave communications. Substituting the transmission equations (11) and (12) we find

$$\Delta\mathbf{x}_{comm}(t) = \frac{1}{\sqrt{2b}} \int_{t-T}^t \mathbf{u}_m(\tau) - \mathbf{v}_s(\tau)\,\mathrm{d}\tau. \tag{38}$$

In steady state, when the wave signals have decayed to zero without any velocity or force inputs, this position error is zero. Hence the desired slave position exactly equals the master location. Without force inputs, the PD controller will guarantee that the actual slave position also converges there.

Unfortunately, this argument is susceptible to errors because it assumes that the transmission and integration are perfect, for example

$$\int_0^{t-T} \mathbf{u}_m(\tau)\,\mathrm{d}\tau \equiv \int_0^t \mathbf{u}_s(\tau)\,\mathrm{d}\tau. \tag{39}$$

Moreover, the resulting algorithm must actually compute the desired slave position by integrating a velocity signal via eq. (33) or eq. (37). Also any initial position offsets will remain undetected and hence unchanged.

In practice, the desired position may drift from its theoretical value and the system may see slow drift between the two robots for several reasons.

- **Discrete sampling.** The algorithms are usually implemented on digital computers with finite sampling rates. Integration is necessarily replaced by a finite summation.
- **Numerical errors.** The data are also represented with finite precision, so that the continual summation may accumulate the effect of many roundoff errors.
- **Data loss.** Should the transmission ever lose any data, the integration will always be shifted by the corresponding amount.

However, we should also point out that given today's digital computers, these errors are typically very small and do not accumulate to noticeable levels until after very long periods of operation.

6.2. Transmitting the Wave Integrals

Just as the wave signals encode velocity and force, their integrals encode position and momentum information. Computing and transmitting these values can provide explicit position information and prevent the above-mentioned problems.

The integrated wave variables are defined as

$$\mathbf{U}(t) = \int_0^t \mathbf{u}\,\mathrm{d}\tau = \frac{b\mathbf{x} + \mathbf{p}}{\sqrt{2b}} \tag{40}$$

$$\mathbf{V}(t) = \int_0^t \mathbf{v}\,\mathrm{d}\tau = \frac{b\mathbf{x} - \mathbf{p}}{\sqrt{2b}} \tag{41}$$

and encode position $\mathbf{x}$ and momentum $\mathbf{p}$, which is the integral of force

$$\mathbf{p} = \int_0^t \mathbf{F}\,\mathrm{d}\tau. \tag{42}$$

The position information can be measured directly without problems. However, the momentum data are not as easy to obtain and need to be integrated numerically. In essence, we have shifted the problem from integrating velocity into position to integrating force into momentum. However, while the same errors may still appear, we place little or no importance on the actual momentum value. Velocity signals are still contained in the original wave variables and do not depend on this new quantity. Fortunately, we may actually circumvent the long-term integration, as noted below.

The communications should transmit both wave and integrated wave signals. To use the added information, the system should be extended as follows. On the slave side, the force is integrated into momentum

$$\mathbf{p}_s = \int_0^t \mathbf{F}_s \,\mathrm{d}\tau, \tag{43}$$

after which the incoming wave integral and momentum determine the desired position

$$\mathbf{x}_{sd} = \frac{\sqrt{2b}\,\mathbf{U}_s - \mathbf{p}_s}{b}, \tag{44}$$

and finally the desired position and momentum specify the returning wave integral

$$\mathbf{V}_s = \frac{b\mathbf{x}_{sd} - \mathbf{p}_s}{\sqrt{2b}}. \tag{45}$$

On the master side, the command wave integral is computed directly from the actual position measurements and the incoming wave integral

$$\mathbf{U}_m = \sqrt{2b}\,\mathbf{x}_m - \mathbf{V}_m. \tag{46}$$

A closer examination shows that this extension actually implements the following behavior

$$\mathbf{x}_{sd}(t) = 2\mathbf{x}_m(t-T) - \mathbf{x}_{sd}(t-2T) - \frac{1}{b}\int_{t-2T}^{t} \mathbf{F}_s(\tau)\,\mathrm{d}\tau, \tag{47}$$

which we can also compute directly. In this alternative form, the position information is transmitted directly and we need not use the momentum. However, we do need to integrate the force $\mathbf{F}_s$ over a finite period of time.

Using either form, the desired slave position is now based on the master position, preventing any drift. Also notice that these developments do not affect passivity. Indeed they only explicitly guarantee what theoretically should be true already. As such, they do not interfere with any other elements of the system and are generally ignored during the following sections. If needed, this type of position feedback via the wave integrals can be added to any wave system.

6.3. Single Channel Transmissions

At first glance, transmitting the wave integral, as proposed above, would seem to require additional channels or bandwidth in the communications. In fact this is not the case, as we can combine a wave signal with its integral into a single quantity. Then, after reception on the other side, the values can be recovered.

This coding/decoding process is depicted in Figure 8 for the right moving wave. It is accurate and numerically stable. In particular, add both values before transmission,

$$\overline{\mathbf{U}}_m = \mathbf{U}_m + \frac{1}{\lambda}\mathbf{u}_m \tag{48}$$

send

$$\overline{\mathbf{U}}_s(t) = \overline{\mathbf{U}}_m(t-T) \tag{49}$$

and separate via a stable first-order filter

$$\mathbf{u}_s = \lambda(\overline{\mathbf{U}}_s - \mathbf{U}_s) \qquad \mathbf{U}_s = \int_0^t \mathbf{u}_s\,\mathrm{d}\tau, \tag{50}$$

where λ is the positive constant bandwidth of the filter.

This makes use of the fact that the two values are closely related via integration or differentiation. The choice of the bandwidth λ is arbitrary and may be used to scale the average magnitudes of $\mathbf{U}$ and $\mathbf{u}$. It should, however, always be significantly less than the sampling rate of the digital implementation.

In effect, the combination $\overline{\mathbf{U}}$ encodes all information required to operate the system—position, velocity, and force—into a single value, and the decoding and wave transformations extract the information as needed.

6.4. Adjusting the Wave Command

The second method for providing absolute position feedback observes the drift error between the two sides of the wave communications and adds a corrective term to the wave command. It can also handle initial position offsets and temporary transmission losses, generating smooth position tracking without sudden step inputs.

Using the notation of Figure 9, we define the actual wave command $\mathbf{u}_m$ based on the uncorrected value $\hat{\mathbf{u}}_m$, which is computed as before by eq. (35), and the corrective term $\Delta\mathbf{u}_m$ as

$$\mathbf{u}_m = \hat{\mathbf{u}}_m + \Delta\mathbf{u}_m. \tag{51}$$

Also we define the actual position difference between the two sides of the communications to be computed on the master side, hence using the delayed slave desired position, as

$$\Delta\mathbf{x}_{actual}(t) = \mathbf{x}_m(t) - \mathbf{x}_{sd}(t-T). \tag{52}$$

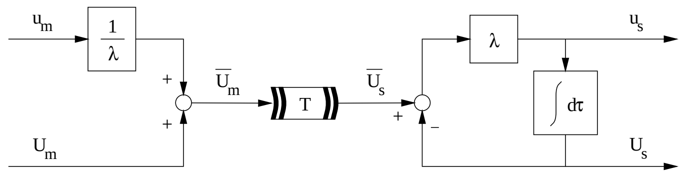


Fig. 8. Transmitting the sum of both wave and integrated wave signals allows all information to flow through a single channel. The receiving side can decompose the signal with a numerically stable first order filter.

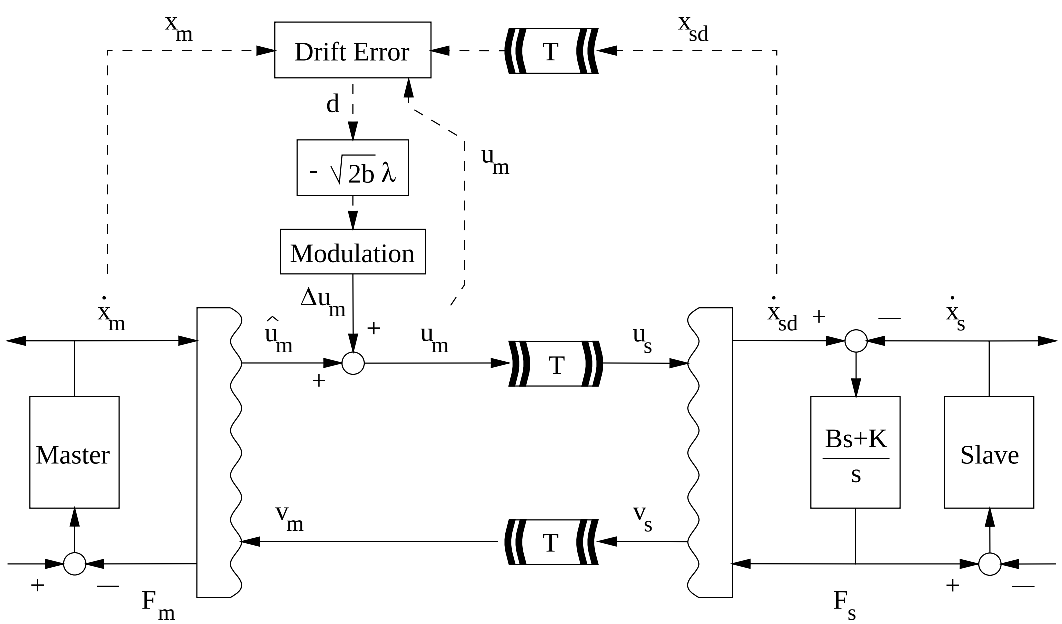


Fig. 9. By adjusting the wave command, the system can provide absolute position feedback and guarantee tracking even with numerical errors and occasional data loss. In addition, initial position differences are slowly removed without sudden changes.

The spring-like characteristics of the transmission may produce a deflection between the two sides, so this position difference need not remain at zero. Instead, we can predict the value for the difference, substituting the wave definitions and transmission equations, and assuming perfect initial conditions and no wave corrections, as

$$\Delta \mathbf{x}_{predicted}(t) = \frac{1}{\sqrt{2b}} \int_{t-2T}^{t} \mathbf{u}_m(\tau)\, d\tau. \qquad (53)$$

The position difference should thus reach zero in steady state when no further forces are applied, i.e., when the wave commands reach zero.

This leads us to the drift error $\mathbf{d}(t)$

$$\mathbf{d}(t) = \Delta \mathbf{x}_{predicted}(t) - \Delta \mathbf{x}_{actual}(t), \qquad (54)$$

which should remain at zero. Finally, the derivative of the drift error $\mathbf{d}(t)$ is related to the corrective wave term $\Delta \mathbf{u}_m$ via

$$\dot{\mathbf{d}}(t) = \frac{1}{\sqrt{2b}}\, \Delta \mathbf{u}_m(t). \qquad (55)$$

We adjust the wave command to change and reduce any existing drift errors, using

$$\Delta \mathbf{u}_m = -\sqrt{2b}\,\lambda\, \mathbf{d}, \qquad (56)$$

where λ is the bandwidth of this new feedback path. Any other value will also work as long as $\Delta \mathbf{u}_m$ and $\mathbf{d}$ are of opposite sign.

Unfortunately, this added feedback path does not contain any power source or any power flow. So as not to disturb the passivity criterion, the corrective term may not introduce any power into the system. Hence, the corrected wave command must be bounded by the uncorrected (original) version

$$\tfrac{1}{2}\mathbf{u}_m^{\mathrm{T}}\mathbf{u}_m \leq \tfrac{1}{2}\hat{\mathbf{u}}_m^{\mathrm{T}}\hat{\mathbf{u}}_m. \qquad (57)$$

In addition, we do not want the final system to confuse the operator, so we require that both corrected and uncorrected wave commands are of the same sign (for each degree of freedom).

We satisfy these requirements by modulating the magnitude of the desired correction (56) in each degree of freedom according to

$$\Delta \mathbf{u}_m = \begin{cases} 0 & \text{if} \quad d\hat{u}_m < 0 \\ -\sqrt{2b}\,\lambda\, d & \text{if} \quad d\hat{u}_m > 0 \quad \text{and} \quad \sqrt{2b}\,\lambda\,|d| < |\hat{u}_m| \\ -\hat{u}_m & \text{if} \quad d\hat{u}_m > 0 \quad \text{and} \quad \sqrt{2b}\,\lambda\,|d| > |\hat{u}_m| \end{cases} \tag{58}$$

and are thereby assured of passivity and hence stability. The corrective term $\Delta \mathbf{u}_m$ and the drift error $\mathbf{d}$ are never of the same sign, so that the drift error will never increase. Instead, it will decrease once the system detects appropriate wave commands and associated power. This "waiting" behavior is a side effect of passivity, where the system cannot force tracking until it receives some power input from the operator.

Figure 9 displays the procedure. The added feedback path is depicted by dashed lines, because it does not contain wave variables or any power flow. The new bandwidth λ may be quite small as the drift errors should be minimal. It is best chosen to remove initial position differences in an acceptable time frame.

In effect we have added a second, higher-level feedback, operating on a slower time-scale and using only "leftover power" unneeded by the primary loop. Its only purpose is to slowly compensate for tracking errors that build up over time or are unobserved by the inner feedback loop (e.g., initial conditions or loss of the wave signal). This may even be compared to supervisory control behavior.

Finally, note that this type of position feedback may also be added to other wave-based systems with an appropriate redefinition of the expected position difference and the drift error.

7. Impedance Matched System

In Section 5 we determined that wave reflections may occur and should be avoided if possible, as they distract the operator and can lead to oscillatory behavior. Such behavior has been compared to an underdamped resonance of the wave communications at its natural frequency. Wave reflections appear when a wave signal hits an element with an impedance unequal to its own wave impedance b. So, to reduce reflections, we try to match the impedance for both master and slave subsystems. This is analogous to using master and slave energy dissipation as damping for the communications resonance.

In the following we discuss such efforts, describe the resulting symmetric teleoperator system and its contact behavior, and finally introduce wave filters. These filters provide an alternative or additional method for reducing reflections and adjusting wave dynamics.

7.1. Impedance Matching Design

Impedance matching can be decomposed into two steps. In the first step, both the master and slave subsystems are configured and tuned independently. Each appears as a pure damper to the wave commands, although their impedance value remains unspecified. This value is selected and adjusted in step two, detailed below, according to the desired task.

In general, impedance matching requires making the dynamics of each subsystem appear as a simple damper to the wave commands. This allows dissipation of all incoming energy associated with the wave commands, and without this energy, no reflections can occur. Only initial conditions, external inputs, and disturbances will create a return wave, providing the user with clean and appropriate information.

The process includes tuning specific to the subsystem and/or hardware, as well as for the expected operating condition. It often necessitates additional control elements. In our case, both master and slave robots operate mostly in free space and present inertial behavior to the system. So, in an uncontrolled situation, the transfer function between velocity and force and thus their apparent impedance is of the form

$$\frac{\mathcal{L}(F)}{\mathcal{L}(\dot{x})} = ms, \tag{59}$$

where $\mathcal{L}(.)$ denotes the Laplace transform. This transfer function is complex valued and cannot be directly matched to the real valued impedance of a damper. Counterbalancing the phase shift in the transfer function requires the use of a capacitive or spring element and hence a PD-type controller. Force control, which the simple teleoperator in Section 5 uses at the master side, is not appropriate. This motivates a symmetric configuration with impedance controllers at both sites. Figure 10 shows the block diagram of transfer functions, depicted for a single degree of freedom.

More specifically, both impedance controllers accept a velocity command from the incoming waves and return forces. The transfer function between the velocity command $\dot{x}_d$ and the controller's feedback force F_c is

$$\frac{\mathcal{L}(F_c)}{\mathcal{L}(\dot{x}_d)} = R + \frac{(ms + D)(Bs + K)}{ms^2 + Ds + Bs + K}. \tag{60}$$

Tuning the impedance controller according to

$$D = \lambda m \qquad B = \lambda m \qquad K = \lambda^2 m \qquad R = b - \lambda m \tag{61}$$

simplifies the dynamics to

$$\frac{\mathcal{L}(F_c)}{\mathcal{L}(\dot{x}_d)} = R + \lambda m = b. \tag{62}$$

Indeed, this is nothing other than a critically damped PD controller, which causes the robot velocity to track the command in a first-order fashion

$$\frac{\mathcal{L}(\dot{x})}{\mathcal{L}(\dot{x}_d)} = \frac{(Bs + K)}{ms^2 + Ds + Bs + K} = \frac{\lambda}{s + \lambda}. \tag{63}$$

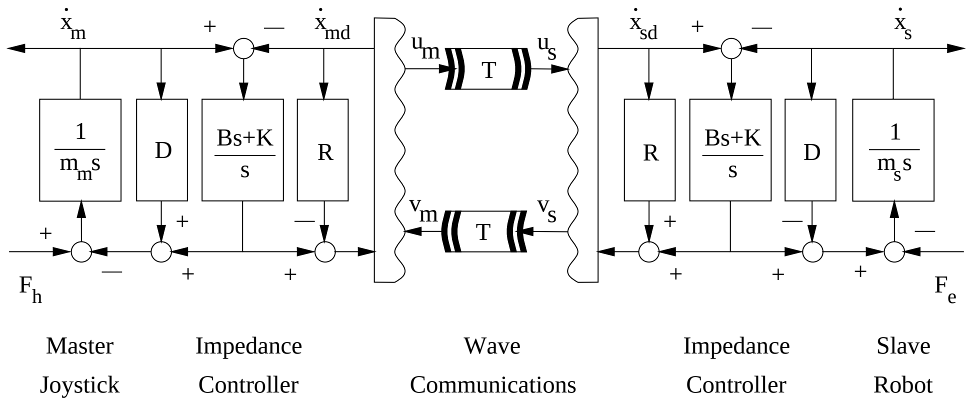


Fig. 10. The symmetric impedance matched teleoperator uses PD control on both sides to eliminate wave reflections.

The bandwidth λ remains independently tunable to best match the amplifiers, unmodeled dynamics or other characteristics of each robot. Other controllers that produce equally simple closed-loop dynamics may also be used. They may even be necessary if the robots exhibit significant nonlinear dynamics or friction. Essential to impedance matching is only that the feedback term from the controller to the wave transformation provides a simple, real valued impedance in normal operating conditions, and that this feedback also encodes the information necessary to operate if conditions change.

Returning to our case, either the master or slave robot may encounter a disturbance inconsistent with motion in free space, arising from contact with the human operator or by touching the remote environment. The disturbance force $F_{disturbance}$ is passed to the wave transformation via the controller's feedback force F_c as

$$\frac{\mathcal{L}(F_c)}{\mathcal{L}(F_{disturbance})} = \frac{\lambda}{s+\lambda}. \tag{64}$$

Notice the similarity between eqs. (63) and (64). This is a side effect of the use of passive strategies, where both signals pass through the same controller elements and are shaped in the same fashion.

7.2. Wave Impedance Tuning and Virtual Tool Interpretation

Having designed and tuned each subsystem independently to appear to the wave transformations as a simple damper, the wave impedance can now be selected based on the desired task.

First, briefly review the symmetric configuration of Figure 10. The wave communications element now acts as a true admittance: it connects the master and slave impedances, observes forces on both sides and updates the desired motions as needed. It is the stiffness of the communications, as derived in Section 4.3, together with the impedance controller stiffness, that determines the final force-feedback behavior.

Also note the introduction of two dissipation elements in both impedance controllers: R and D. The damping D is used to create a critically damped response, necessary for the dynamic simplification. The damping R is added to match the plain controller impedance to the wave impedance and help dissipate the communications resonance if needed.

Together the entire system can be related to the virtual tool via the mechanical parameters of total inertia, damping, and steady-state stiffness (Niemeyer and Slotine 1997a):

$$M = M_m + bT + M_s \tag{65}$$

$$B = 2b \tag{66}$$

$$K^{-1} = K_m^{-1} + \left(\frac{b}{T}\right)^{-1} + K_s^{-1}. \tag{67}$$

This perspective clearly shows how the time delay T deteriorates both the inertial and stiffness properties, while the wave impedance b trades off the two aspects.

To tune the wave impedance, we must examine the desired task. For free space motions with no expected contact or other disturbance forces, reduce b as far as possible to minimize damping and inertial effects and allow quick moves. During this task no wave responses or reflections will appear, and the user can work freely, as though there was no force feedback at all.

When we expect contact with the remote environment, we increase b. Higher values of b provide more resistance to motion, which will prevent the manipulator from creating high impact forces. Also the increased steady-state stiffness ensures that contact forces are fed back to the master in close proximity to the actual contact location, providing good spatial resolution.

In essence, the wave impedance assumes the role of a crude predictor of the task and environment, allowing the system to tune itself to respond in the best possible fashion. The only immediate feedback to the user (path 1 in Figure 6) is via b in the wave transformation, so that b is literally a zero-order approximation and prediction of the environment, distinguishing free and constrained motion. The user can adjust this prediction efficiently by anticipating upcoming changes in the interaction, but expanded wave predictors could also be used to automate the process.

Finally notice the lower limit for the wave impedance b from eq. (61)

$$b \geq B = D = \lambda m. \tag{68}$$

To ensure impedance matching, the wave impedance must always be greater than or equal to the master or slave dissipation, because a passive controller can only add damping. If the lower limit on b is too high, we have to retune the master/slave controllers to a lower bandwidth with lower dissipation requirements.

7.3. Contact Behavior

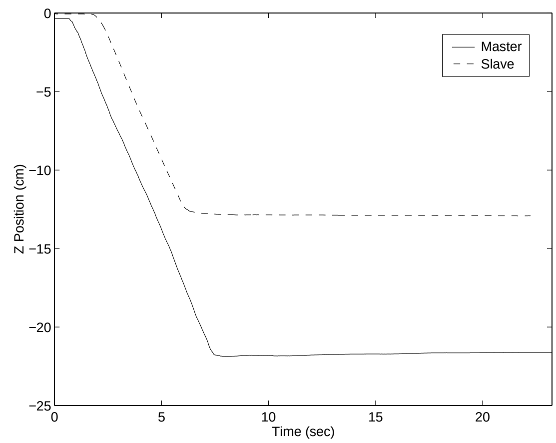
The impedance matching process was performed under the assumption that both robots move as pure inertias in free space, eliminating all returning waves. When contact is made, this model is no longer valid, and the returning wave signals carry the contact information.

To study the extreme case, consider contact with a rigid environment. We find the returning wave v_s governed by

$$\frac{\mathcal{L}(v_s)}{\mathcal{L}(u_s)} = -\frac{K_s}{2bs + K_s}, \tag{69}$$

which is a low-pass filter of bandwidth $K_s/2b$. Increasing the wave impedance will therefore lower the filter bandwidth and prevent the reflections from becoming oscillatory and disorienting the operator. This observation is consistent with the above (on-line) tuning process, which calls for a high wave impedance for good force clarity during contact operations.

Hence the impedance matched teleoperator, together with appropriate on-line tuning of the wave impedance, can achieve good performance both in free space and in contact. For example, see Figure 11 which shows a force applied to the environment after the slave manipulator reaches a rigid contact. The wave impedance was tuned high in this test and the minor reflections disappear quickly.



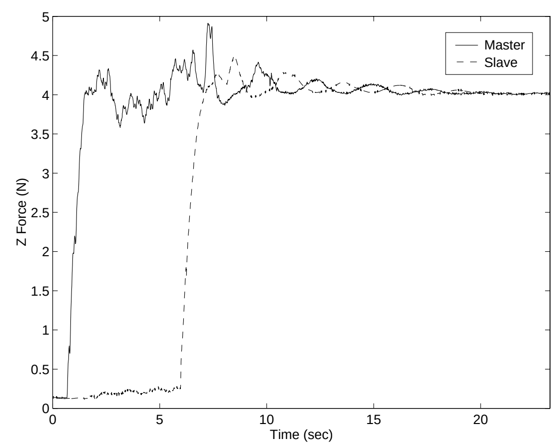


Fig. 11. A contact force may be applied and felt by the operator even for a delay of $T = 1.0$ s. The upper graph of master and slave positions shows initial, delayed tracking and a steady-state deflection after the slave has made contact. The lower graph of master and slave forces demonstrates that the force applied to the master is transferred to the slave contact force after impact.

7.4. Wave Filtering

Finally, if the operator behavior or environment contacts vary widely, or if the impedance matching process cannot be completed or fails for other reasons, wave reflections can also be reduced with wave filters. Wave filters smooth the system behavior regardless of operating conditions or task. However, they also reduce tracking performance and should therefore be used sparingly, and only if reflections become disruptive.

Inserting a filter into the wave transmission path between master and slave site does not affect passivity. Indeed the wave variables are constructed to be unaffected by delays or phase lag. The only requirement on the filters is that their gain must be limited below unity, so, for example, linear filters may not be underdamped. Filtering creates a smoother behavior by eliminating the high-frequency components often seen in the wave reflections. Generally the filter constants should be chosen such that the bandwidth is close to the actual time delay which dominates the closed-loop behavior.

For example, using first-order linear filters would update the wave communications of eqs. (11) and (12) to

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{u}_s(t) + \lambda \mathbf{u}_s(t) = \lambda \mathbf{u}_m(t - T) \tag{70}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{v}_m(t) + \lambda \mathbf{v}_m(t) = \lambda \mathbf{v}_s(t - T), \tag{71}$$

where λ might be set to $1/T$.

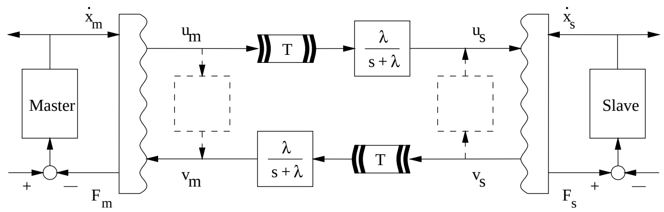


Fig. 12. Adding wave filters and designing in the wave domain may ultimately remove the need for explicit master and slave PD controllers.

However, unlike impedance matching, the filters also reduce some of the feedback signals we want the operator to observe, because they cannot and do not distinguish between reflections and important wave signals. As such they may also reduce performance, as demonstrated with the system parameters

$$M = M_m + bT + \frac{b}{\lambda} + M_s \tag{72}$$

$$B = 2b \tag{73}$$

$$K^{-1} = K_m^{-1} + \left(\frac{b}{T}\right)^{-1} + (b\lambda)^{-1} + K_s^{-1}. \tag{74}$$

Wave filtering increases inertia and reduces stiffness, much like the pure delay.

This approach is analogous to directly damping the wave communications resonance, instead of relying on the dissipation in the master/slave subsystems to absorb wave reflections and oscillations. The communications system removes some of the wave energy as it is transmitted and is no longer lossless.

To salvage the high-frequency data lost by filtering, the unfiltered wave may still be presented to the operator in another form. For example, placing the signal on an auditory or tactile channel will allow the operator to hear or feel important high-frequency events such as impact or stick/slip.

Finally, notice that we have hereby begun to add controllers directly in the wave domain. Ultimately, as depicted in Figure 12, this strategy could eliminate the need for explicit master/slave PD controllers and the entire system could be constructed in wave space (Niemeyer and Slotine 1997b).

8. Conclusions

The use of wave variables in the data transmission of a teleoperator, as well as for analysis and design, leads to a system that operates predictably and reliably even for large delays. This architecture demonstrates many benefits, as follows.

- The use of wave variables provides robustness to delays of any magnitude. For zero delay, the system automatically reverts to a classic teleoperator configuration. For small delays, the system remains transparent and stable; local controllers can be designed without change.
- For larger delays, appropriate design procedures, including impedance matching and possibly wave filtering, follow the design goal of a virtual tool. Under this paradigm the wave variable approach inherently adjusts system performance to account for delay-induced limitations.
- The wave impedance or impedance matrix provides an on-line tuning mechanism to trade off the system parameters of reflected inertia and stiffness. This choice selects between the extrema of a light but flexible setup and a stiff yet heavy configuration. The wave impedance can and should be adjusted by the user to enable either fast motion or sensitive force feedback as best suited to the current task.

  Equivalently, the wave impedance encodes a crude approximation of the expected remote environment. Having the user adjust this parameter based on the actual observed environment allows the system to behave most appropriately. This capability also suggests future extensions with more sophisticated wave-based predictors.
- Wave variables implement a type of symmetric and hybrid encoding, which removes the need for distinct position or force control algorithms and unifies admittance and impedance elements. This flexibility makes them well suited for implementation and application to various tasks.
- When utilizing the wave integrals, the system can combine all relevant information—position, velocity, and force—into a single quantity. This dense coding can reduce transmission requirements.
- A supervisory-type higher-level feedback loop may be added to overcome initial conditions, temporary transmission losses, and numerical drift which are uncompensated by the primary loop. Operating on a slower

time-scale and using only leftover power, this feedback ensures position tracking without interfering with the basic design.

- Impedance matching allows for distinct and optimal tuning of local and remote controllers to their respective robots and anticipated operating configurations, minimizing wave reflections. Optional wave filters provide additional means of shaping wave responses. The wave impedance remains tunable to the desired task.

- Wave variables also provide a framework which we have been able to extend to fluctuating or variable communication delays, as they appear in Internet telemanipulation. This brings the promise of interaction with remote environments or other users to a much larger audience.

- Wave variables lack a direct physical manifestation and are less familiar than position and force signals. Nevertheless, wave variables mimic natural phenomena and provide important properties that may be useful to other analysis and controls applications as well. Early results indicate that designing a control system entirely in the wave domain may lead to new insights and strategies.

Using this philosophy we have been able to establish and operate a telerobot with a constant round trip delay of 2 s, as well as over the Internet with delays varying from 50 ms to 1 s. We hope that these developments will extend the use of force reflecting teleoperation.

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