Basics on Physical Modelling

Summer School on

Port-Hamiltonian modelling and passivity-based control of physical systems. Theory and applications

CeUB, Bertinoro, July 6-8, 2017

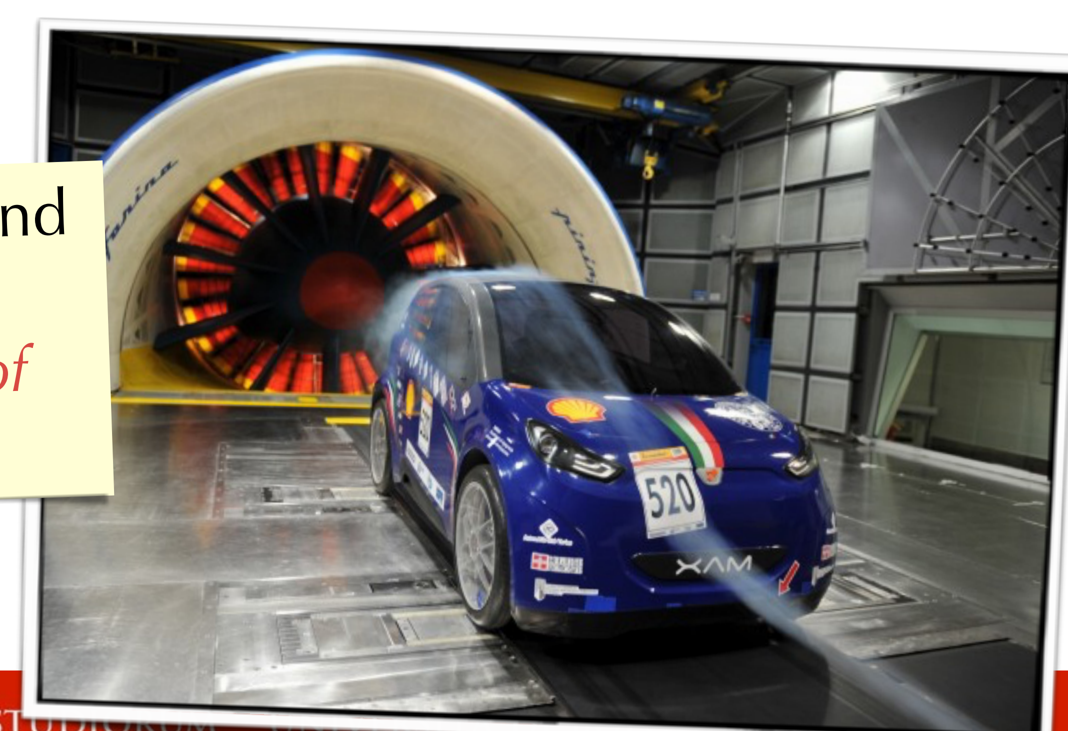
Introduction

- The word *system* describes a variety of concepts, so it is hard to give a meaningful definition, but here two basic assumptions hold
  - A system is assumed to be an entity *separable from the rest* of the universe (the environment of the system) by means of a physical or conceptual *boundary* between what is considered to be part of the system and what represents an *external disturbance* or *command* originating from outside
  - A system is composed of *interacting parts,* with the *reticulation* of a system into its component parts is something that requires skill and art
- The art and science of system modelling has to do with the *construction of a model* complex enough to represent the relevant aspects of the real system but not so complex as to be untreatable

Models of systems

- The focus is on the *dynamics* of the system, since systems of all kinds can exhibit counterintuitive behaviour when considered statically
  - A good understanding of dynamic response is crucial to the design of a *controller* for mechatronic systems
- *Models* of systems are simplified, abstracted constructs used to predict their behaviour (e.g., scaled physical models)
- Here we deal with another kind of model, i.e. the *mathematical* one
  - Because a model must be a simplification of reality, there is a great deal of art in the *construction* of models, and the trick is to reach the “right” level of complexity, or simplicity

No system can be modelled exactly and any system designer needs to have a *procedure for constructing a variety of system models* of varying complexity

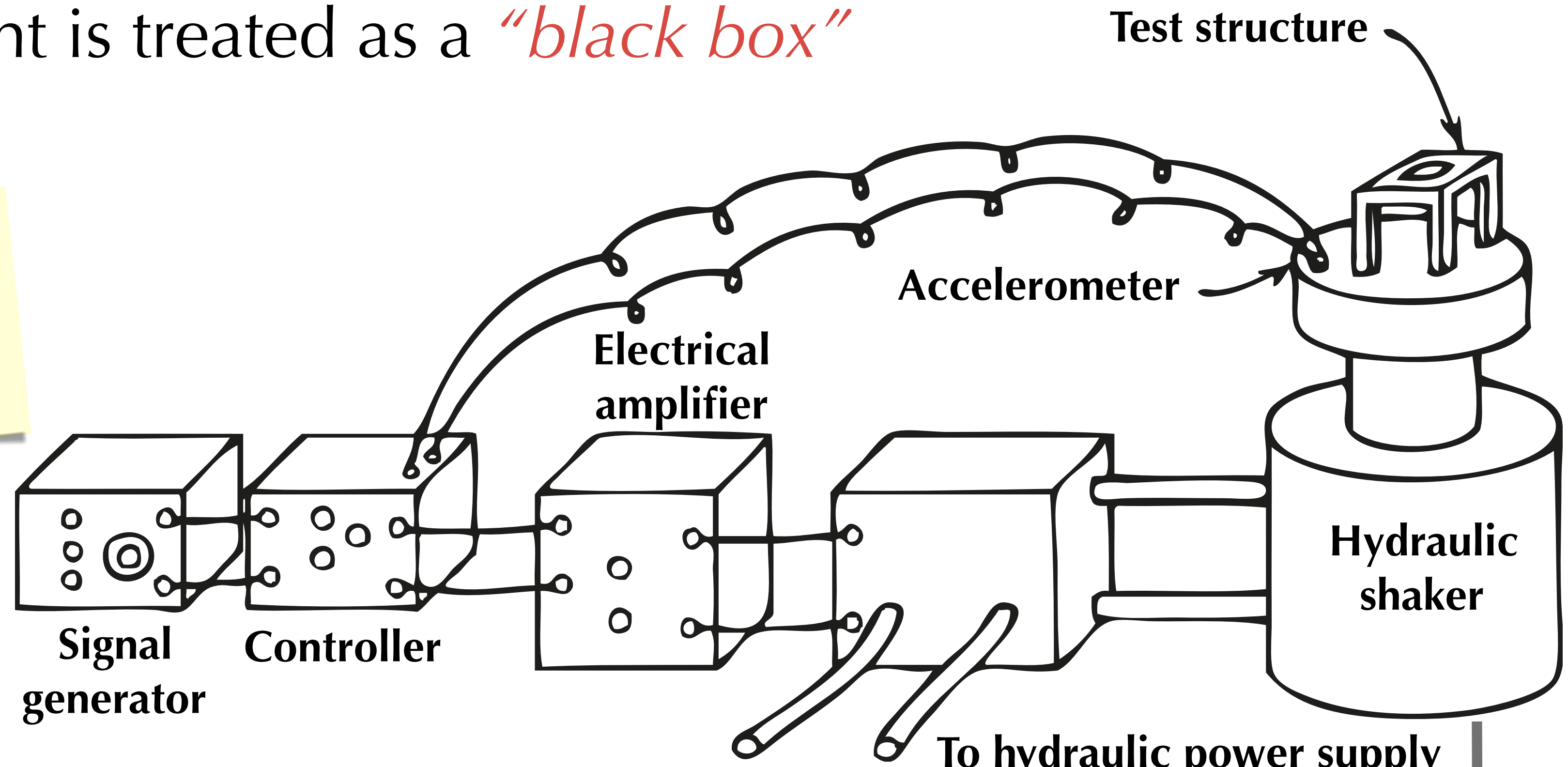


Models of systems

- System models will be constructed using a *uniform notation* for all types of physical systems
  - Models belonging to apparently diverse branches of engineering science can all be expressed using the notation of *bond graphs* based on *energy and information flow*
  - This allows one to study the *structure of the system,* i.e. the nature of the parts and the manner in which the parts interact
  - *Analogies* between various types of systems are made evident
  - Experience in one field can be extended to other fields

(Sub-)Systems & Components

- To model a system, it is necessary to *break it up* into smaller parts, and then to *assemble* the system model from the parts
  - The breaking up of the system is accomplished in *several stages*
  - *Subsystems ➤➤ Components ➤➤ Elements*
  - The *hierarchy* of components is not absolute
- One needs to know *how the component interacts* with other components and one must have a *characterisation* of the component
  - Otherwise a component is treated as a *"black box"*



It is not necessary to provide a *detailed model* of all the sub-systems!!

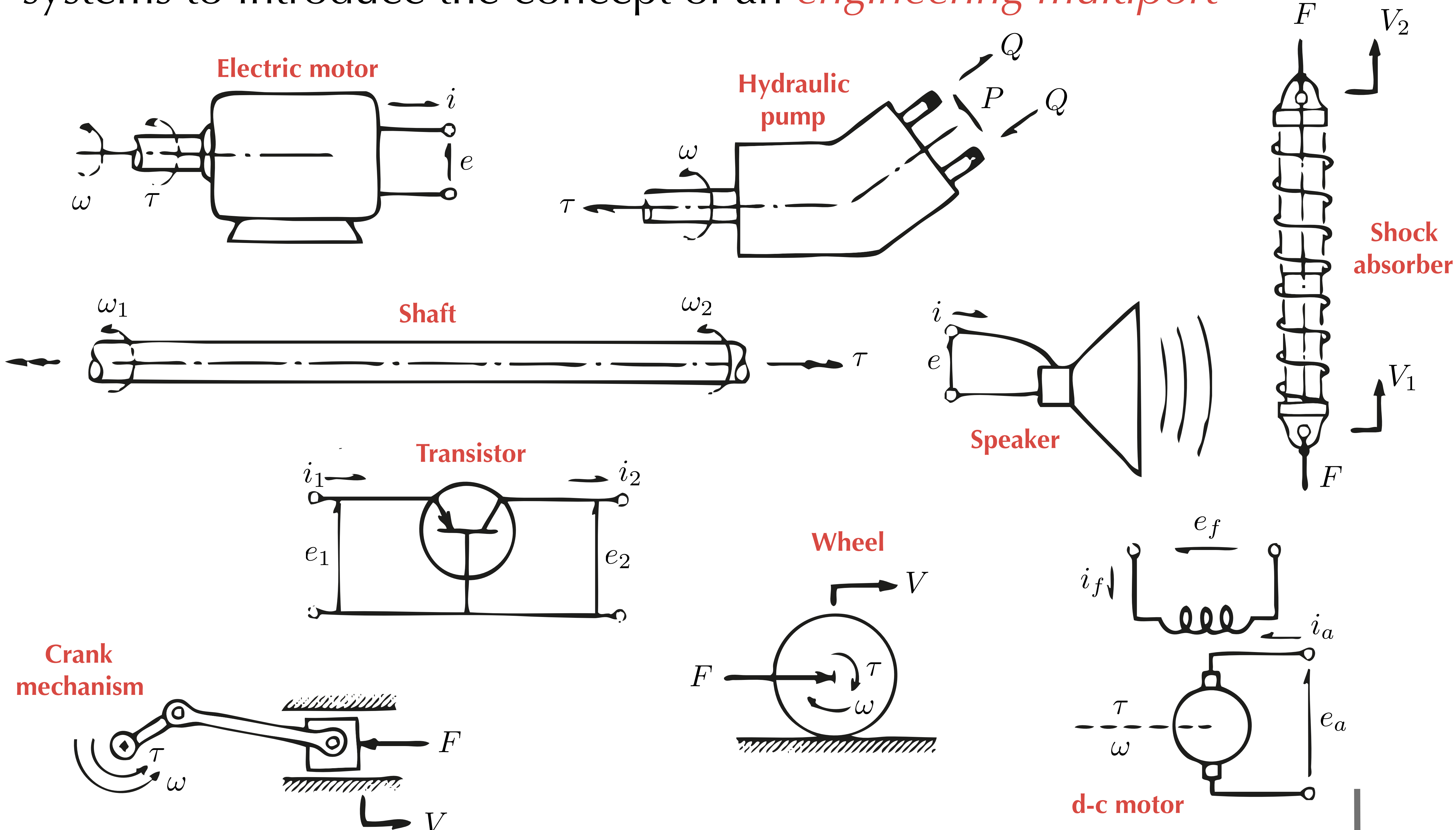
Engineering multiports

- We start now with the first steps toward the development of system modelling techniques for engineering systems involving power interactions
  - *Major subsystems* are identified, and the means by which the subsystems are *interconnected* are studied

  - The fact that interacting physical systems must transmit *power* then power is used to *unify* the description of interconnected subsystems
  - A uniform classification of the variables associated with power and energy is established, and *bond graphs* showing the interconnection of subsystems are introduced
  - Finally, the notions of *inputs, outputs,* and *pure signal flows* are discussed

Engineering multiports

- We give a collection of subsystems or components of engineering systems to introduce the concept of an *engineering multiport*

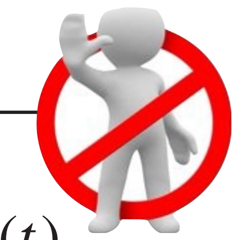


Engineering multiports

- Places at which subsystems can be interconnected are where *power can flow* between the subsystems, and such places are called *ports*
  - Physical subsystems with one or more ports are called *multiports*
  - The variables listed for the previous multiports are called *power variables,* because the product of the two variables considered as functions of time is the *instantaneous power* flowing between the two multiports
  - Since power could flow in either direction, a *sign convention* for the power variables will be established
- Since power interactions are always present when multiports are connected, it is useful to classify the power variables in a *universal scheme* and to describe all multiports in a *common language*
  - All power variables are called either ***effort*** or ***flow***

Engineering multiports

Domain	Effort, $e(t)$	Flow, $f(t)$
Mechanical translation	Force component, $F(t)$	Velocity component, $V(t)$
Mechanical rotation	Torque component, $\tau(t)$	Angular velocity component, $\omega(t)$
Hydraulic	Pressure, $P(t)$	Volume flow rate, $Q(t)$
Electric	Voltage, $e(t)$	Current, $i(t)$



Some *effort* and *flow* quantities

- The *power flowing through a port* can be expressed as the product of an effort and a flow variable:

$$P(t) = e(t) f(t)$$

- Two other types of variables *(energy variables)* turn out to be important in describing dynamic systems:
  - *Momentum:* $p(t) = p_0 + \int_{t_0}^{t} e(\tau)\,\mathrm{d}\tau$
  - *Displacement:* $q(t) = q_0 + \int_{t_0}^{t} f(\tau)\,\mathrm{d}\tau$

These relations can be equivalently written in *derivative* form

$$\frac{\mathrm{d}p}{\mathrm{d}t}(t) = e(t) \qquad \frac{\mathrm{d}q}{\mathrm{d}t}(t) = f(t)$$

Engineering multiports

- The *energy flow* is the time integral of the power flow:

$$E(t) = \int^t P(\tau)\,\mathrm{d}\tau = \int^t e(\tau) f(\tau)\,\mathrm{d}\tau$$

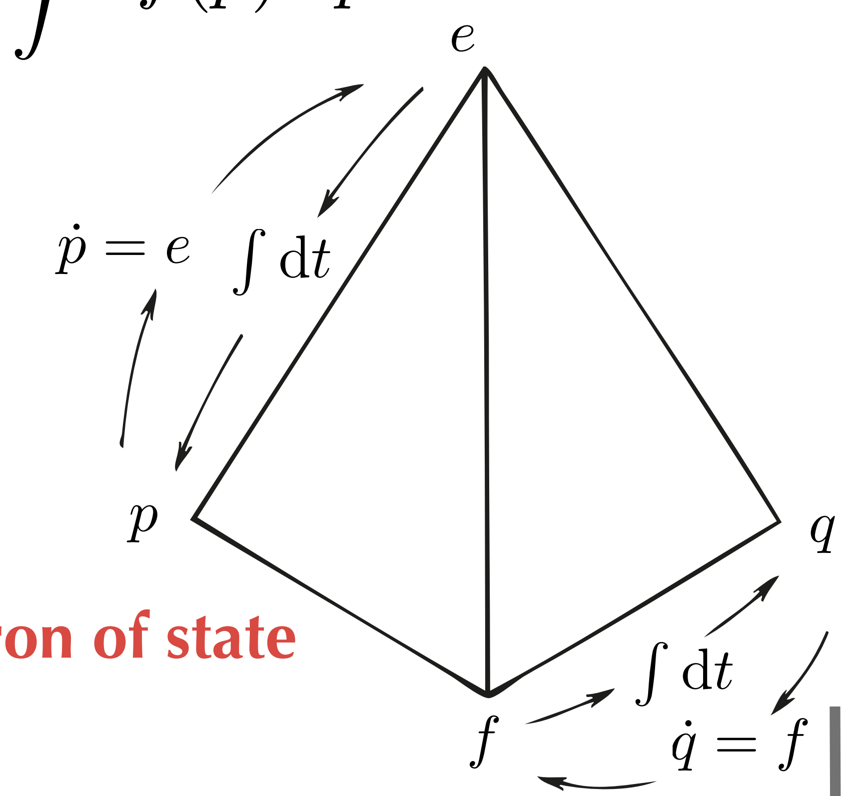
$$E(t) = \int^t e(\tau)\,\mathrm{d}q(\tau) = \int^t f(\tau)\,\mathrm{d}p(\tau)$$

This should explain why momentum and displacement are the *energy variables*

- There are cases where *an effort is a function of a displacement,* or a *flow is a function of a momentum,* which implies that

$$E(q) = \int^q e(\bar{q})\,\mathrm{d}\bar{q} \qquad\qquad E(p) = \int^p f(\bar{p})\,\mathrm{d}\bar{p}$$

The *only* types of variables that will be needed to model physical systems are represented by the *power* and *energy variables*



The tetrahedron of state

Engineering multiports

Generalized Variables	Mechanical Translation	SI Units
Effort, e	Force, F	newtons (N)
Flow, f	Velocity, V	meters per second (m/s)
Momentum, p	Momentum, P	N-s
Displacement, q	Displacement, X	m
Power, P	$F(t)V(t)$	watts (N-m/s $=$ W)
Energy, $\mathbf{E}$	$\int^{x} F\,dx$, $\int^{p} V\,dP$	joules (N-m $=$ J)

Power and Energy Variables for *Mechanical Translational* Systems

Generalized Variables	Mechanical Rotation	SI Units
Effort, e	Torque, τ	newton-meters (N-m)
Flow, f	Angular velocity, ω	radians per second (rad/s)
Momentum, p	Angular momentum, p_τ	N-m-s
Displacement, q	Angle, θ	rad
Power, P	$\tau(t)\omega(t)$	N-m/s $=$ W
Energy, $\mathbf{E}$	$\int^{\theta} \tau\,d\theta$, $\int^{p_\tau} \omega\,dp_\tau$	N-m $=$ J

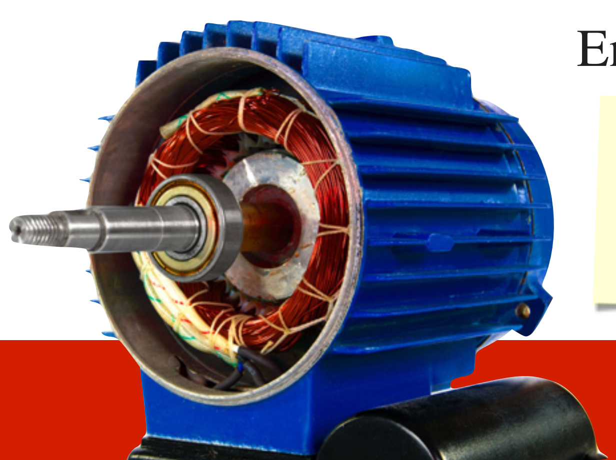
Power and Energy Variables for *Mechanical Rotational* Systems

Engineering multiports

Generalized Variables	Hydraulic Variables	SI Units
Effort, e	Pressure, P	newtons per square meter Pa = (N/m^2)
Flow, f	Volume flow rate, Q	cubic meters per second (m^3/s)
Momentum, p	Pressure momentum, p_p	N-s/m^2
Displacement, q	Volume, V	m^3
Power, P	$P(t)Q(t)$	N-m/s = W
Energy, **E**	$\int^v P\,dV,\ \int^{p_p} Q\,dp_p$	N-m = J

Power and Energy Variables for *Hydraulic* Systems

Generalized Variable	Electrical Variable	Units
Effort, e	Voltage, e	volt (V) = newton-meter per coulomb (N-m/C)
Flow, f	Current, i	ampere (A) = coulomb per second (C/s)
Momentum, p	Flux linkage variable, λ	V-s
Displacement, q	Charge, q	$C = A - s$
Power, P	$e(t)i(t)$	V-A = W = N-m/s
Energy, **E**	$\int^q e\,dq,\ \int^\lambda i\,d\lambda$	V-A-s = W-s = N-m = J



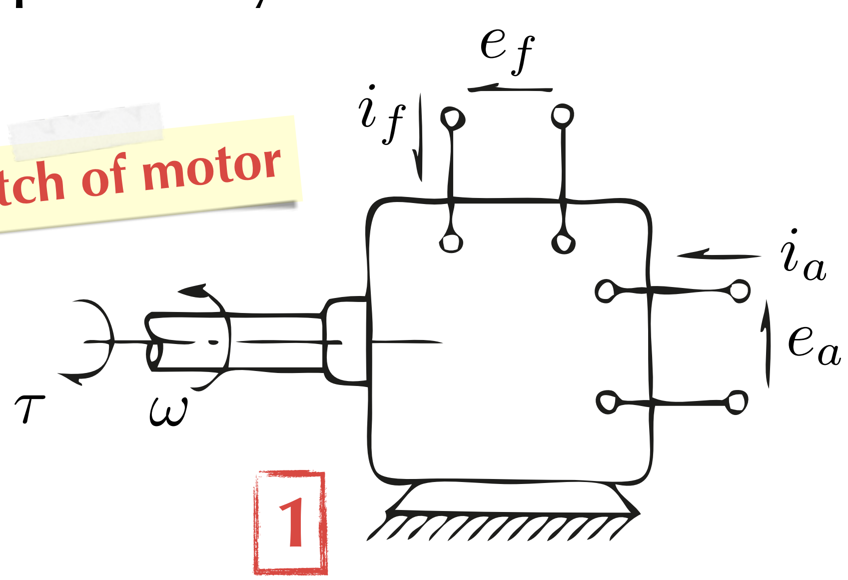
Power and Energy Variables for *Electrical* Systems

…and what about thermodynamic systems??

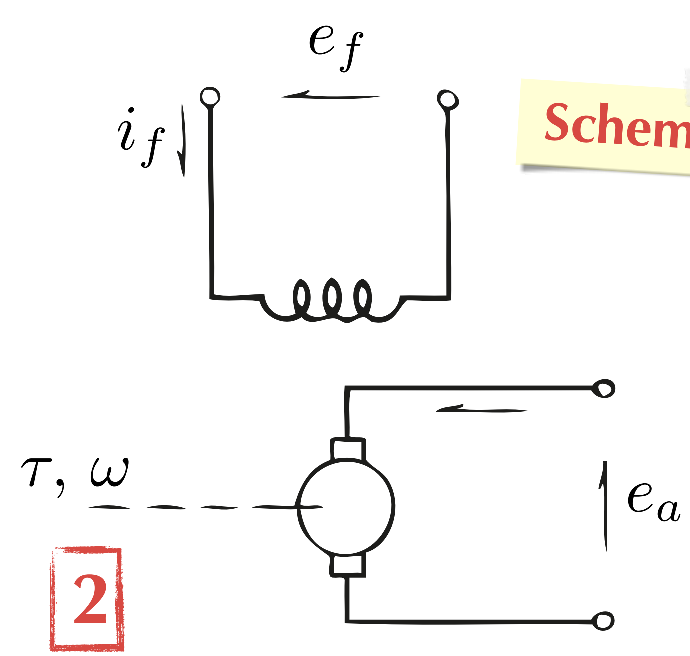
Ports, bonds & power

- Multiport elements can be *connected* to other multiport *to form systems,* and power can flow through the connected ports
  - We develop now a *universal way* to represent multiports and systems of interconnected multiports based on power and energy variables
- The separately excited d-c motor:

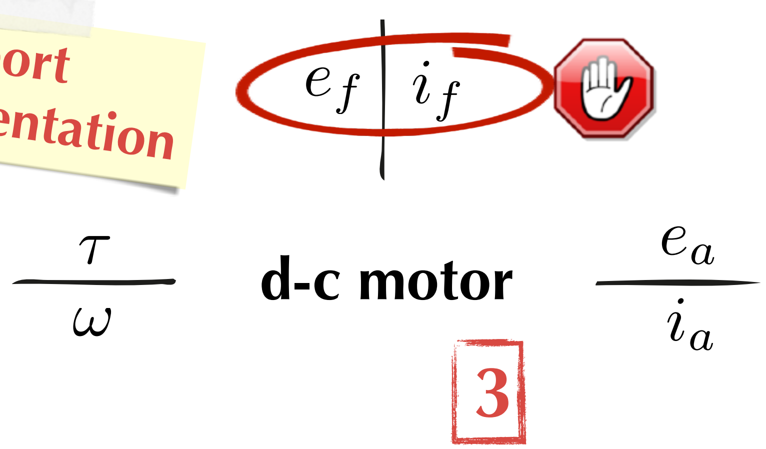
Sketch of motor



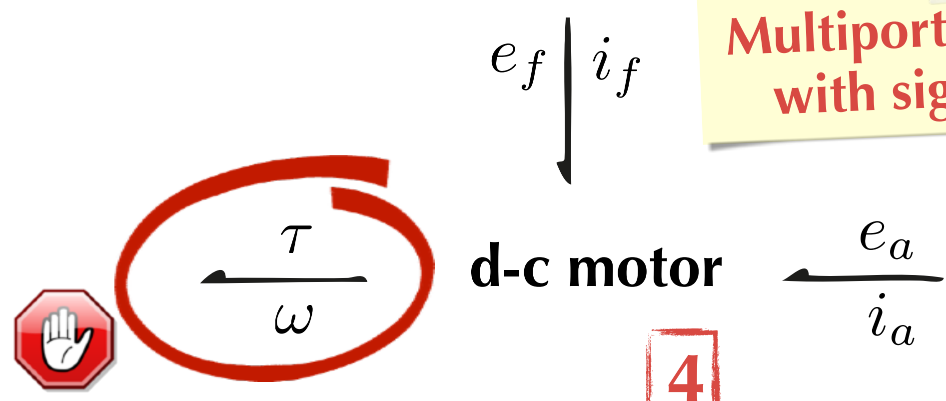
Schematic diagram



Multiport representation

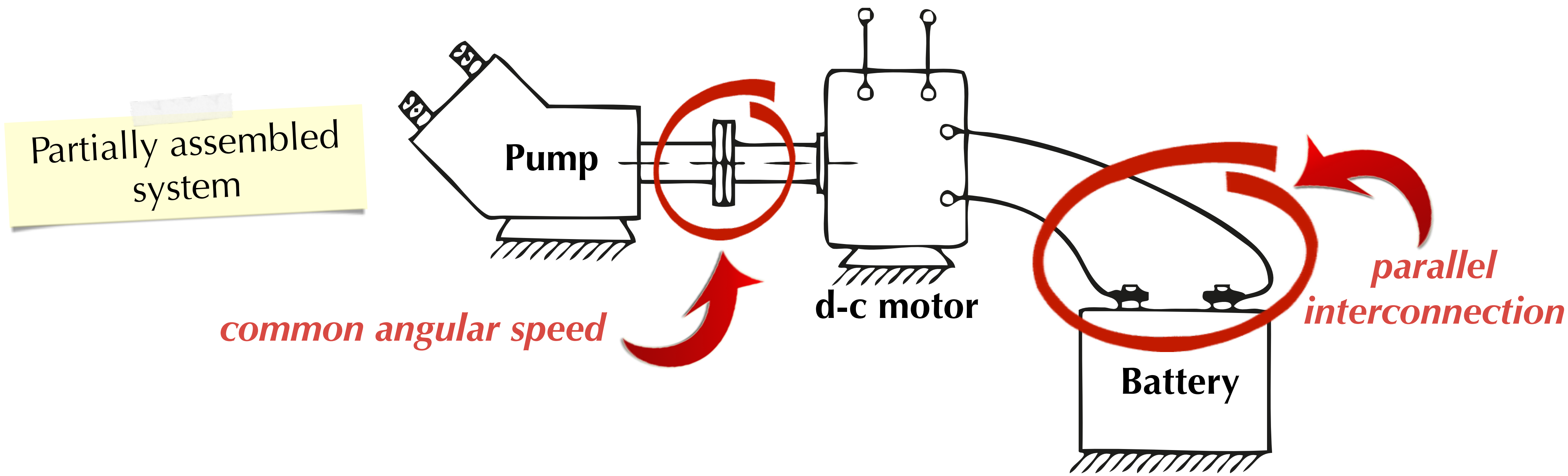


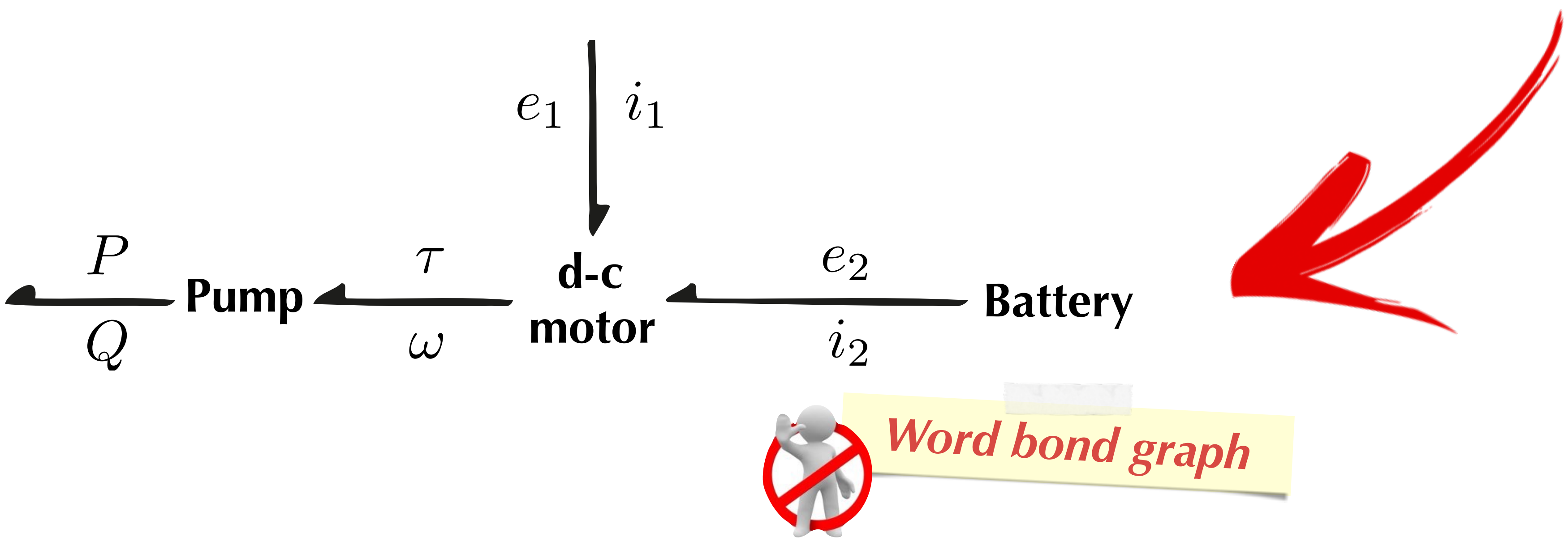
Multiport representation with sign convention

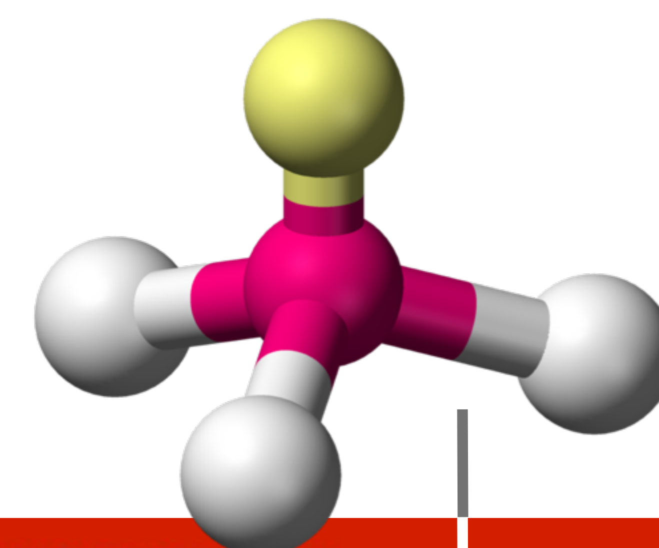


Ports, bonds & power

- When two multiports are coupled so that the *effort and flow variables become identical,* the two multiports are said to have a common bond, in analogy to the bonds between component parts of molecules

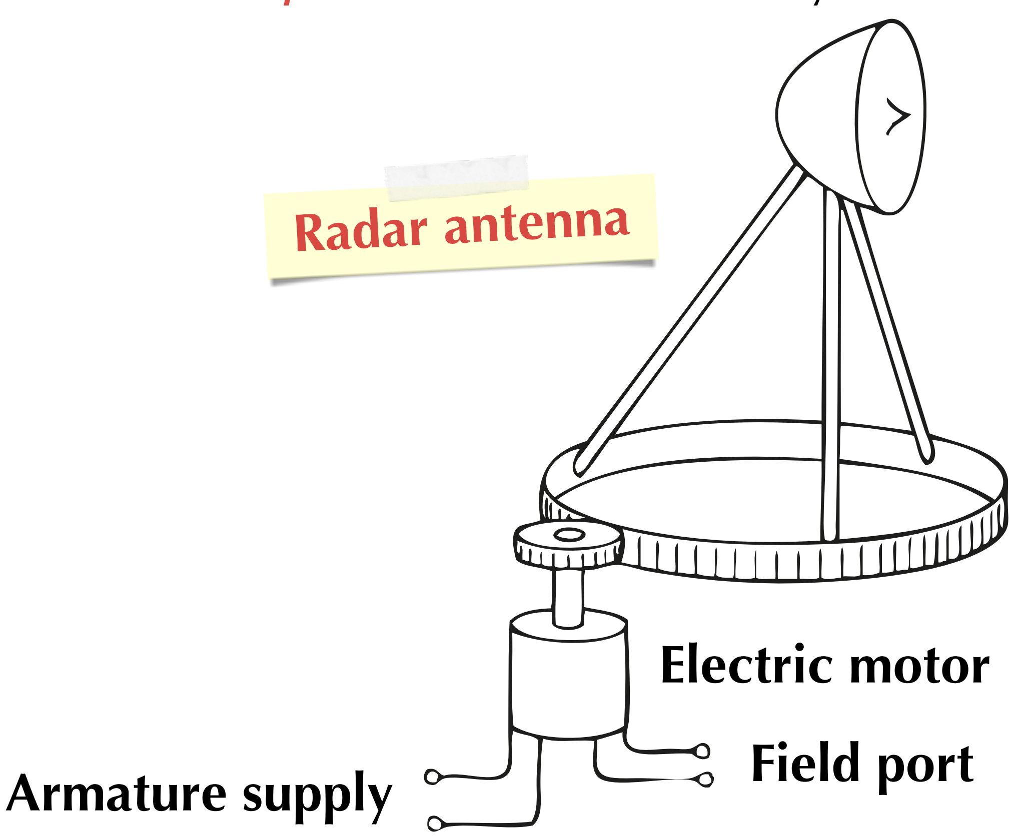


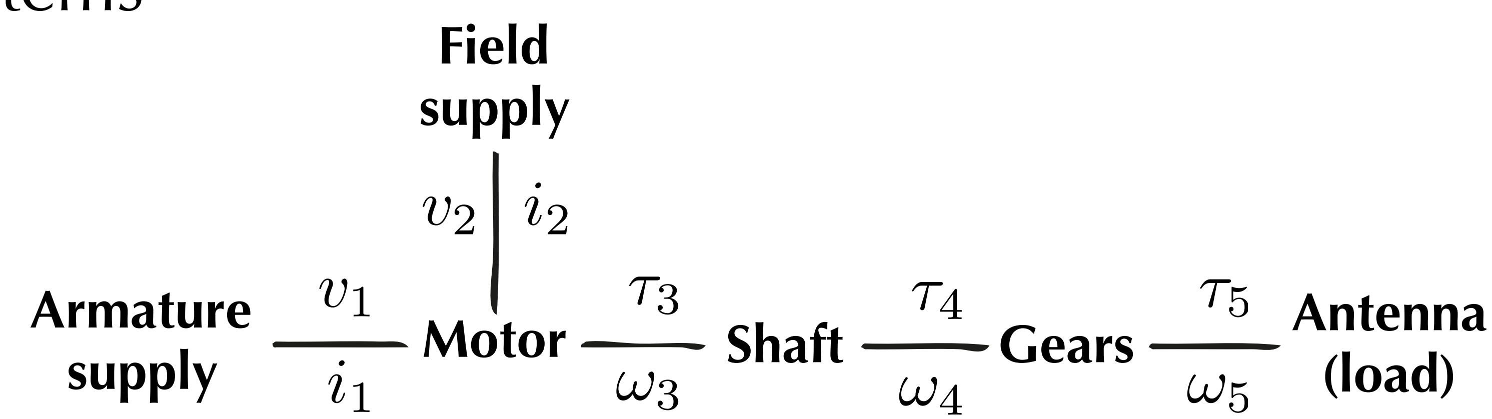




Bond graphs

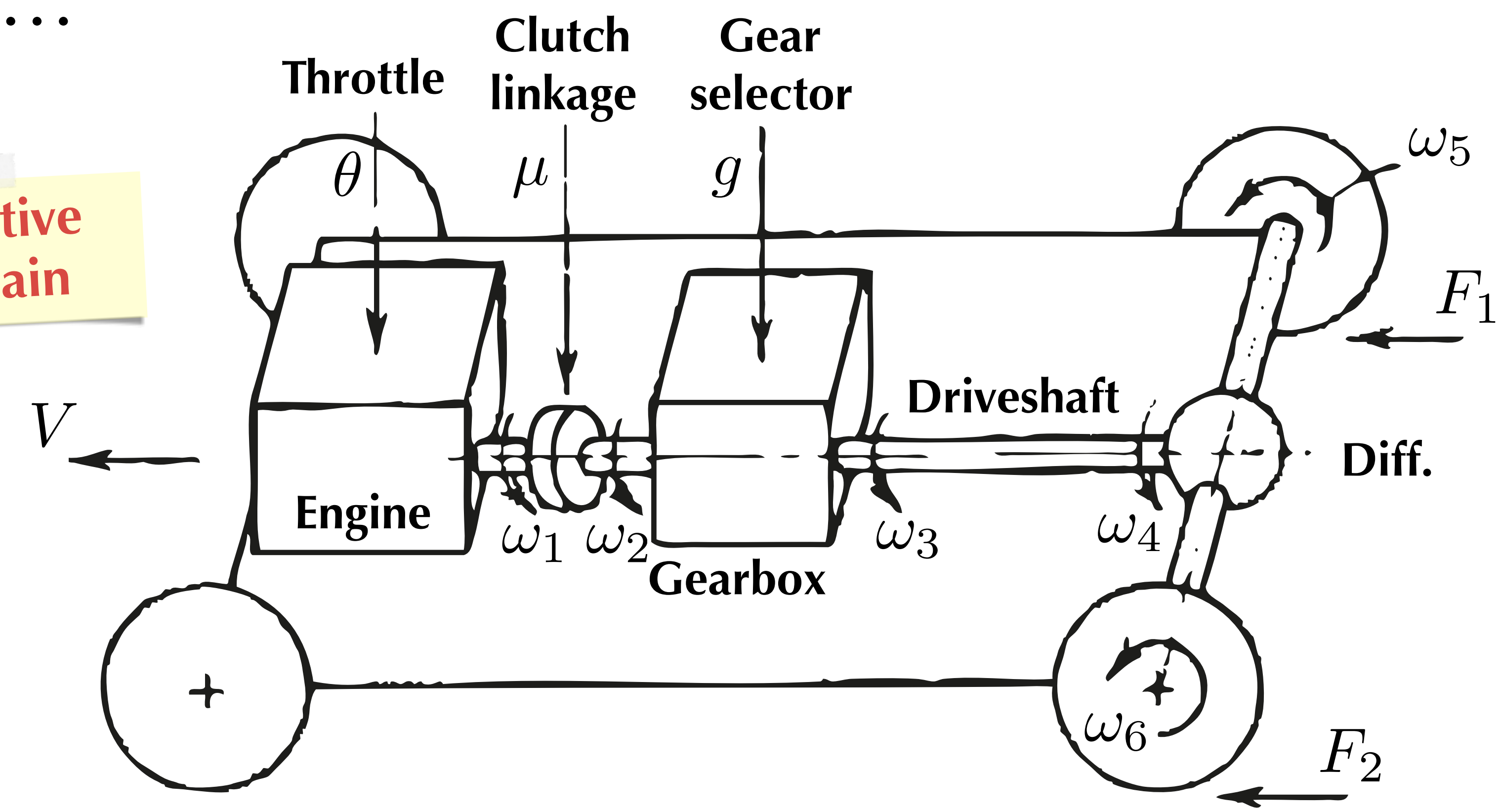
- A *bond graph* consists of subsystems linked together by *power bonds*
- When major sub-systems are represented by words, then the graph is called a *word bond graph*
  - Such a bond graph establishes multiport subsystems, the way in which the subsystems are bonded together, the effort and flow variables at the ports of the subsystems, and *sign conventions for power interchanges*
  - The word bond graph serves to make some *initial decisions* about the *representation* of dynamic systems



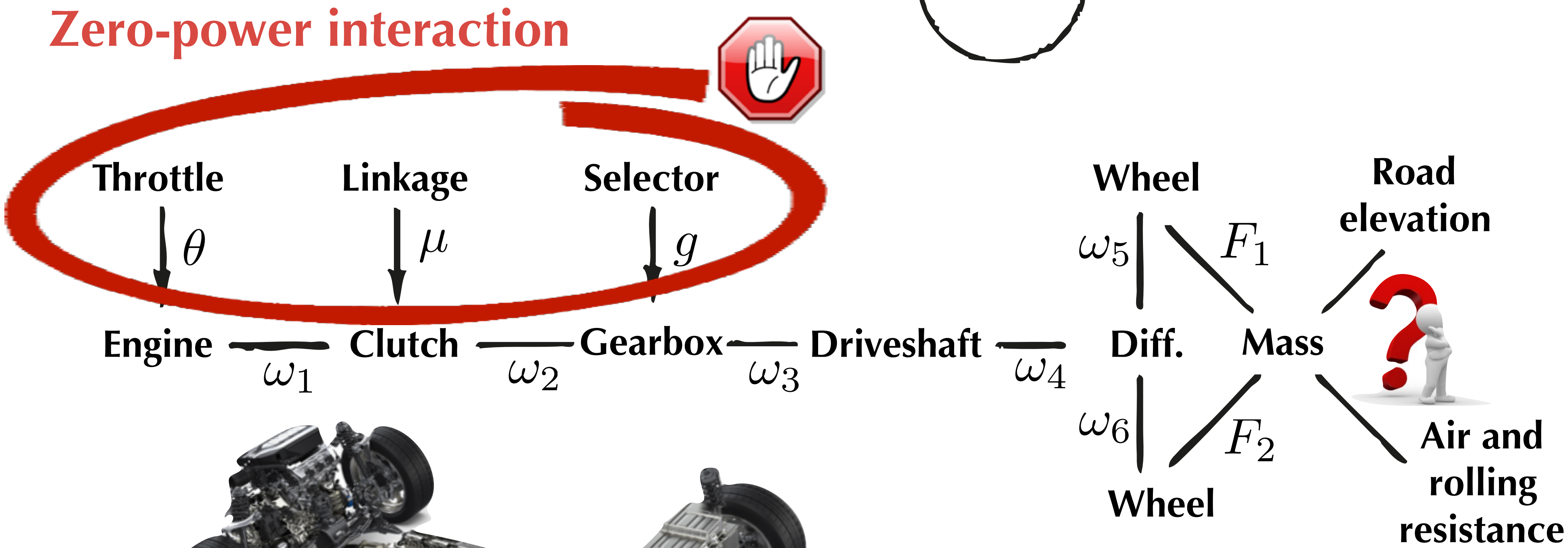


- A more complex example…

Automotive drive train



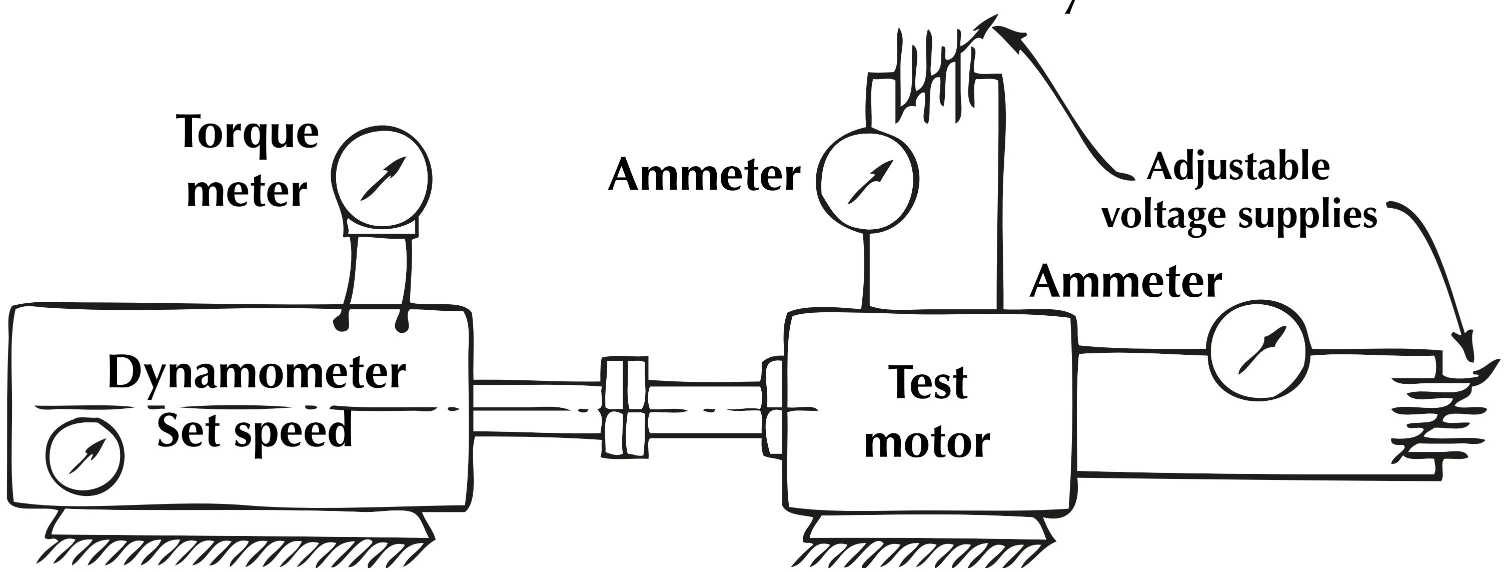
Zero-power interaction

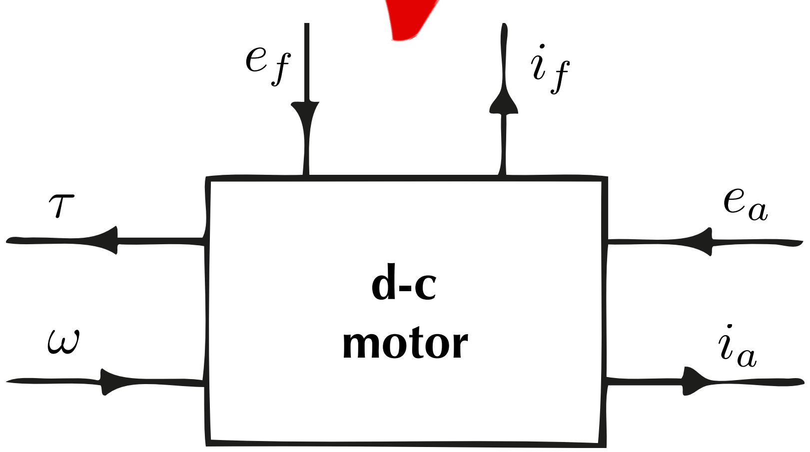


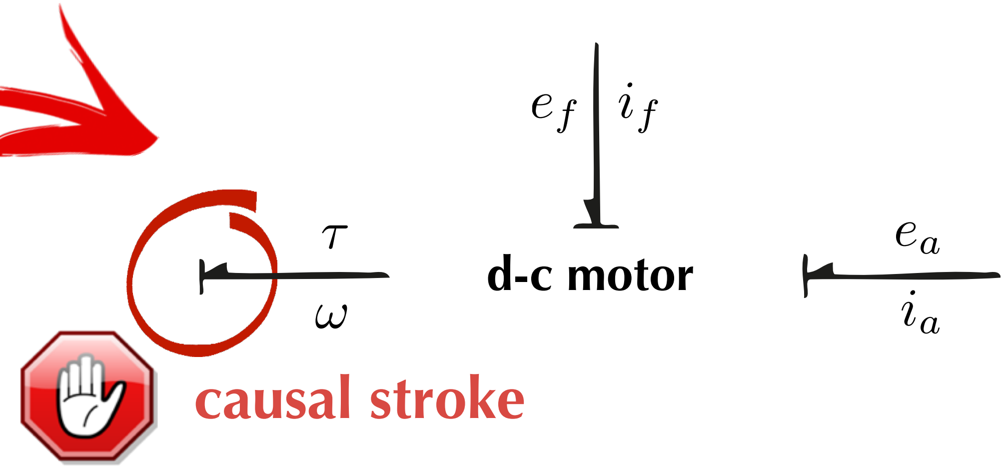
Inputs, outputs & signals

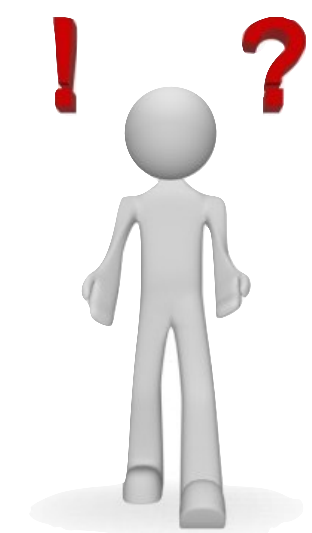
- In performing experiments, the notions of *input* and *output* arise
  - The same concepts will carry over when *"mathematical" models*
  - One must make a *decision* about what is to be done at the ports. At each port, both an effort and a flow variable exist, and *one can control either one but not both* of these variables simultaneously

Experiments on a d-c motor



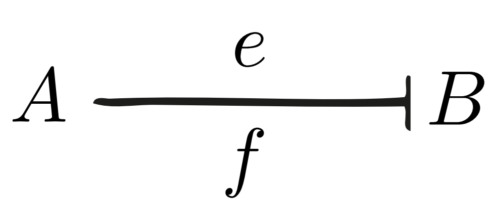


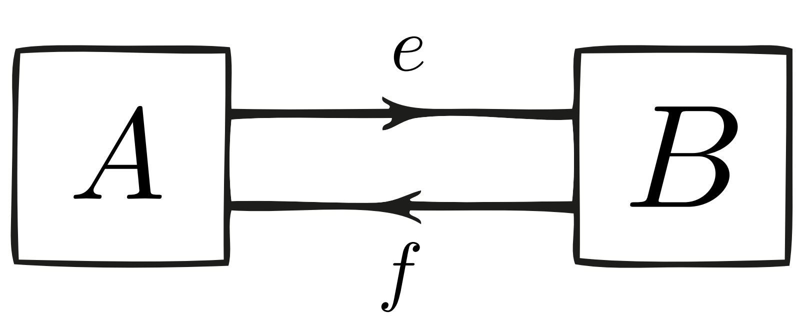




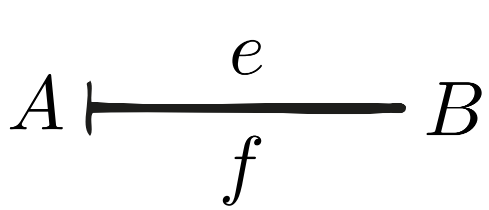
Inputs, outputs & signals

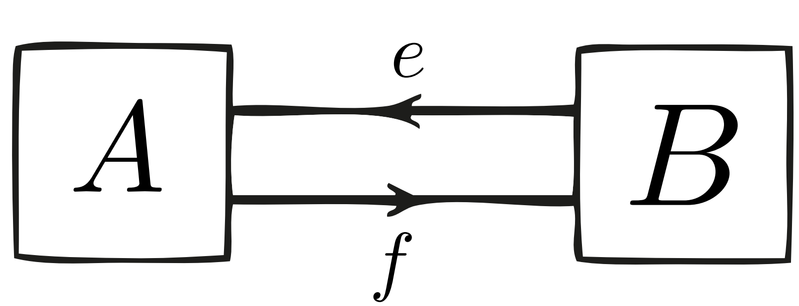
- To know which of the effort and flow signals at a port is the input of the multiport, *only one* piece of information has been supplied
- In bond graphs the way in which inputs and outputs are specified is by means of the *causal stroke*
  - The causal stroke is a short, perpendicular line made at one end of a bond
  - *It indicates the direction in which the effort signal is directed*
  - The end of a bond that does not have a causal stroke is the end toward which the flow's signal arrow points





effort is output of A, input to B
flow is output of B, input to A





effort is output of B, input to A
flow is output of A, input to B

Inputs, outputs & signals

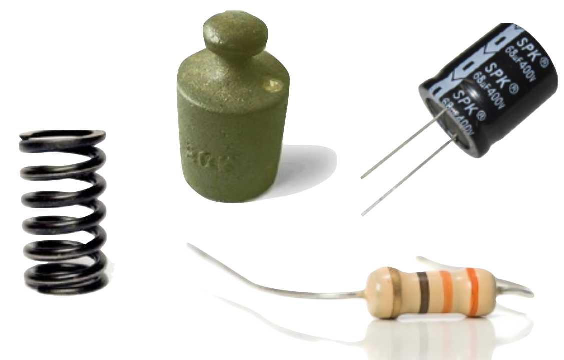
- But, what about *pure signal flow,* or the transfer of *information* with negligible power flow??
  - Multiports in principle all transmit *finite power* when interconnected
  - In many cases, systems are designed that *only one* of the power variables is important, i.e. a *single signal* is transmitted between two subsystems
    - Electronic amplifier
    - Ideal ammeter
    - *Control system* (ideal actuator)
- When the system is dominated by *signal interactions,* then either an effort or a flow may be suppressed at many interconnection points
  - In this case, the bond degenerates to a single signal *(active bond)*

$$A \xrightarrow{e} B \quad = \quad A \overset{e}{\underset{f}{\rule[0.5ex]{3em}{0.4pt}|}} B$$

The flow, f , has a *negligible* effect on A

Basic multiport elements

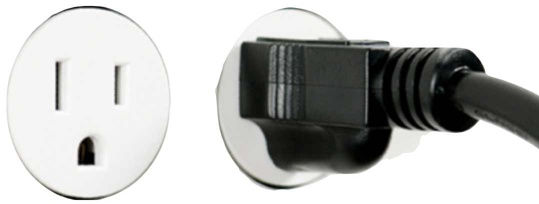
- Real devices can be considered as subsystems from the point of view of power exchanges and external port variables
- It's time to present a *basic set of multiport elements* that can be used to model subsystems in detail
  - Idealised mathematical models of *real components,* or…
  - "Entities" used to model *physical effects* in a device



- Only a few basic types of multiport elements are required in order to represent models in a *wide variety of energy domains*
  - The bond graph notation often allows one to *visualise* aspects of the system more easily than with the state equations or with some graphical notation designed for a single energy domain / signal flow
  - The search for a bond graph model of a complex system frequently increases one's *understanding* of the physical system

Basic 1-port elements

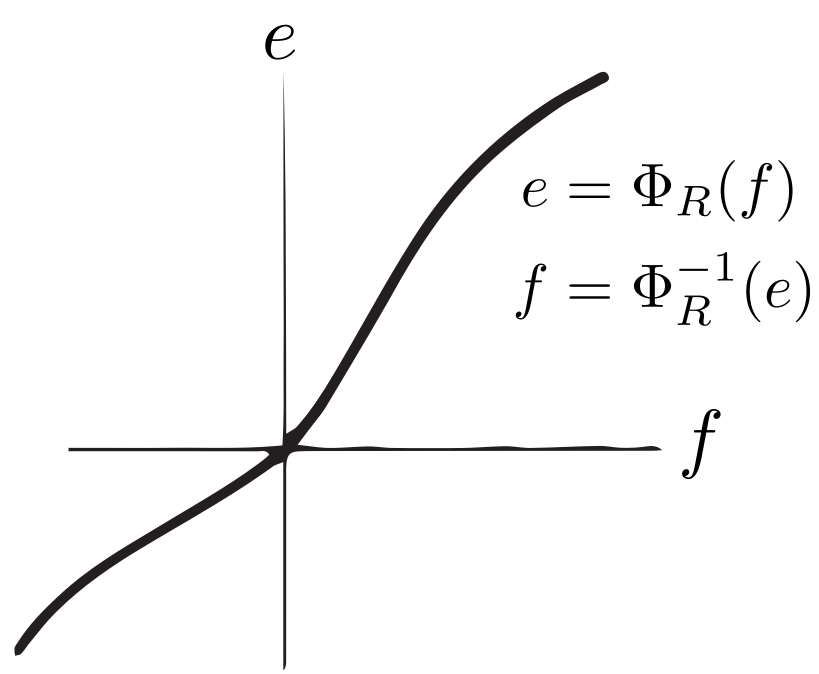
- A *1-port element* has a single pair of effort and flow variables
  - This does not mean that it is a simple system!!



- In the sequel, we deal with the most primitive 1-ports
  - Energy *dissipation*
  - Energy *storage*
  - Energy supply

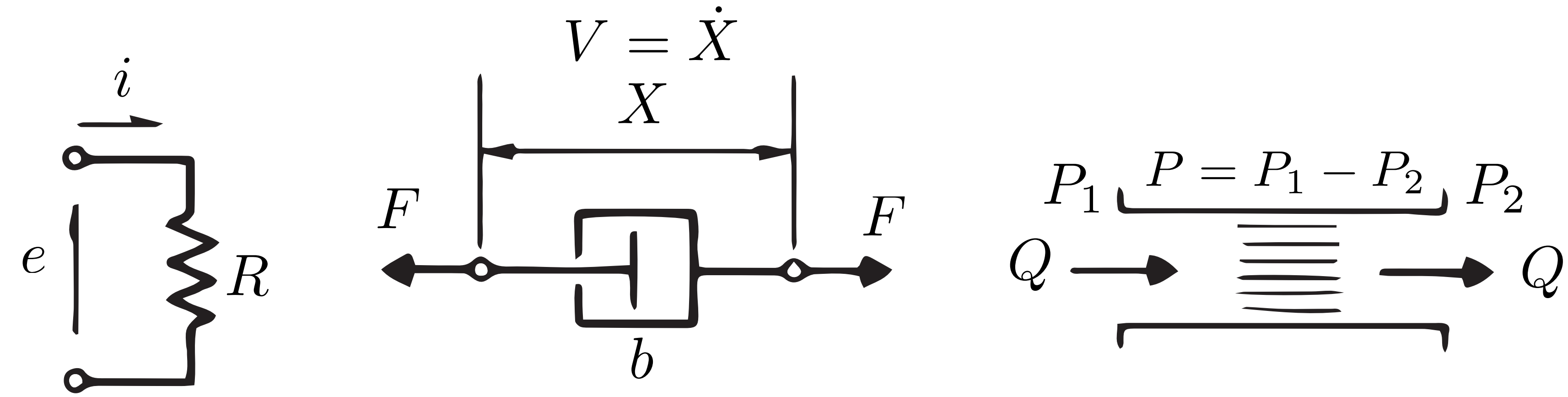
- The *1-port resistor* is an element in which the effort and flow variables at the single port are related by a static function

$$\xrightarrow[f]{e} R$$



Nonlinear
1-port resistor

Basic 1-port elements



$$\xrightarrow[i]{e} R\!:\!R \qquad \xrightarrow[V]{F} R\!:\!b \qquad \xrightarrow[Q]{P} R$$

$e = Ri$

$F = bV$

$$P_1 - P_2 = \frac{\rho}{2A^2} Q|Q|$$

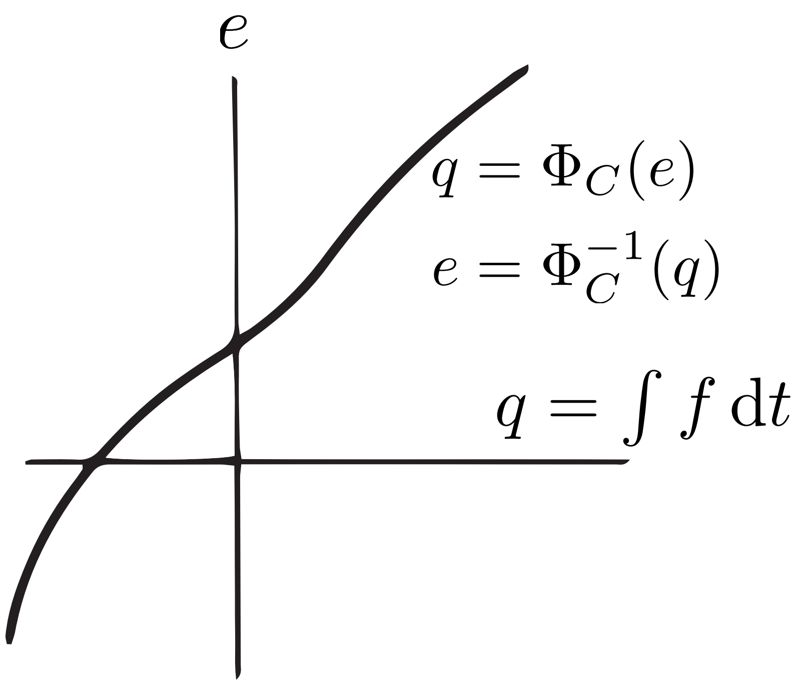
- Usually, *resistors dissipate energy,* i.e. power flows into the resistor but never comes out of it (disappears…)
  - Power flows into the port when the *product of e and f is positive* according to the sign convention shown, so power is always dissipated if the constitutive relation lies in the first and third quadrants of the *e–f* plane
  - When a resistive element is assumed to be *linear,* it is conventional to indicate this on the bond graph by appending a *colon (:)* next to R
  - For passive (power dissipating) resistors, establish the power sign convention by means of a *half-arrow pointing toward the resistor*

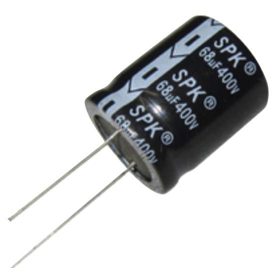
Basic 1-port elements

- Next consider a 1-port device in which a static constitutive relation exists between an effort and a displacement
  - Such a device *stores* and gives up energy *without loss*
  - In bond graph terminology, an element that relates e to q is called a *1-port capacitor* or *compliance*
  - In physical terms, a capacitor is an idealisation of devices as springs, torsion bars, electrical capacitors, gravity tanks, and hydraulic accumulators
  - There are idealised *linear* compliance elements as well as *nonlinear* ones

Nonlinear
1-port capacitor

$$\xrightarrow[f]{e} C$$





The *linear compliance* is defined as the inverse of the slope of the e versus q curve

Basic 1-port elements

- With the same sign convention of the resistor, we have that

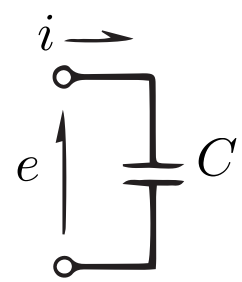
$$E(t) = \int_0^t e(\tau) f(\tau)\,\mathrm{d}\tau + E_0$$

$$f\,\mathrm{d}t = \mathrm{d}q$$

$$e = e(q)$$

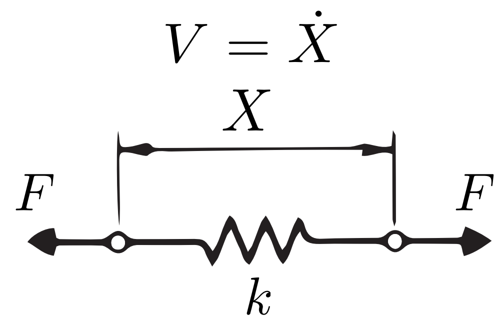
$$E(q) = \int_{q_0}^{q} e(\bar{q})\,\mathrm{d}\bar{q} + \cancel{E_0}$$

- The *conservation of energy for C* is obvious
  - The power flow into the port reverses and power flows out of the port
  - During the process, *no energy is lost,* i.e. it is conserved



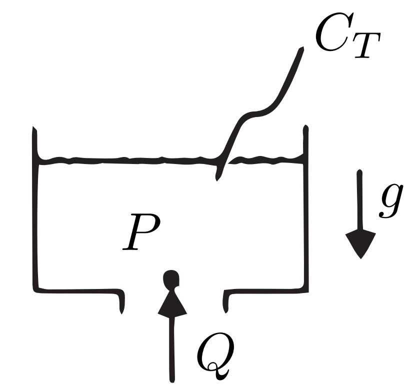
$$\xrightarrow[i]{e} C : C$$

$$e = \frac{q}{C}$$



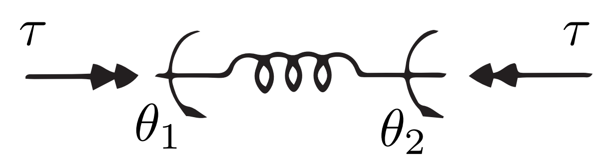
$$\xrightarrow[V]{F} C : 1/k$$

$$F = kx$$



$$\xrightarrow[Q]{P} C : C_T$$

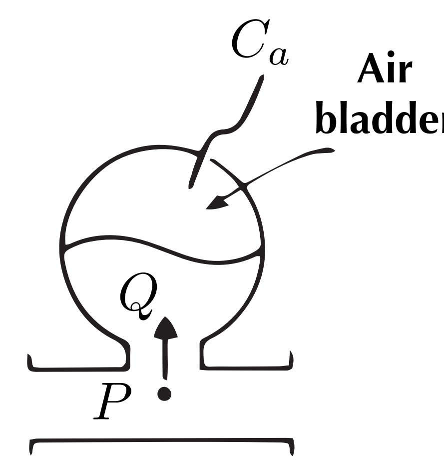
$$P = C_T V$$



$$\theta = \theta_1 - \theta_2$$

$$\omega = \dot{\theta}$$

$$\xrightarrow[\omega]{\tau} C : 1/k_r$$



$$\xrightarrow[Q]{P} C : C_a$$

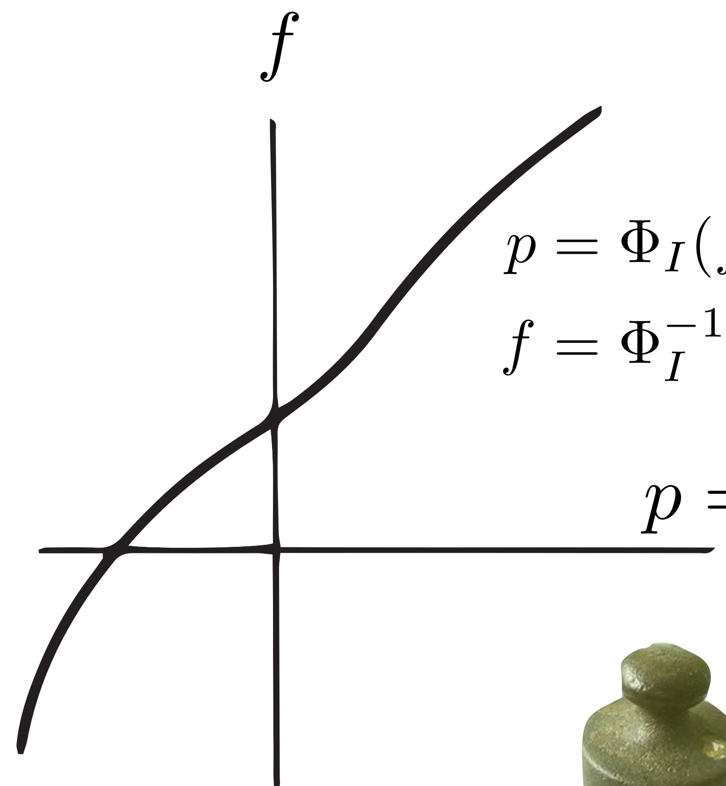
$$P = \frac{P_0 V_0^{\gamma}}{V^{\gamma}}$$

Basic 1-port elements

- The *dual* of the capacitor is the *inertia,* an energy-storing 1-port with the momentum *p* related by a static constitutive law to the flow *f*
  - It is used to model *inductance* effects in electrical systems, and *mass* or *inertia* effects in mechanical or fluid systems

Nonlinear
1-port inertia

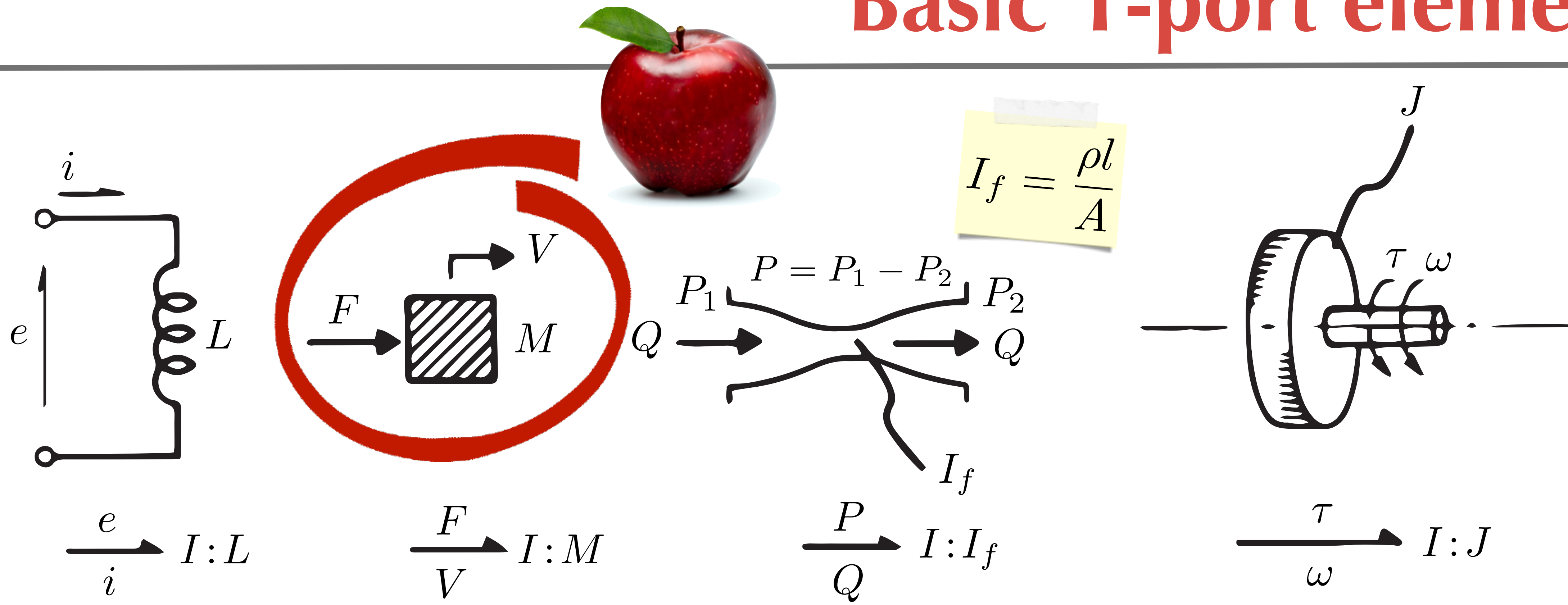
$$\xrightarrow[f]{e} I$$



$$p = \Phi_I(f)$$
$$f = \Phi_I^{-1}(p)$$
$$p = \int e \, \mathrm{d}t$$

The *"linear inertia"* is defined as the inverse of the slope of the *f* versus *p* curve

Basic 1-port elements



☑ The inertia is an *energy storing element,* and as before we have

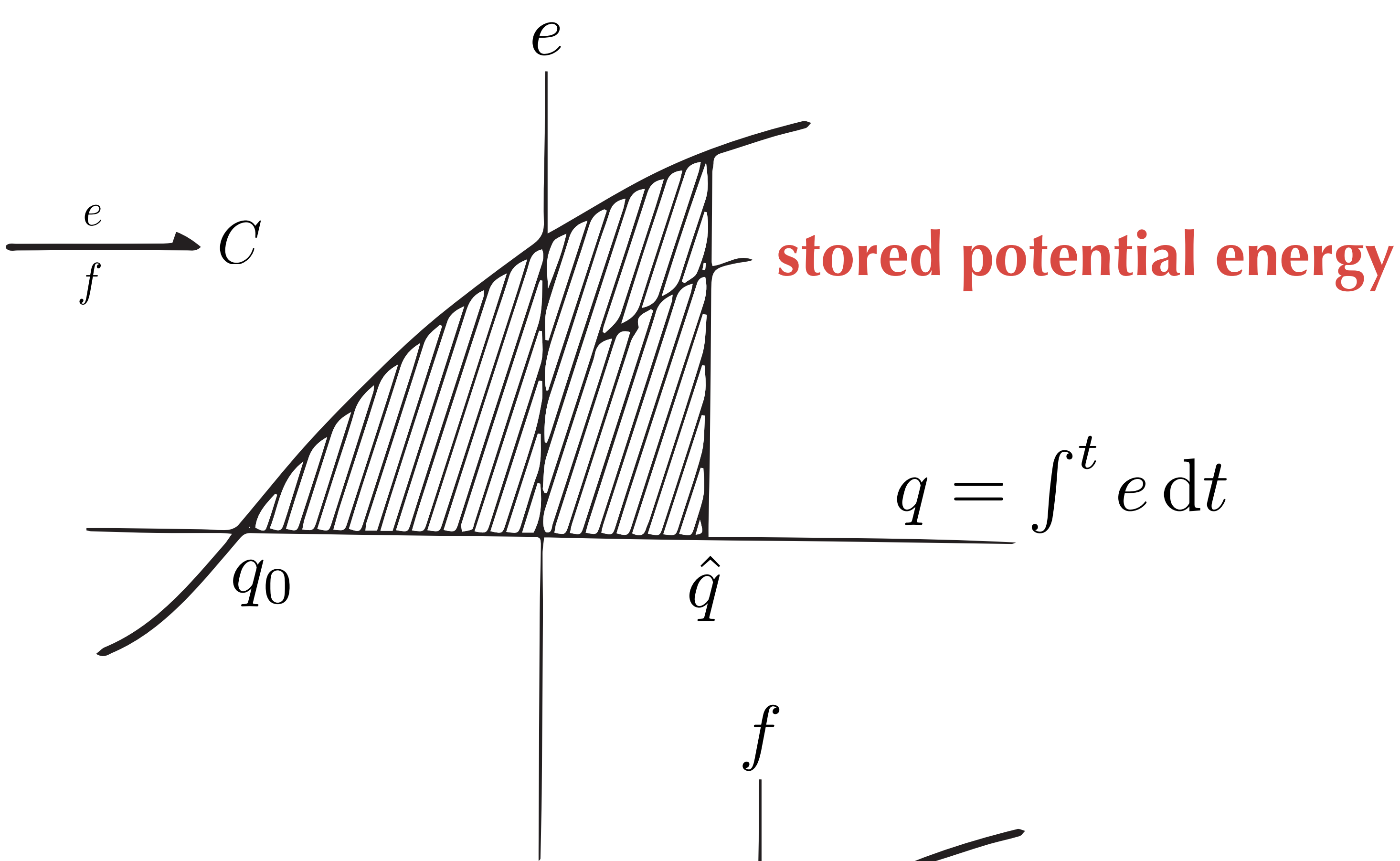
$$E(t) = \int_0^t e(\tau) f(\tau)\, \mathrm{d}\tau + E_0 \qquad E(p) = \int_{p_0}^{p} f(\bar{p})\, \mathrm{d}\bar{p} + \cancel{E_0}$$

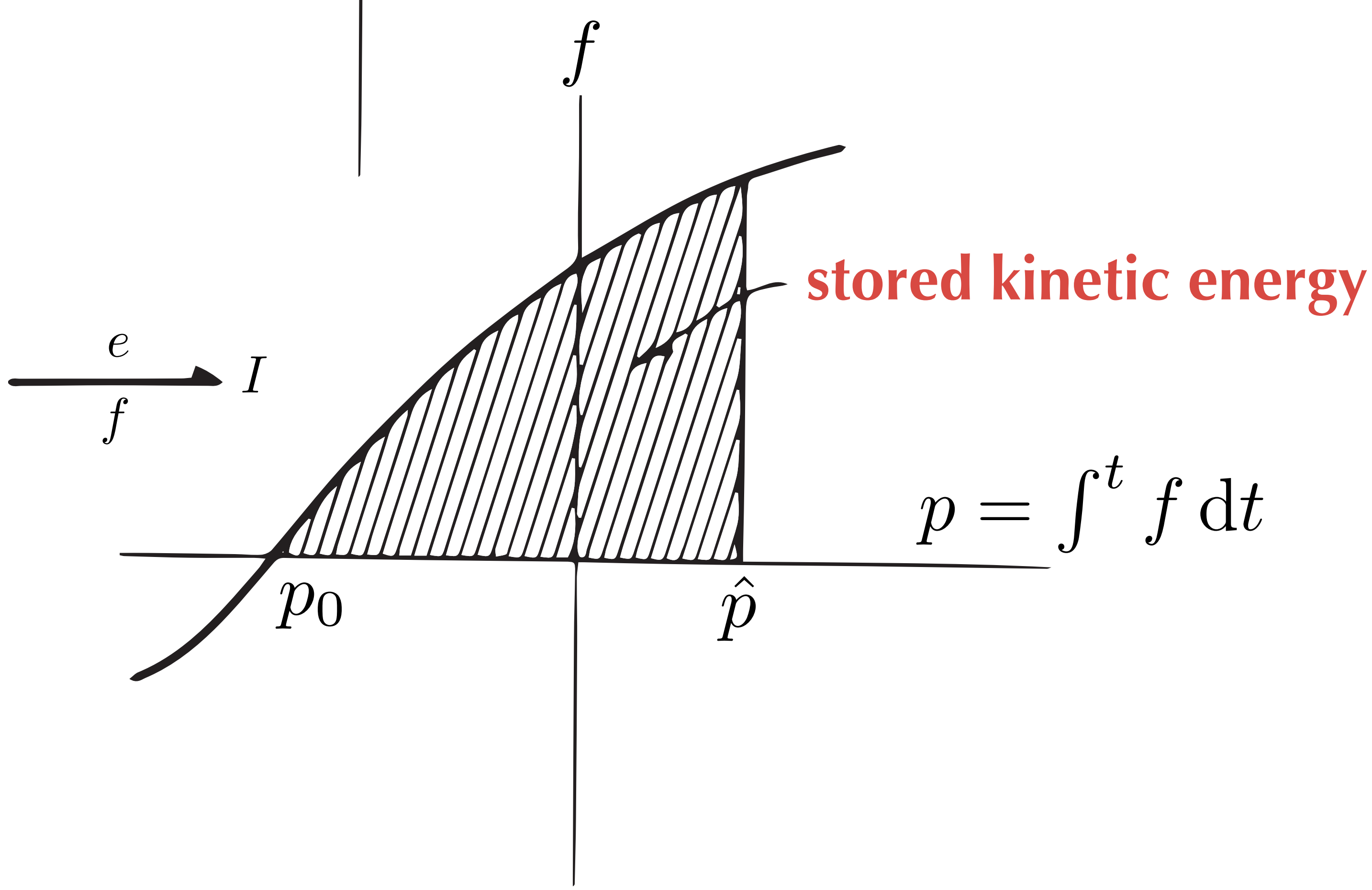
$$e\, \mathrm{d}t = \mathrm{d}p$$

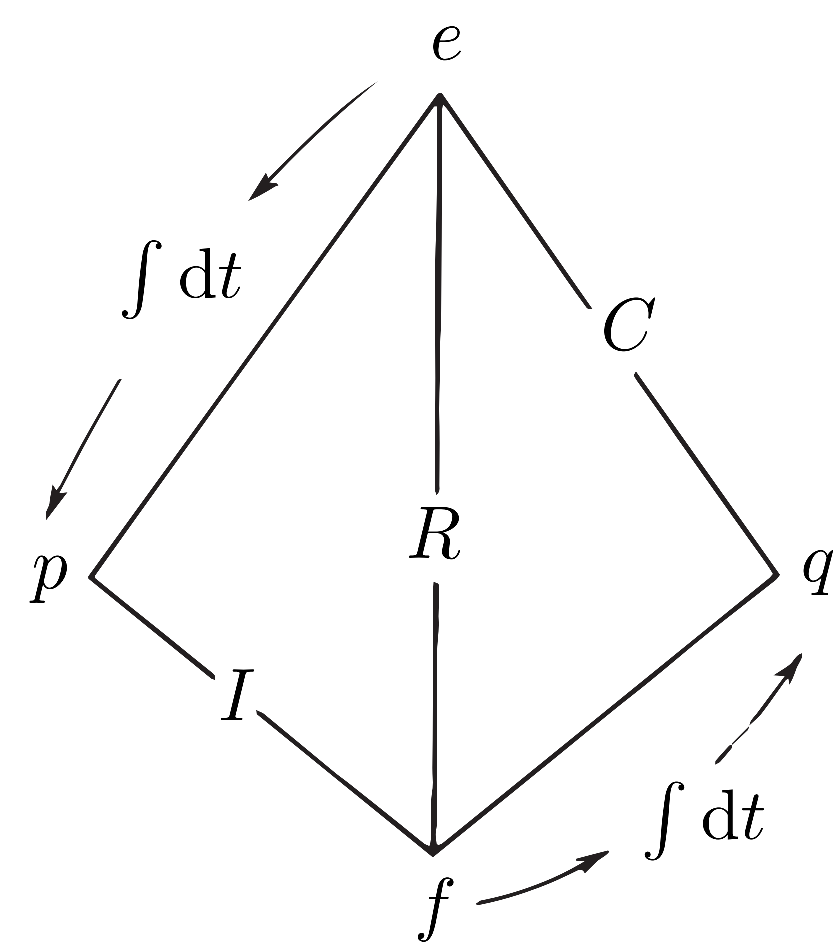
$$f = f(p)$$

☑ Let's give a closer look to the *stored energy!*

Basic 1-port elements







In the *linear case* we have:

$$E_q(q) = \frac{q^2}{2C} \qquad E_p(p) = \frac{p^2}{2I}$$

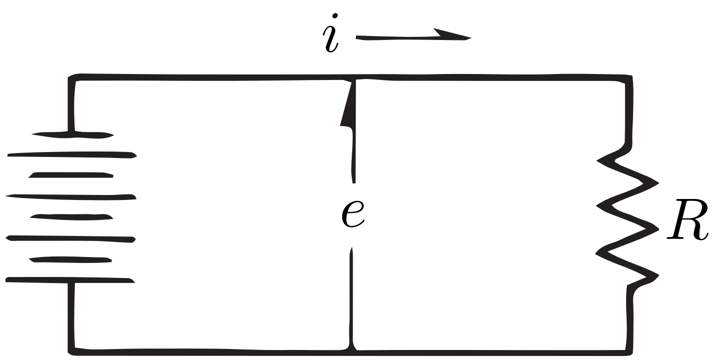
Basic 1-port elements

- The *effort source* and the *flow source* are idealised versions of voltage supplies, pressure sources, vibration shakers, constant-flow systems…
  - An effort or flow is either maintained reasonably *constant,* or constrained to be some particular *function of time*

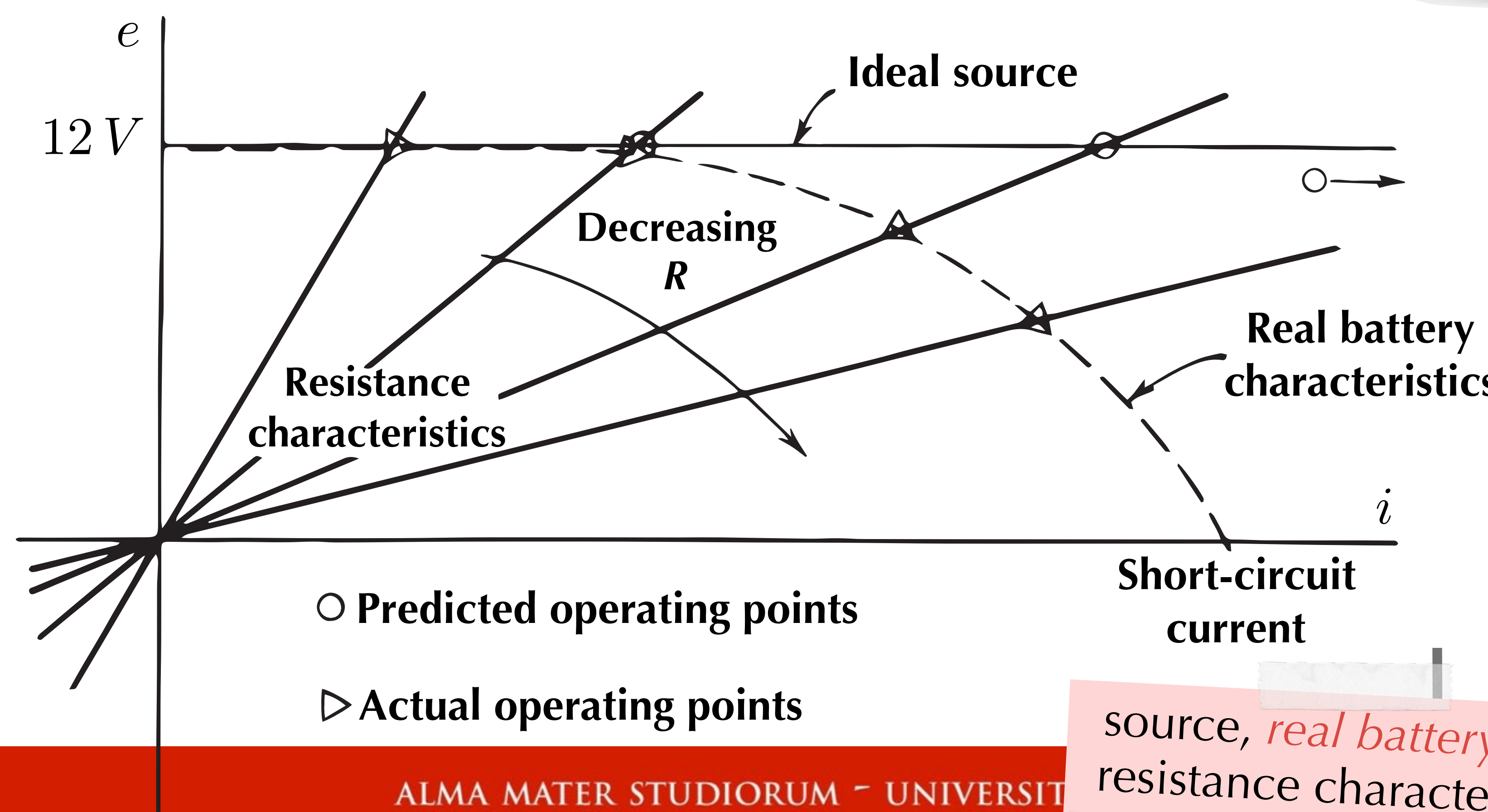
$$S_e \xrightarrow[f]{e} \qquad\qquad S_f \xrightarrow[f]{e}$$

Source elements are thought of as *supplying power to a system*

  - A source maintains one of the power variables constant or a specified function of time *no matter how large the other variable may be*



$$S_e \xrightarrow[i]{e} R$$



source, *real battery*, and resistance characteristics

Basic 2-port elements

- One might expect that it would be necessary to define more basic types of 2-ports than 1-ports, but, in fact, *only two basic types of 2-ports* need to be defined here
  - We need to discuss here only those that cannot be modelled using the basic 1-ports and *other elements* to be defined later

- The 2-ports to be discussed here are *ideal* in the specific sense that power is conserved
  - Let us consisted a conservative 2-port with a *through power* sign convention:

$$\xrightarrow[f_1]{e_1} TP \xrightarrow[f_2]{e_2} \qquad e_1(t)f_1(t) = e_2(t)f_2(t)$$

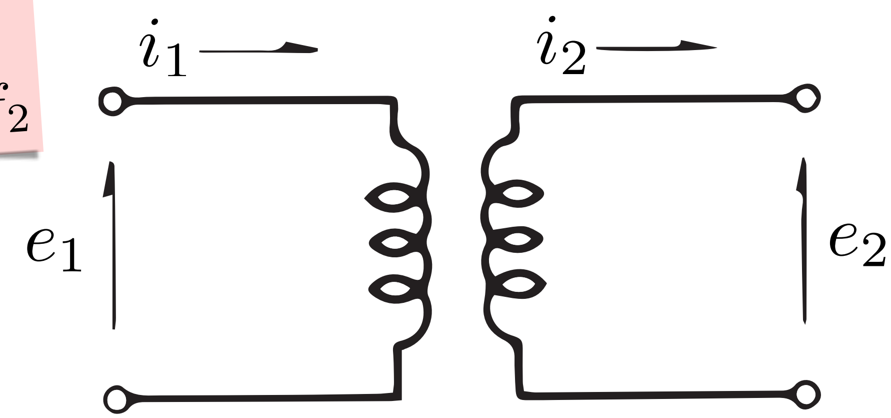
  - *Transformer (TF):* $e_1 = me_2, \; mf_1 = f_2$
  - *Gyrator (GY):* $e_1 = rf_2, \; rf_1 = e_2$

Basic 2-port elements

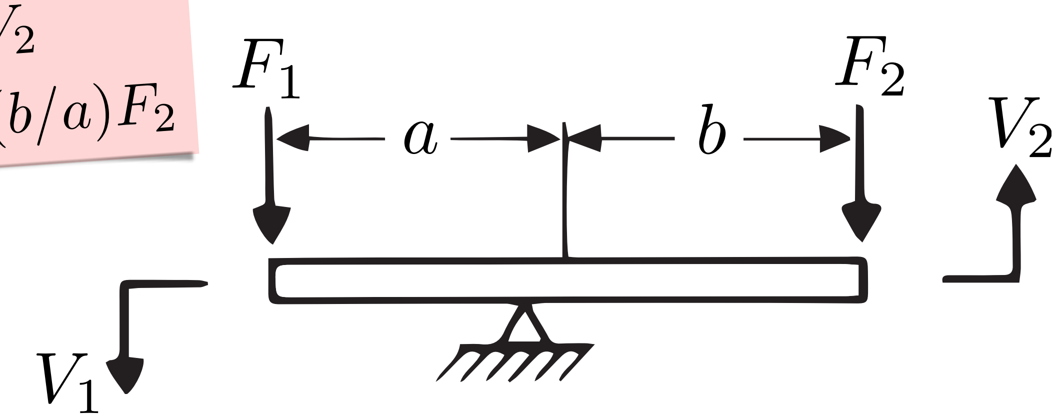
$$\xrightarrow[f_1]{e_1} TF \xrightarrow[f_2]{e_2}$$

☑ Some examples of transformer…

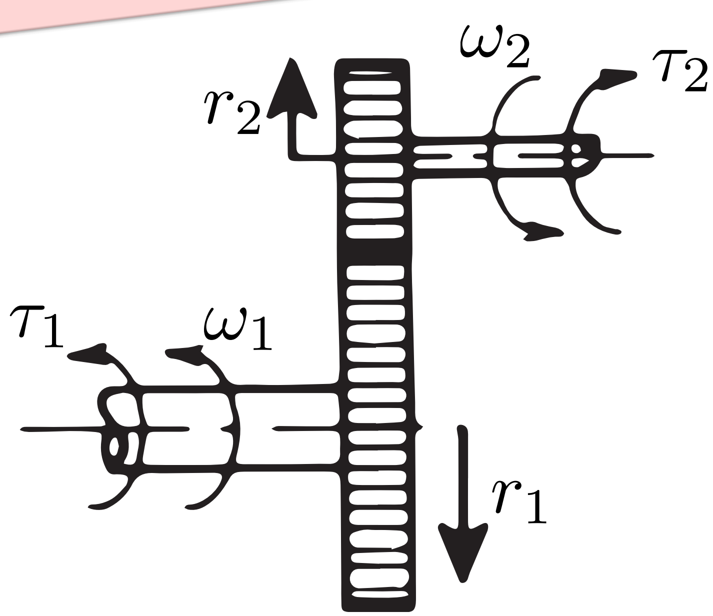
$N e_1 = e_2$
$f_1 = N f_2$

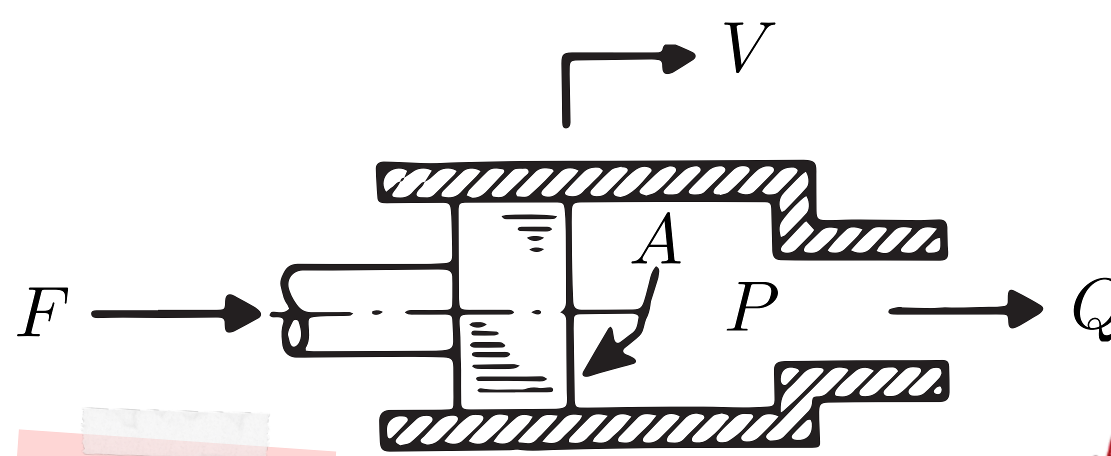


$(b/a)V_1 = V_2$
$F_1 = (b/a)F_2$



$(r_1/r_2)\omega_1 = \omega_2$
$\tau_1 = (r_1/r_2)\tau_2$



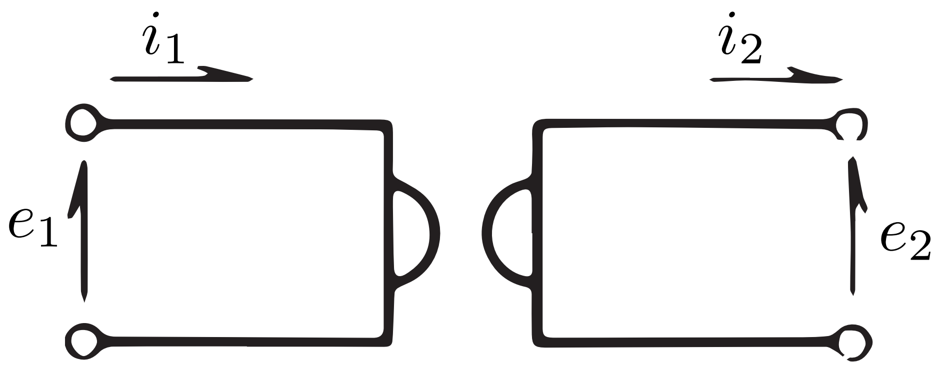


$F = AP$
$AV = Q$

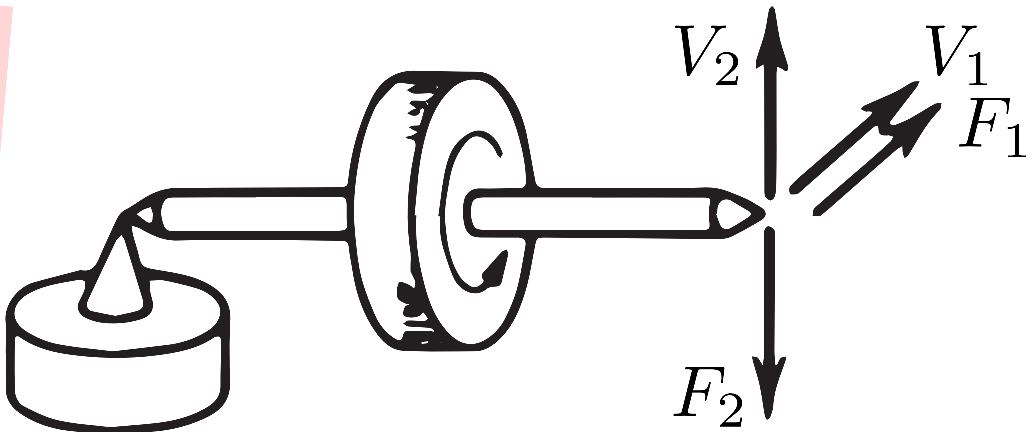
Basic 2-port elements

$$\xrightarrow[f_1]{e_1} GY \xrightarrow[f_2]{e_2}$$

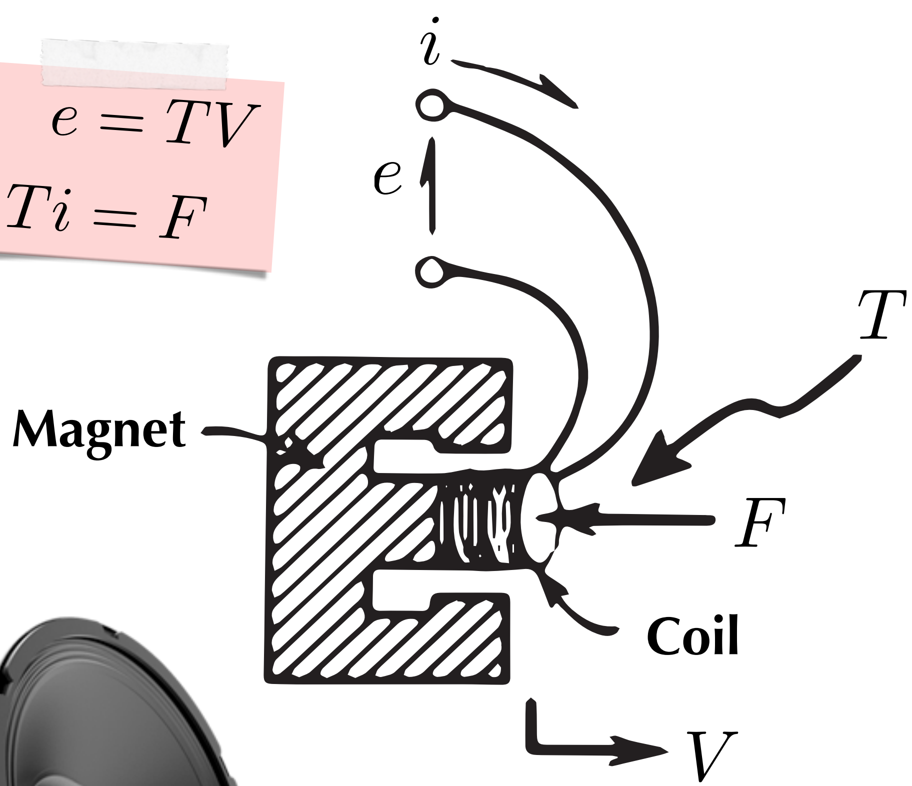
☑ Some examples of gyrator…



$$e_1 = N f_2$$
$$N f_1 = e_2$$



$$e = TV$$
$$Ti = F$$



If the rotor spins very rapidly, a push in the direction of F_1 will yield a proportional velocity V_2. Similarly, a force F_2 will result in a velocity V_1. The *counterintuitive behaviour of the gyroscope* is predicted by the gyroscope equations. For example, if the gravity force is in the direction of F_2, then the device precesses in a horizontal path.

Basic 2-port elements

- ☑ The gyrator seems to be a *mysterious element!!*
  - ✱ Before the significance of the gyrator was recognised, it was common to make equivalent electrical network diagrams for electromechanical or electro-hydraulic systems *using only transformers,* but this is *not correct!!*
  - ✱ A gyrator is a *more fundamental element* than a transformer, e.g. two gyrators cascaded are equivalent to a transformer

$$\xrightarrow[f_1]{e_1} GY \xrightarrow[f_2]{e_2} GY \xrightarrow[f_3]{e_3} \quad = \quad \xrightarrow[f_1]{e_1} TF \xrightarrow[f_3]{e_3}$$

  - ✱ It is also important to realise that the gyrator essentially interchanges the roles of effort and flow, so *it allows to map C-elements into I-elements*

$$\xrightarrow[f_2]{e_2} GY \xrightarrow[f_1]{e_1} I \quad = \quad \xrightarrow[f_2]{e_2} C$$

$$\xrightarrow[f_2]{e_2} GY \xrightarrow[f_1]{e_1} C \quad = \quad \xrightarrow[f_2]{e_2} I$$

- ☑ No need to have *two* different kinds of energy storage elements

Basic 2-port elements

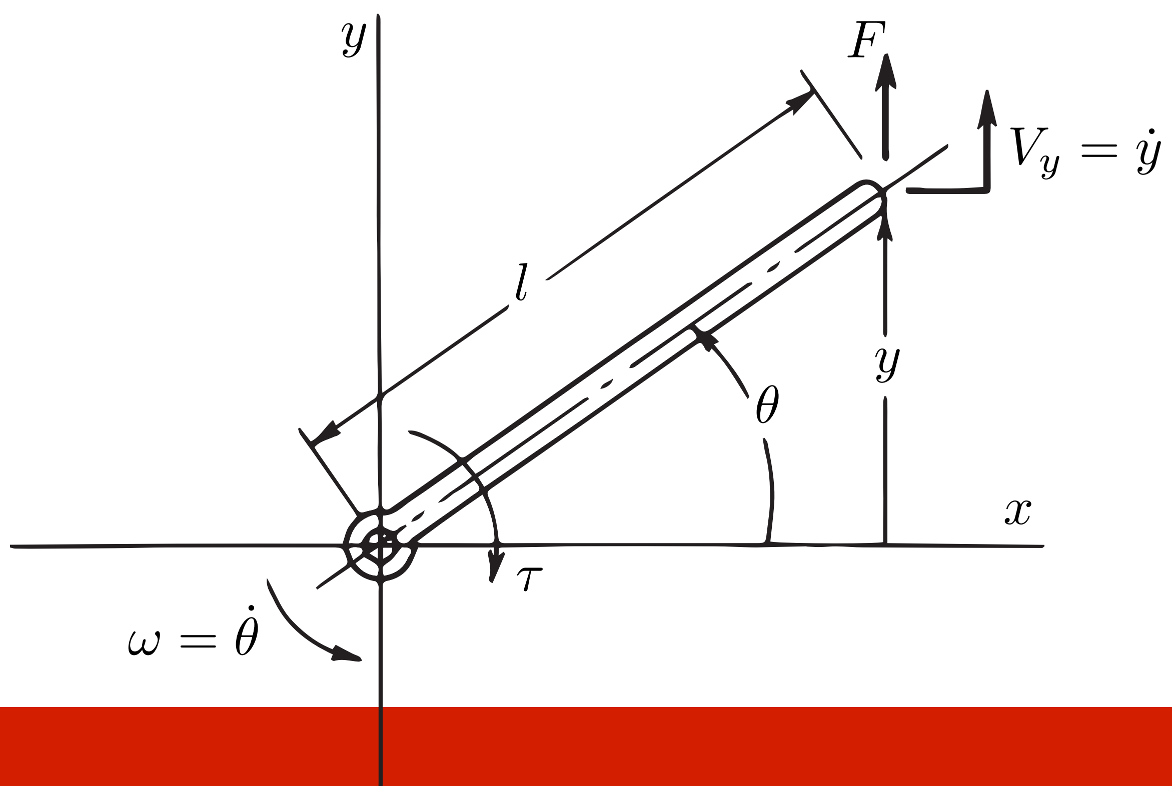
☑ The scaling ratio for a *TF* or a *GY* can be *time-dependent:*

$$\xrightarrow[f_1]{e_1} \overset{\displaystyle m \downarrow}{MTF} \xrightarrow[f_2]{e_2} \qquad \xrightarrow[f_1]{e_1} \overset{\displaystyle r \downarrow}{MGY} \xrightarrow[f_2]{e_2}$$

Modulated TF and GY

- Many physical devices may be modelled by the modulated 2-ports. For example, the *electrical autotransformer* contains a mechanical wiper, which, when moved, *alters the turns ratio* between the primary and secondary coils: this alteration takes no power

- In mechanics, the *MTF* is particularly important and may be used to represent *geometric transformations* or *kinematic linkages*



$$y = l \sin \theta$$

$$V_y = (l \cos \theta)\omega$$

$$(l \cos \theta) F = \tau$$

$$\xrightarrow[V_y]{F} \overset{\displaystyle l\cos\theta \downarrow}{MTF} \xrightarrow[\omega]{\tau}$$

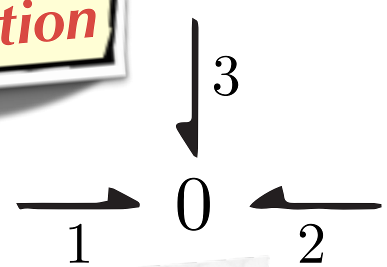
3-port junction elements

$$e_1 f_1 + e_2 f_2 + e_3 f_3 = 0$$

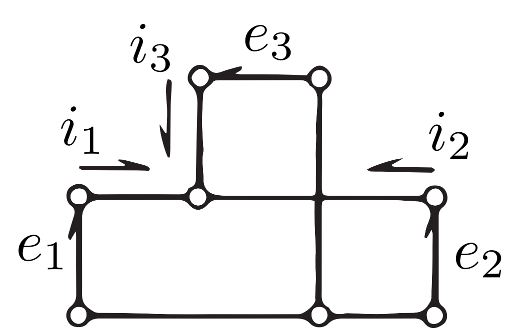
☑ These 3-port, power conserving elements serve to interconnect other multiports into subsystems or system models

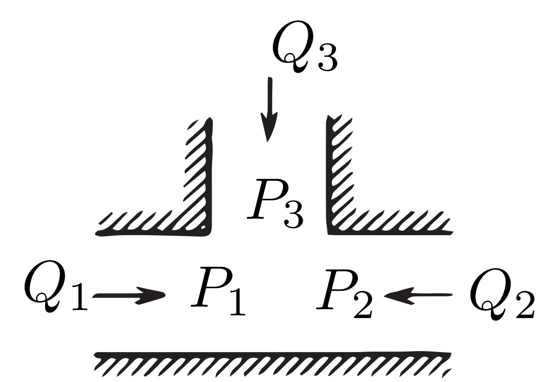
- They are the most fundamental ideas behind bond graph formalism, and…
- …an abstraction the *parallel* and *series connections* of electrical circuits
- They appear in *all* physical domain

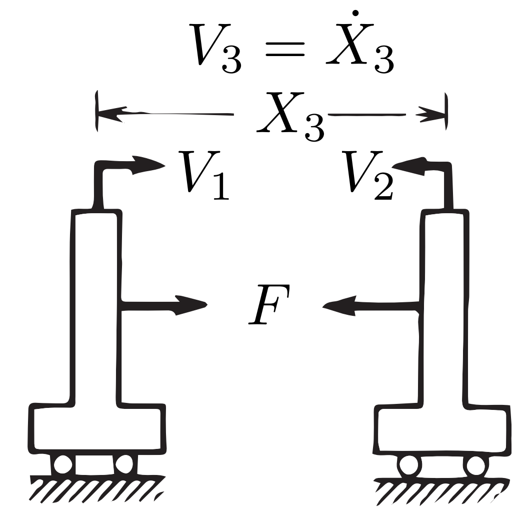
0-junction



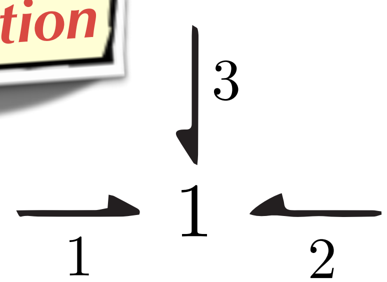
$$e_1(t) = e_2(t) = e_3(t)$$
$$f_1(t) + f_2(t) + f_3(t) = 0$$



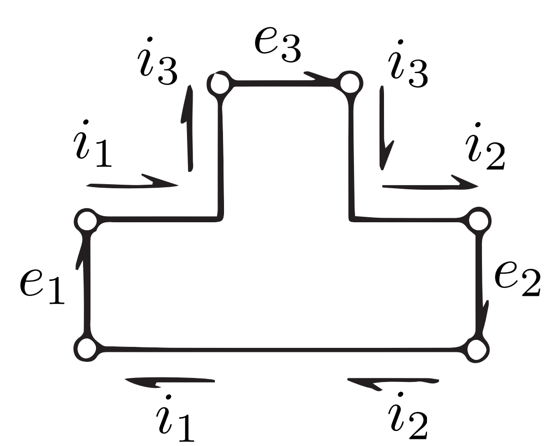


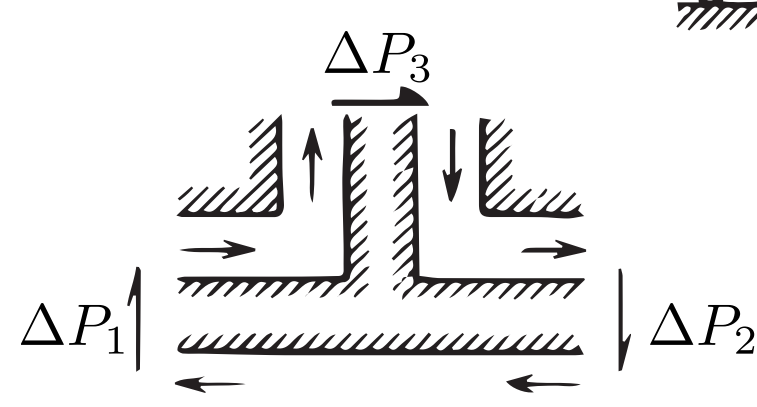


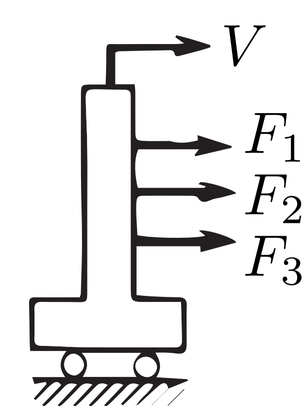
1-junction



$$e_1(t) + e_2(t) + e_3(t) = 0$$
$$f_1(t) = f_2(t) = f_3(t)$$







$$V = V_1 = V_2 = V_3$$

3-port junction eleme

Electrical circuits

* *0-junction:* Kirchhoff's current law for a node where three conductors join
* *1-junction:* Kirchhoff's voltage law written along a loop in which a current flows and experiences three voltage drops

Mechanical systems

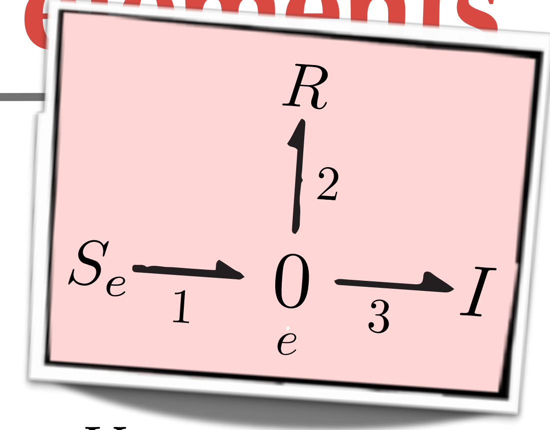
* *0-junction:* geometric compatibility for a situation involving a single force and three velocities that algebraically sum to zero
* *1-junction:* dynamic equilibrium of forces associated with a single velocity; when an inertial element is involved, the junction enforces Newton's law for the mass element

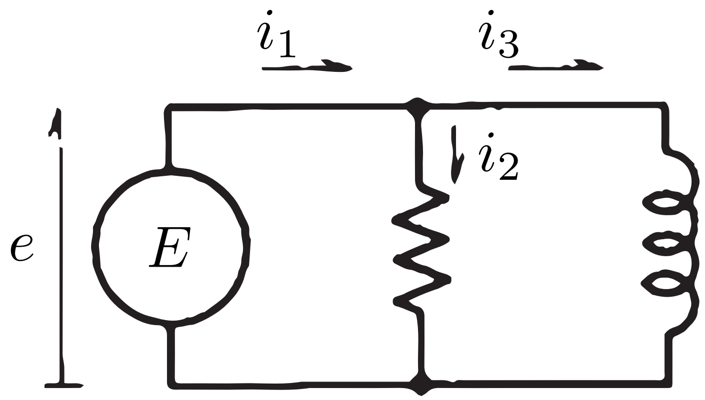
Hydraulic systems

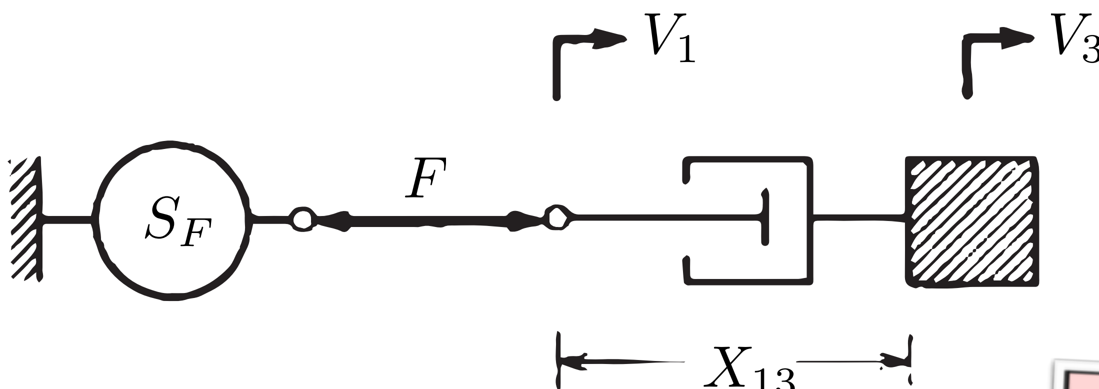
* *0-junction:* conservation of volume flow rate at a point where three pipes join
* *1-junction:* the sum of pressure drops around a circuit involving a single flow must sum algebraically to zero

3-port junction elements

- Some example systems involving 3-port junctions
  - *Systems using 0-junctions*

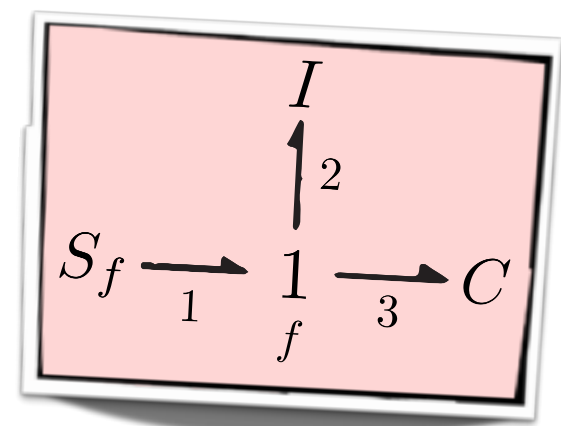


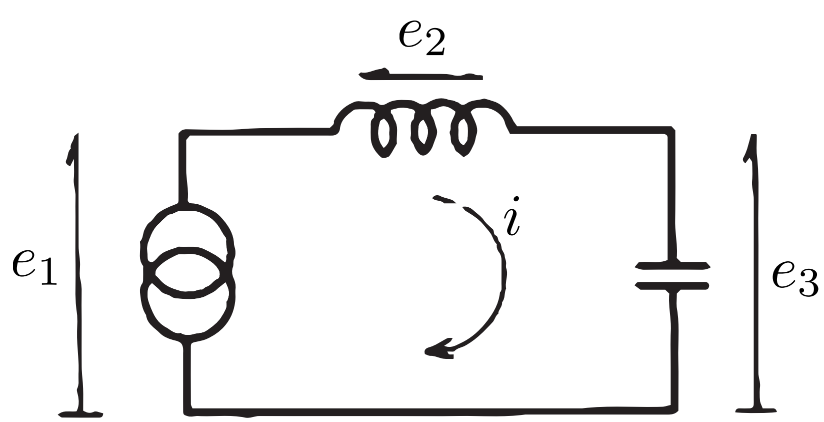


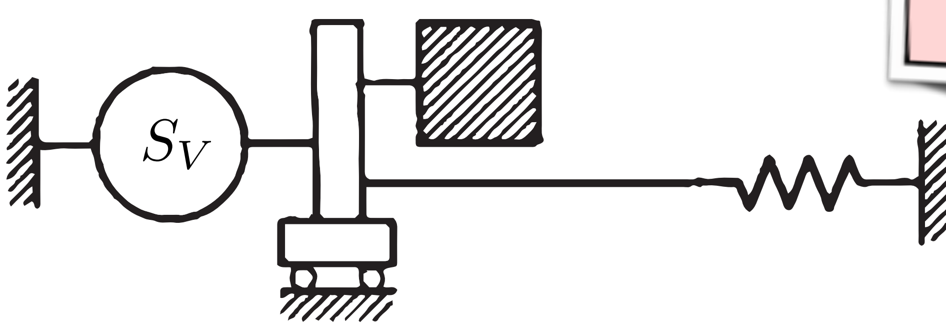


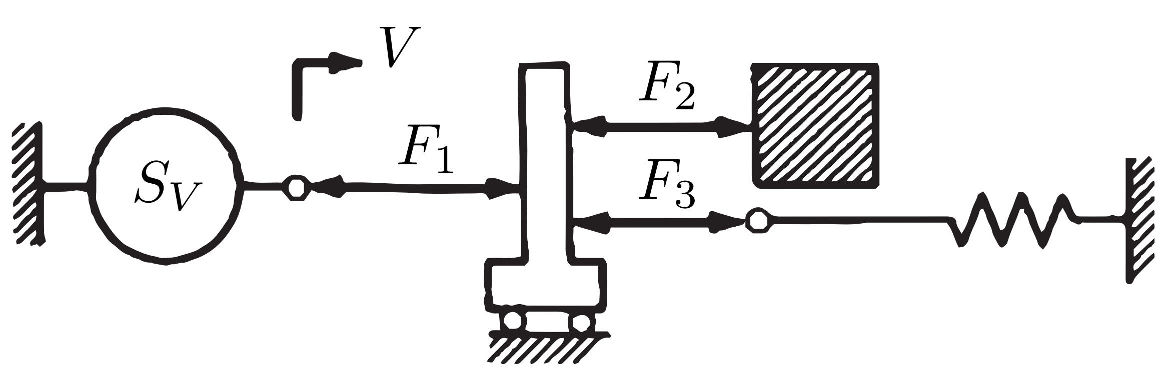
$$V_2 = -\dot{X}_{13} = V_1 - V_3$$

  - Systems using 1-junctions





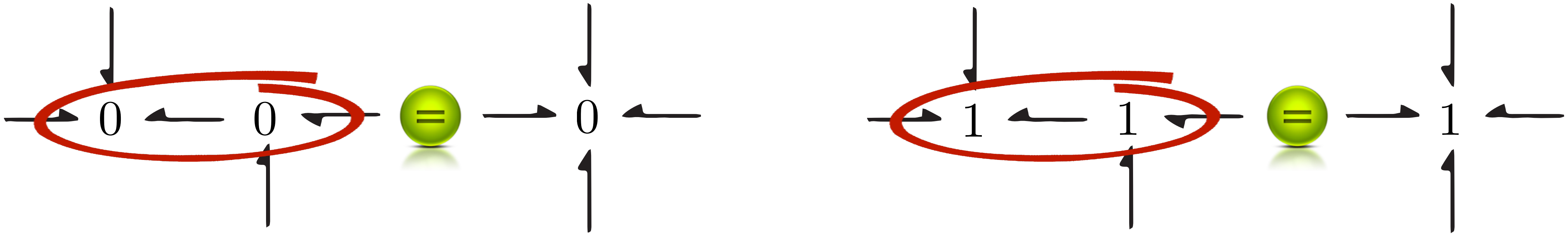




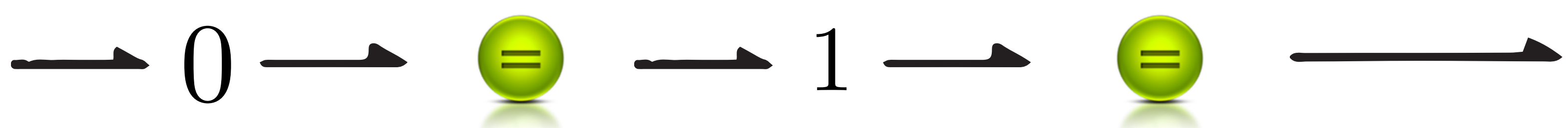
Note the *sign convention* for the junctions

3-port junction elements

- The generalisation of 3-port junction to *n-port* is quite immediate



- The following identities hold:



- This implies that, with some sign patterns, the 2-port 0- and 1-junctions serve to reverse the sign definition of an effort or flow

Causality for basic 1-ports

- For *effort* and *flow sources,* causality is trivial!
- The *1-port resistor* is normally *indifferent* to the causality imposed upon it, so there are two possibilities:

$$e = \Phi_R(f) \qquad\qquad f = \Phi_R^{-1}(e)$$

- The constitutive law of *1-port capacitor* is a static relation between effort and displacement, which means that

$$e = \Phi_C^{-1}\left(\int^t f \, \mathrm{d}\tau\right) \qquad\qquad f = \frac{\mathrm{d}}{\mathrm{d}t}\Phi_C(e)$$

integral causality

$$q = \int^t f \, \mathrm{d}\tau$$

- Dually, the constitutive law of *1-port inertia* is a static relation between flow and momentum

$$f = \Phi_I^{-1}\left(\int^t e \, \mathrm{d}\tau\right) \qquad\qquad e = \frac{\mathrm{d}}{\mathrm{d}t}\Phi_I(f)$$

integral causality

$$p = \int^t e \, \mathrm{d}\tau$$

Causality for basic 1-ports

- To summarise…

Element	Acausal Form	Causal Form	Causal Relation
Effort source	$S_e \rightharpoonup$	$S_e \rightharpoonup\!\!\mid$	$e(t) = E(t)$
Flow source	$S_f \rightharpoonup$	$S_f \mid\!\!\rightharpoonup$	$f(t) = F(t)$
Resistor	$R \leftharpoonup$	$R \leftharpoonup\!\!\mid$	$e = \Phi_R(f)$
		$R \mid\!\!\leftharpoonup$	$f = \Phi_R^{-1}(e)$
Capacitor	$C \leftharpoonup$	$C \leftharpoonup\!\!\mid$	$e = \Phi_C^{-1}\left(\int^t f\,dt\right)$
		$C \mid\!\!\leftharpoonup$	$f = \frac{d}{dt}\Phi_C(e)$
Inertia	$I \leftharpoonup$	$I \mid\!\!\leftharpoonup$	$f = \Phi_I^{-1}\left(\int^t e\,dt\right)$
		$I \leftharpoonup\!\!\mid$	$e = \frac{d}{dt}\Phi_I(f)$

Causality for basic 2- & 3-ports

- For basic *2-ports,* there are two possible causal assignments
  - For example, in a *TF,* as soon as one of the *e*'s or *f*'s has been assigned as an input, the other *e* or *f* is *constrained* by the constitutive relation
- Similar considerations are valid also for 3-ports *(junctions)*

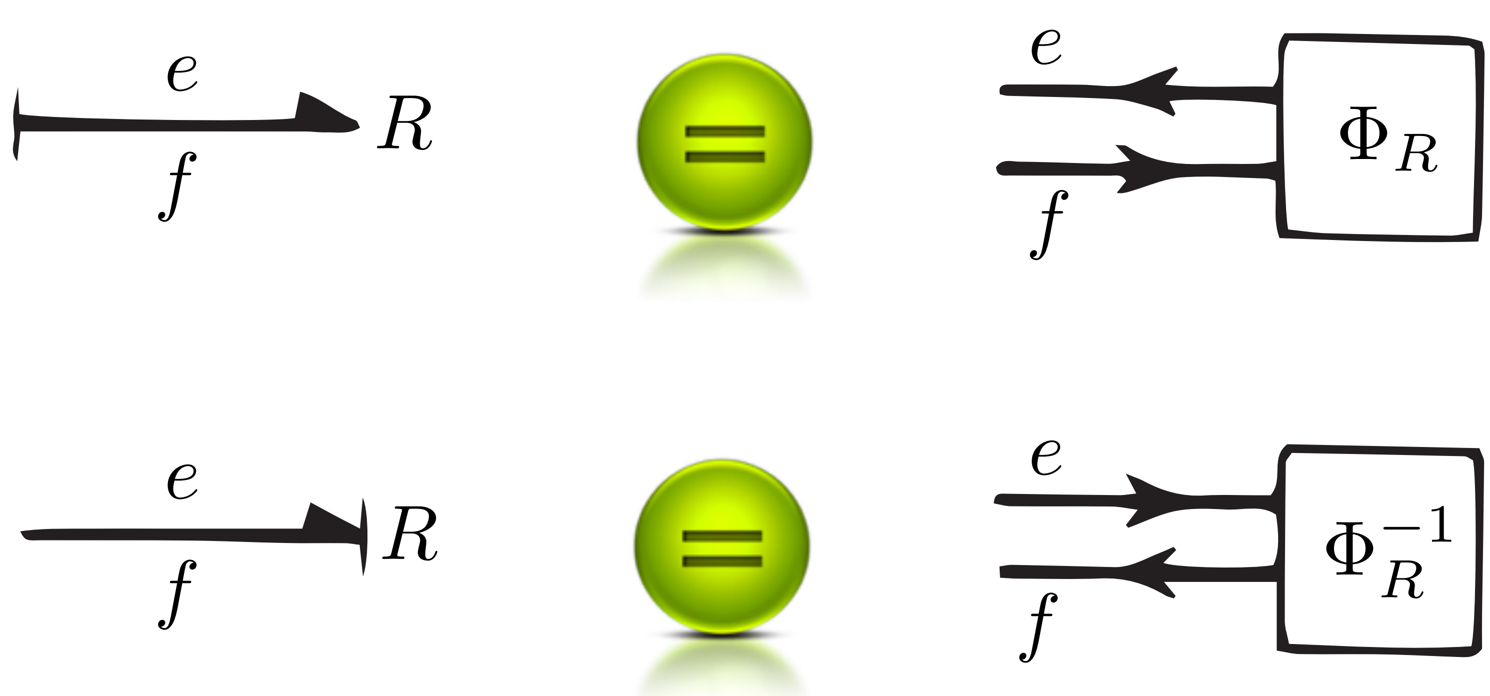
Element	Acausal Graph	Causal Graph	Causal Relations
Transformer	$\overset{1}{\rightharpoonup} TF \overset{2}{\rightharpoonup}$	$\overset{1}{\vdash\!\rightharpoonup} TF \overset{2}{\vdash\!\rightharpoonup}$	$e_1 = me_2$ $f_2 = mf_1$
		$\overset{1}{\rightharpoonup\!\dashv} TF \overset{2}{\rightharpoonup\!\dashv}$	$f_1 = f_2/m$ $e_2 = e_1/m$
Gyrator	$\overset{1}{\rightharpoonup} GY \overset{2}{\rightharpoonup}$	$\overset{1}{\vdash\!\rightharpoonup} GY \overset{2}{\rightharpoonup\!\dashv}$	$e_1 = rf_2$ $e_2 = rf_1$
		$\overset{1}{\rightharpoonup\!\dashv} GY \overset{2}{\vdash\!\rightharpoonup}$	$f_1 = e_2/r$ $f_2 = e_1/r$
0-Junction	$\overset{1}{\rightharpoonup} 0 \overset{2}{\rightharpoonup}$, $3 \upharpoonleft$	$\overset{1}{\rightharpoonup\!\dashv} 0 \overset{2}{\rightharpoonup\!\dashv}$, $3 \upharpoonleft\!\perp$	$e_2 = e_1$ $e_3 = e_1$ $f_1 = -(f_2 + f_3)$
1-Junction	$\overset{1}{\rightharpoonup} 1 \overset{2}{\rightharpoonup}$, $3 \upharpoonleft$	$\overset{1}{\vdash\!\rightharpoonup} 1 \overset{2}{\leftharpoonup\!\dashv}$, $3 \top\!\upharpoonleft$	$f_2 = f_1$ $f_3 = f_1$ $e_1 = -(e_2 + e_3)$

Some remarks on causality

- Although the causal considerations have been stated for all the basic multiports defined so far, it can hardly be clear as to what all the *implications of causality* are
- The study of causality is very important, and *bond graphs are uniquely suited to this study*
  - Only when some real system models have been assembled is it clear why causal information is so important
- Using the rules of causality, it is possible to *predict many important features of these systems* even before the constitutive laws for all the elements have been decided upon
  - Predict the *order of the model* before any equations are written
  - Support for writing the *state equations* or in setting up a ***system simulation***

Causality and block diagrams

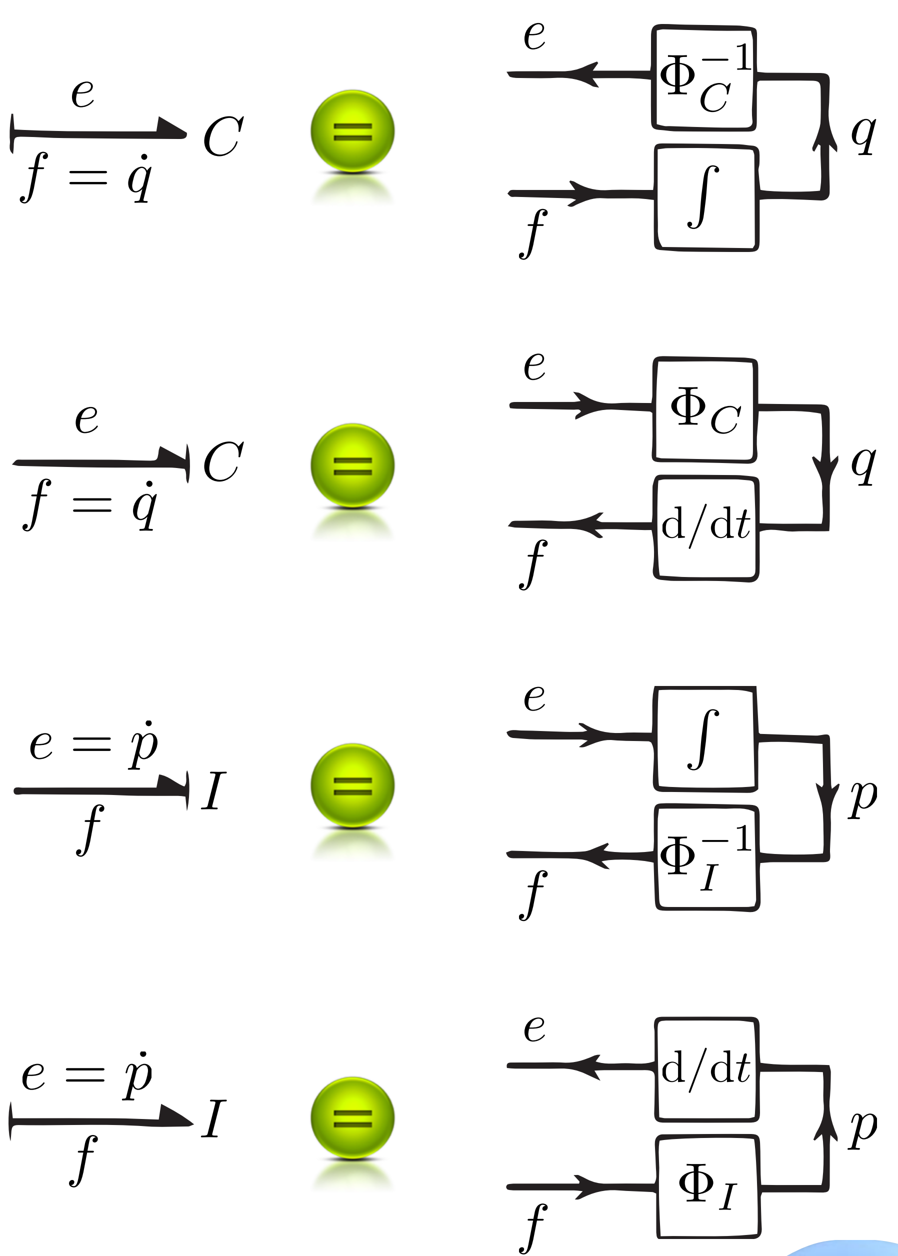
- *Block diagrams* indicate how input and output signals flow
  - The blocks show how the signals are *transformed*
  - Block diagrams are *inherently causal* since they show how an input signal is transformed into an output signal
  - *When causal strokes are added* to a bond graph, one may represent the information in the bond graph by a block diagram
  - Block diagrams are more complex graphically than bond graphs because *a single bond implies two signal flows* on a block diagram

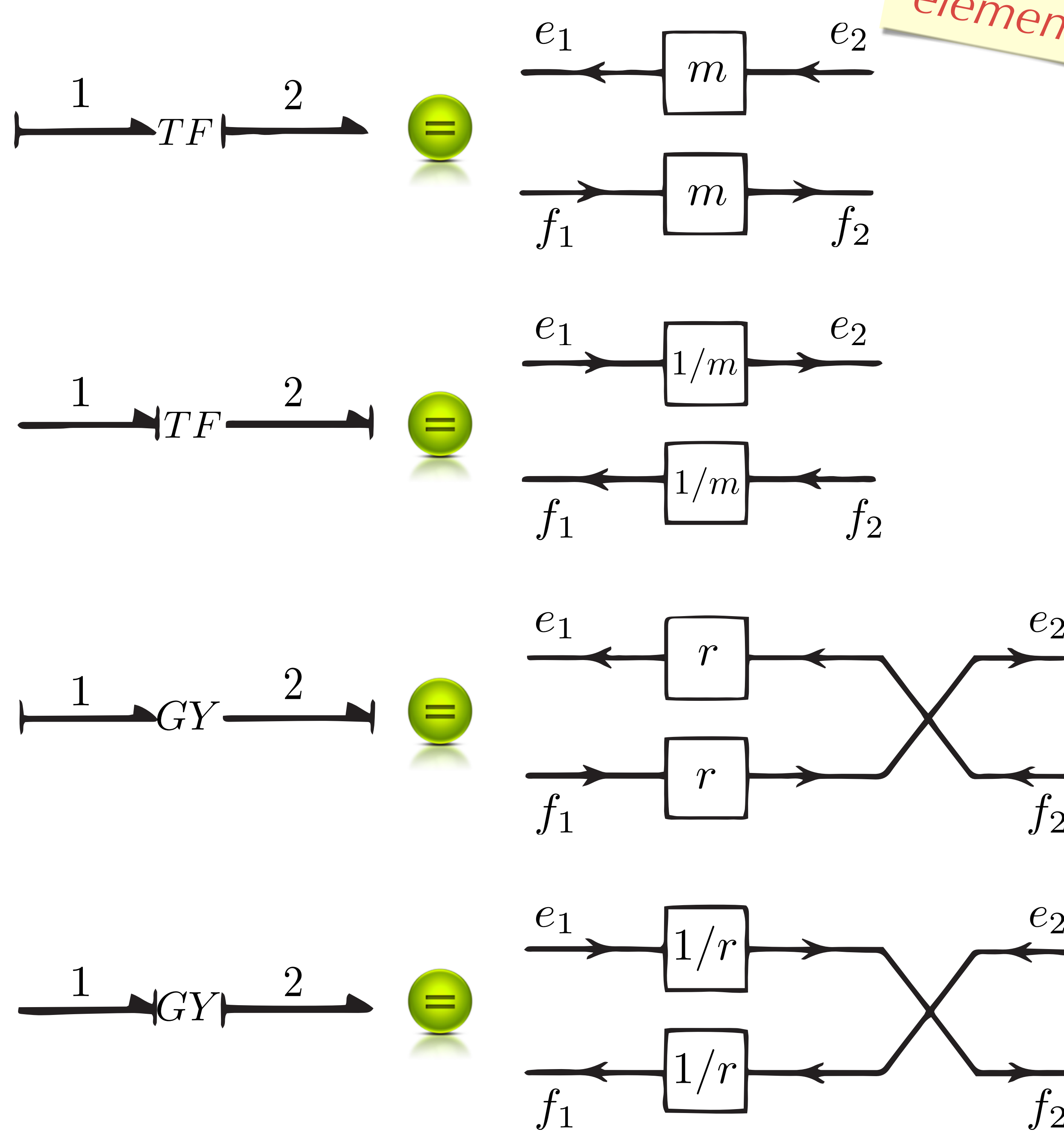


1-port resistor

Causality and block diagrams

2-port elements

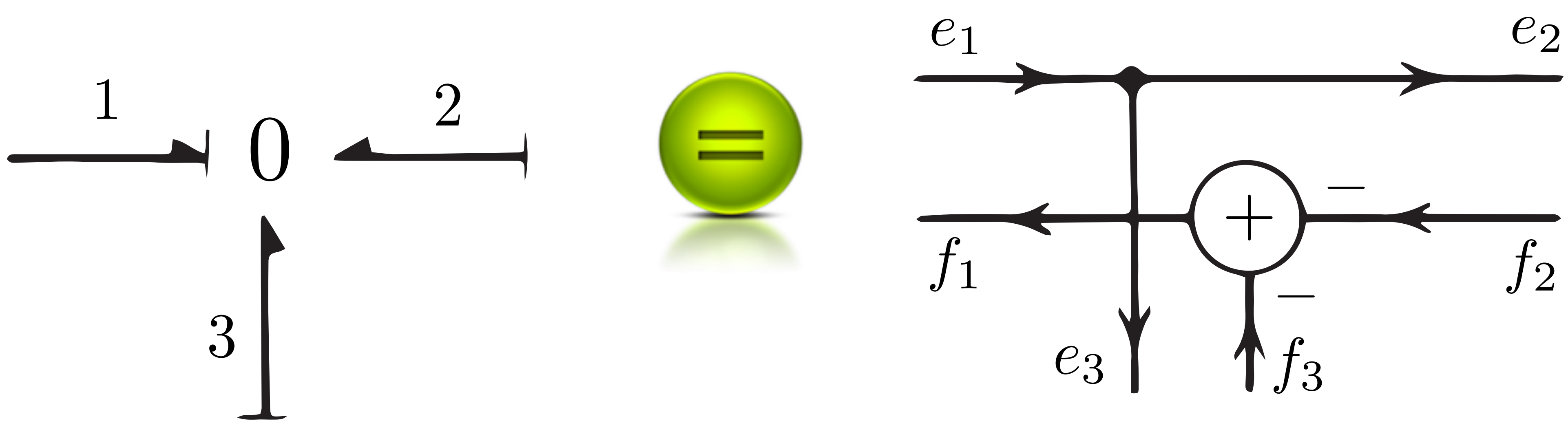


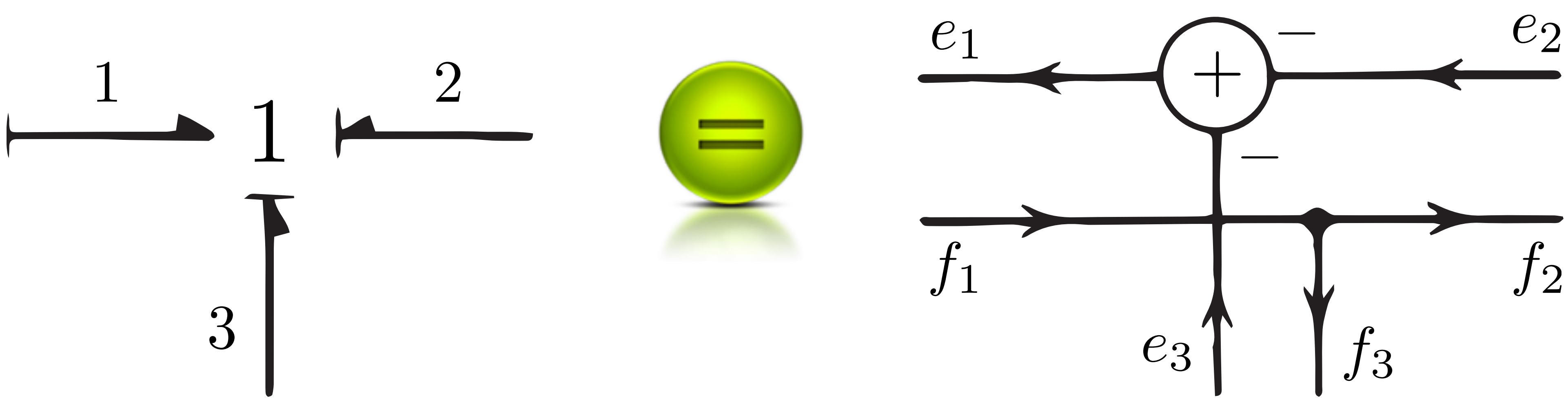


1-port
capacitor & inertia

Causality and block diagrams

3-port junctions



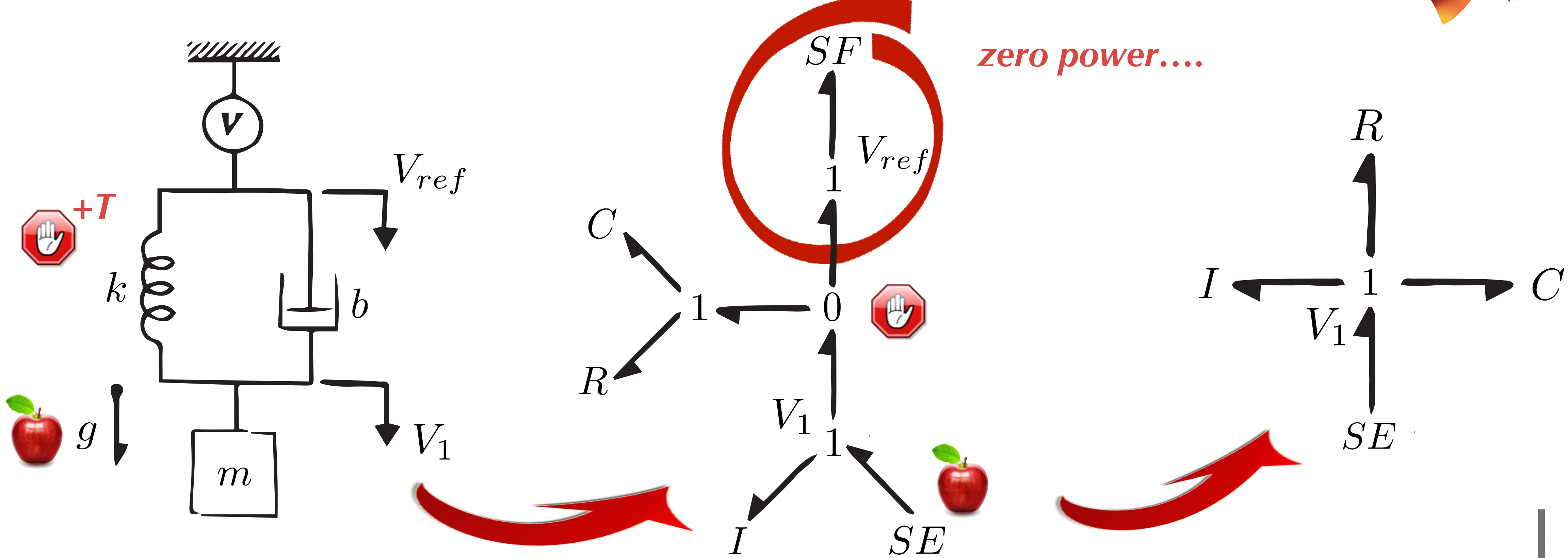


Mechanical systems

- *Mechanical systems* are those composed of such components as masses, springs, dampers, levers, flywheels, gears, shafts, and so forth
- When dynamic systems are put together from these components, we must *interconnect* rotating and translating inertial elements with axial and rotational springs and dampers, and we must appropriately account for the *kinematics* of the systems
- Main topics:
  - Mechanical *translation*
  - Mechanical systems that *rotate*
  - Dynamics of *plane motion*

Mechanics of translation

- Construction procedure for electrical systems:
  - Identify the important node voltages and represent them with *0-junctions*
  - Create the appropriate voltage drops with *1-junctions*
  - Select the *reference* voltage (ground)
- For mechanical systems, the procedure is *dual*
  - Representing system *velocities* using 1-junctions
  - Create the appropriate *relative velocities* using 0-junctions



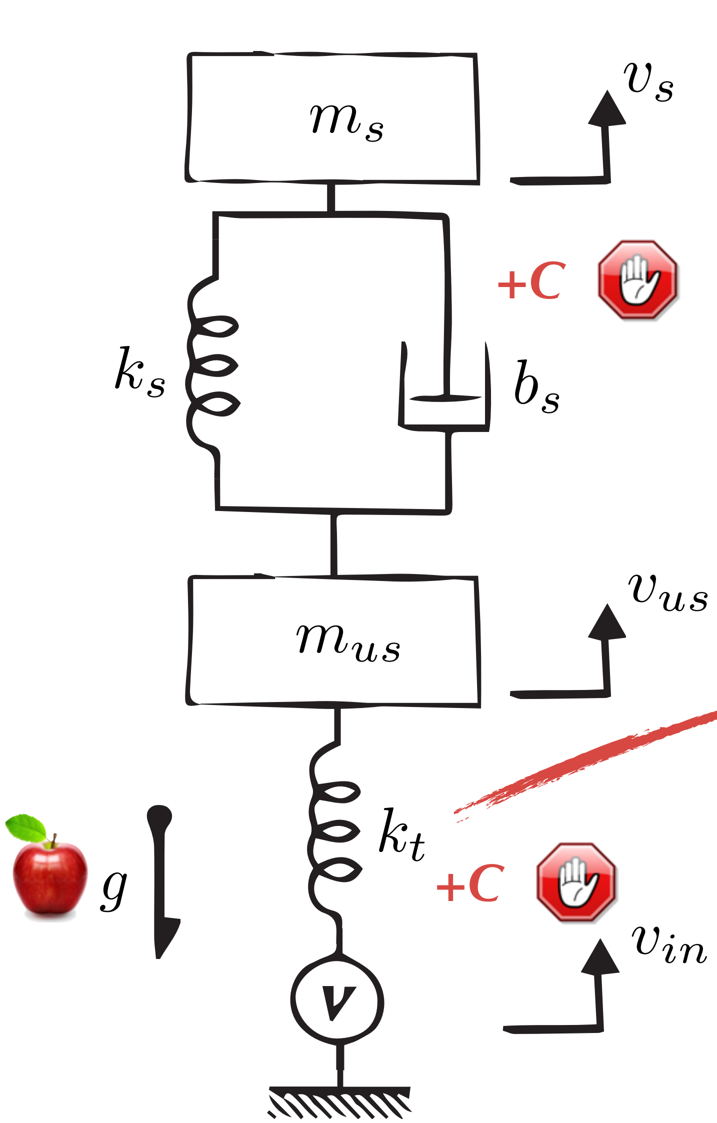
Mechanics of translation

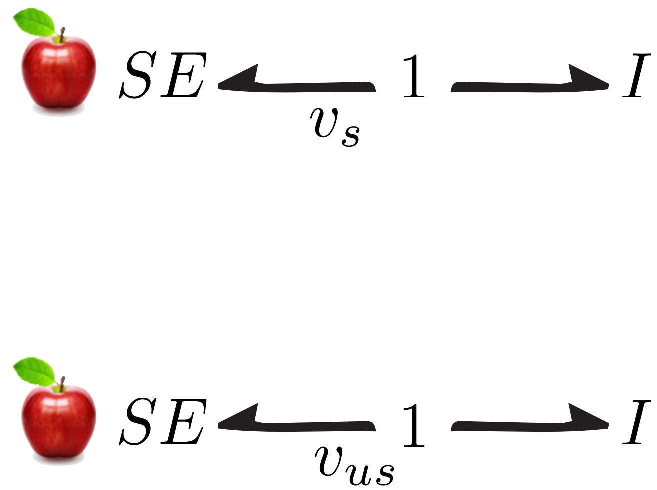
- The *spring* and *damper* react to the *relative velocity* across them
- The velocities on the *C*- and the *R*-elements bonds represent the *rate of extension* of these elements
  - When forces are *positive in tension,* power will flow toward the elements

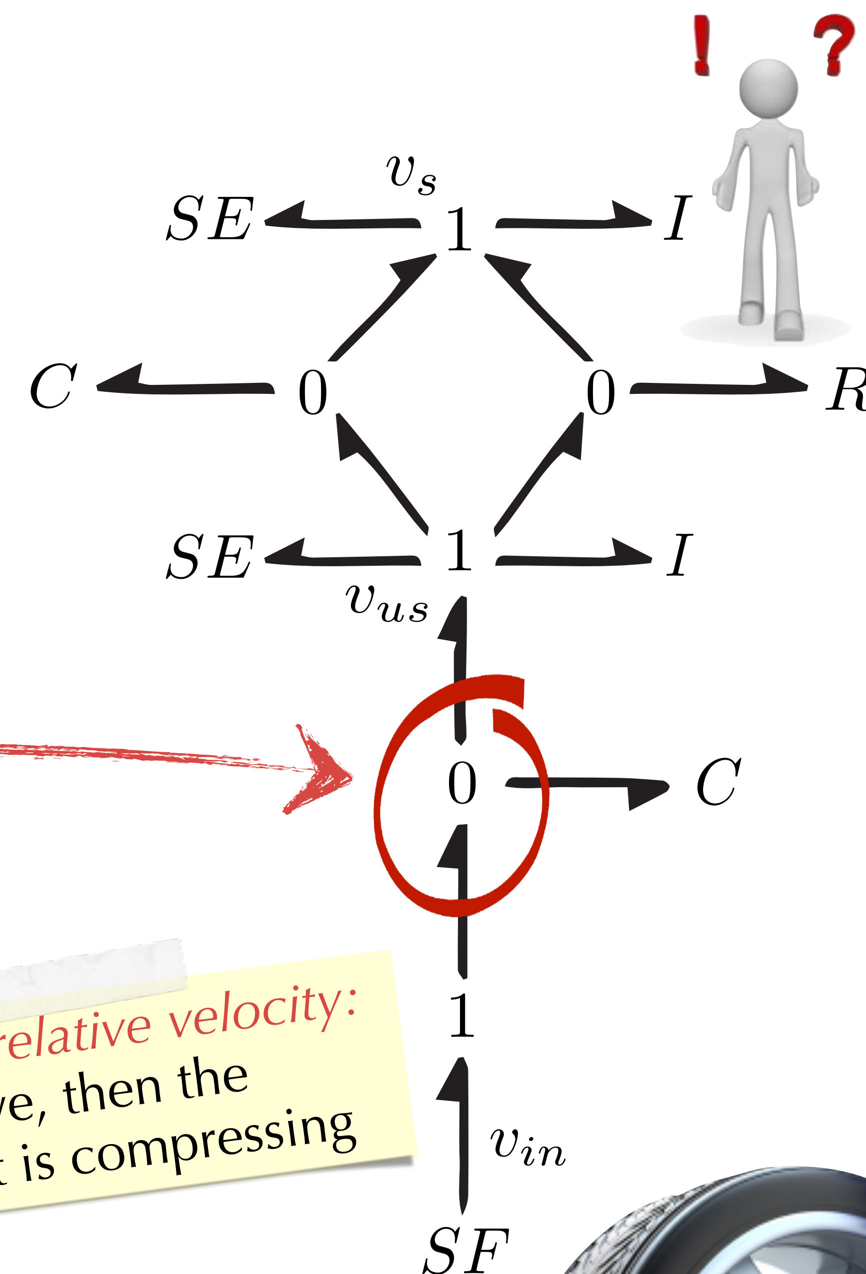
Construction procedure for Mechanical Translation

- Use arrows and symbols to indicate the *positive direction* of *absolute velocity* components. State whether *force-generating elements* are *positive* in tension or compression using symbols such as +*T* or +C
- Use *1-junctions* to represent each *distinct velocity*
- Attach to each 1-junction any element that relates to the absolute velocity represented by the junction, usually *inertias*
- Use *0-junctions* to establish *relative velocities,* with sign-convention so that the connecting elements are positive in compression or tension
- Eliminate the bonds with zero velocity

☑ Example: the *quarter car* model







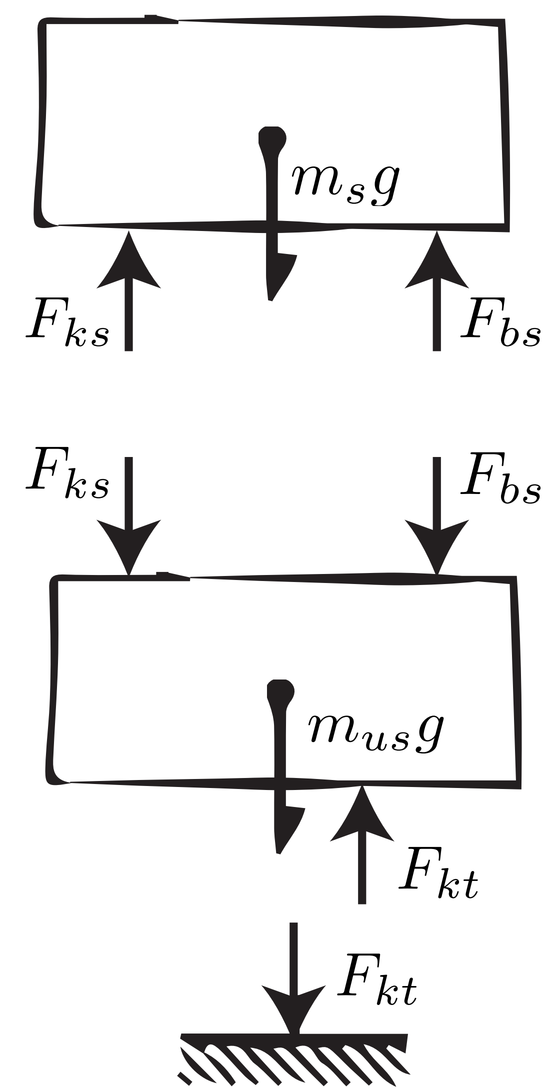
Correct *relative velocity*: if positive, then the element is compressing

m_s is the *sprung mass* (one-quarter of the body mass)
m_{us} is the *unsprung mass* (tire, wheel, and some part of the brakes and suspension)
k_s, b_s are the *suspension spring* and *damper* constants
k_t is the *tire stiffness*

Mechanics of translation

- We paid attention *only* to establishing the *proper velocity components,* and *no attention to the forces*
  - The beauty of *power conservation in junctions* is that we need only to constrain the velocities, and the forces will *automatically* be balanced

- Consider the free-body diagram for the quarter-car
  - Masses are isolated
  - Forces (springs, dampers, and gravity) are exposed

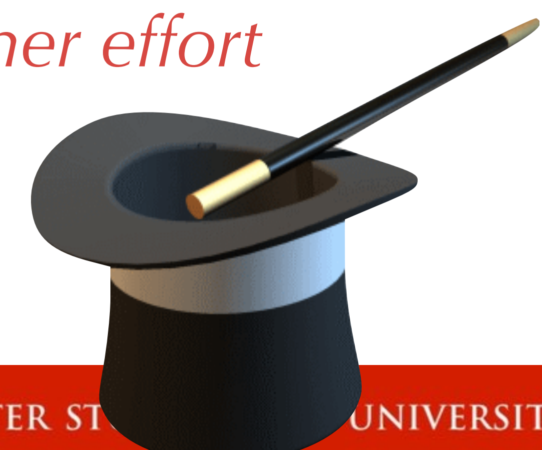


- *Newton's law* for the masses:

$$F_s = F_{ks} + F_{bs} - m_s g$$

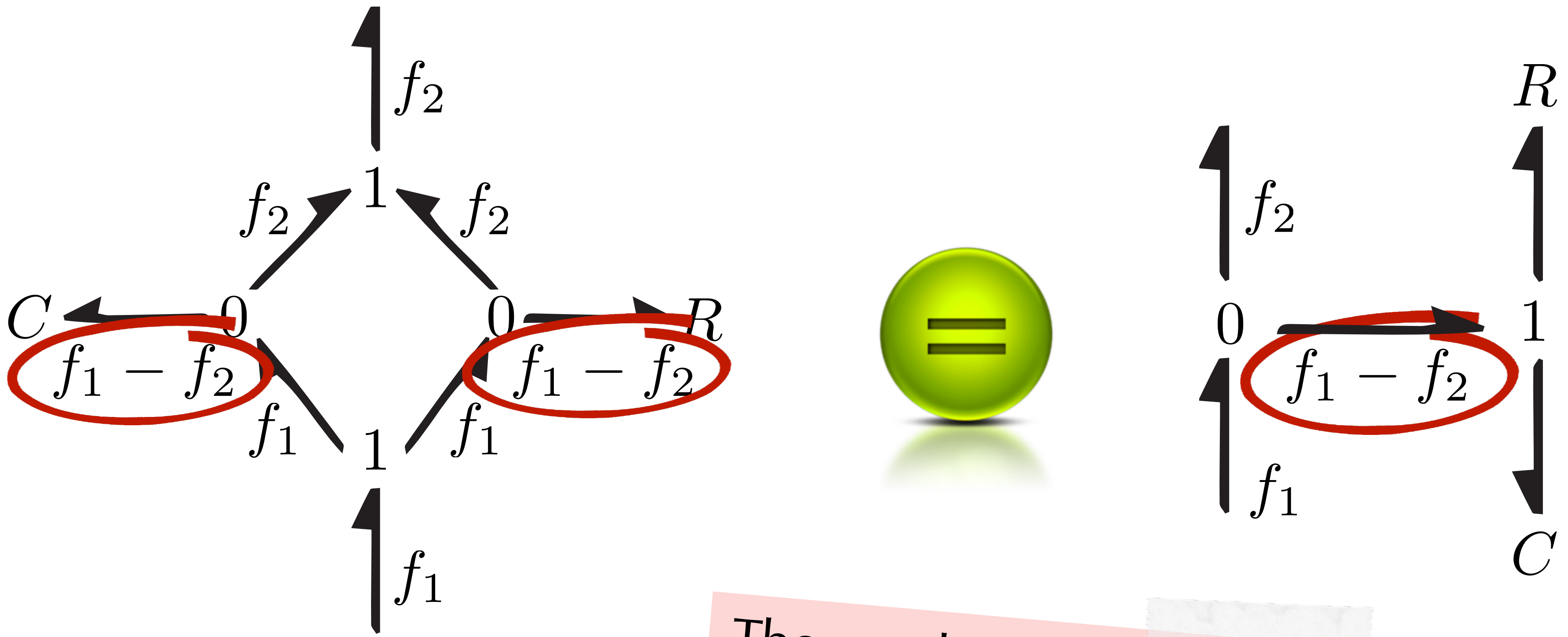
$$F_{us} = F_{kt} - F_{ks} - F_{vs} - m_{us} g$$

- Note that the *force balance* is realised by 1-junctions!!
  - The forces were constrained *without further effort* after enforcing the velocity constraints
  - No *explicit* action-reaction principle



Mechanics of translation

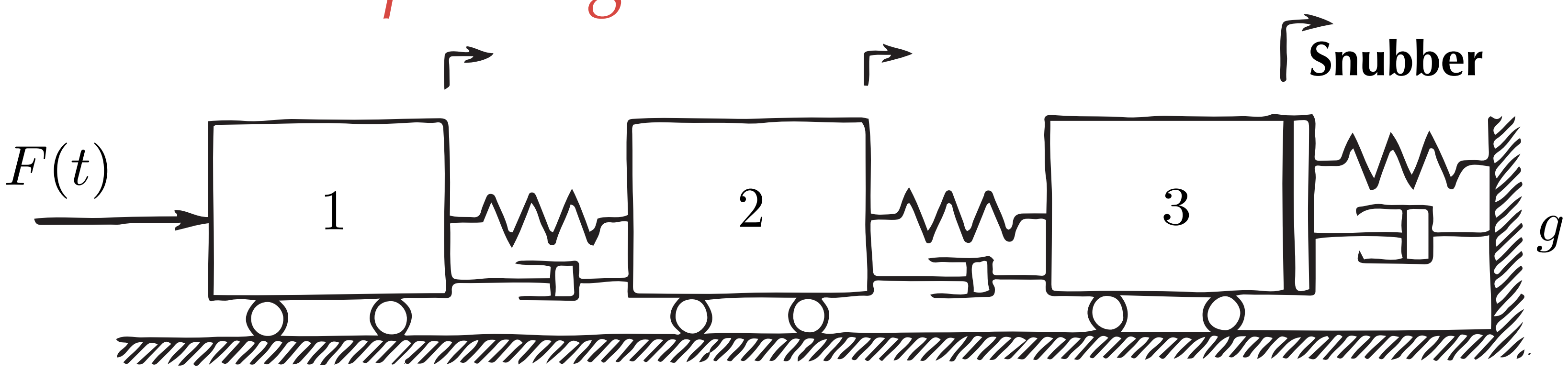
- The construction procedure requires to determine a relative velocity for both the spring and the damper using 0-junctions
  - The result is correct and the very common resulting bond graph structure is what is called a *reducible loop*
  - You saw it already for an electrical system
  - The structure can be *simplified*

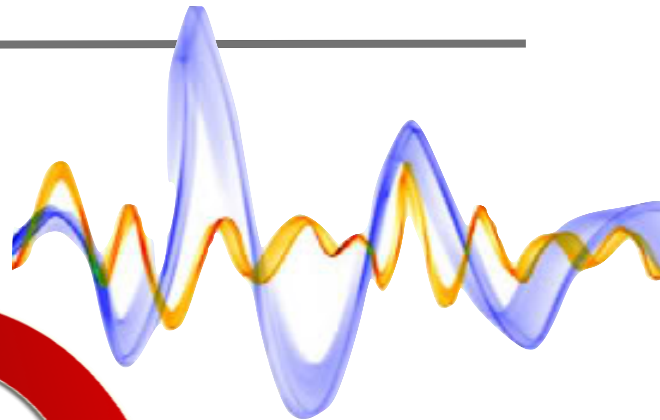


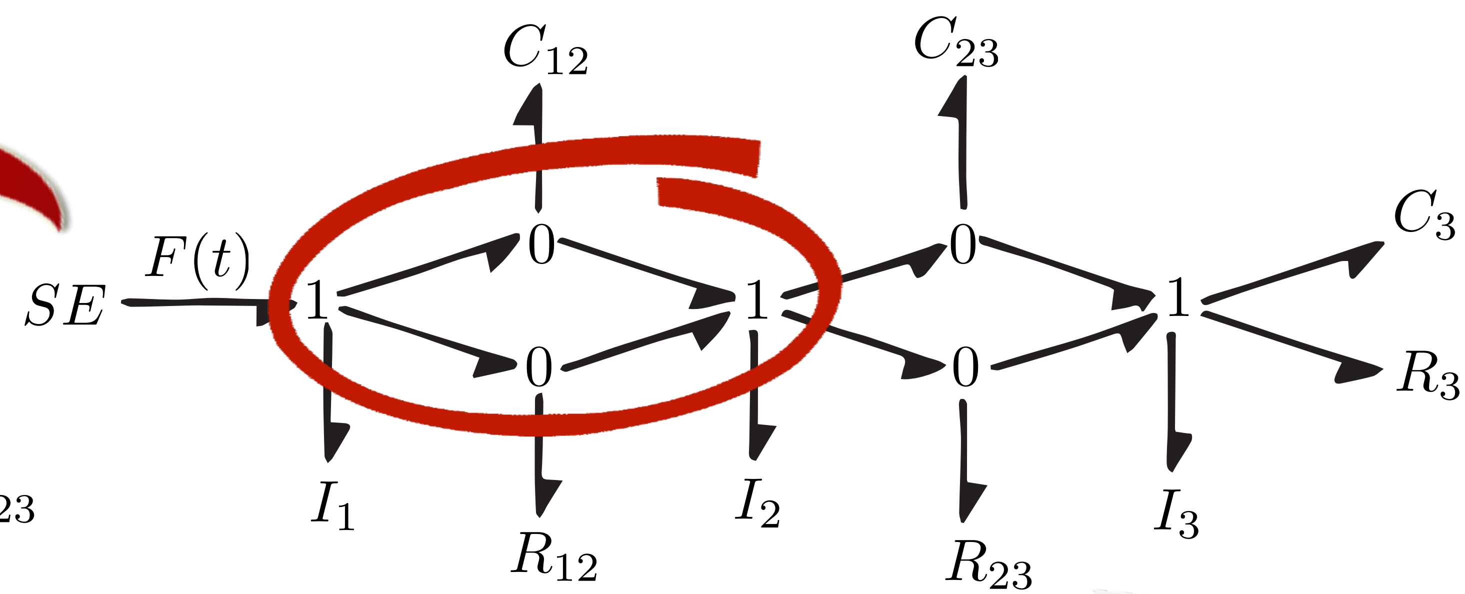
The result is a slightly simpler bond graph emphasising that *several elements have exactly the same relative velocity*

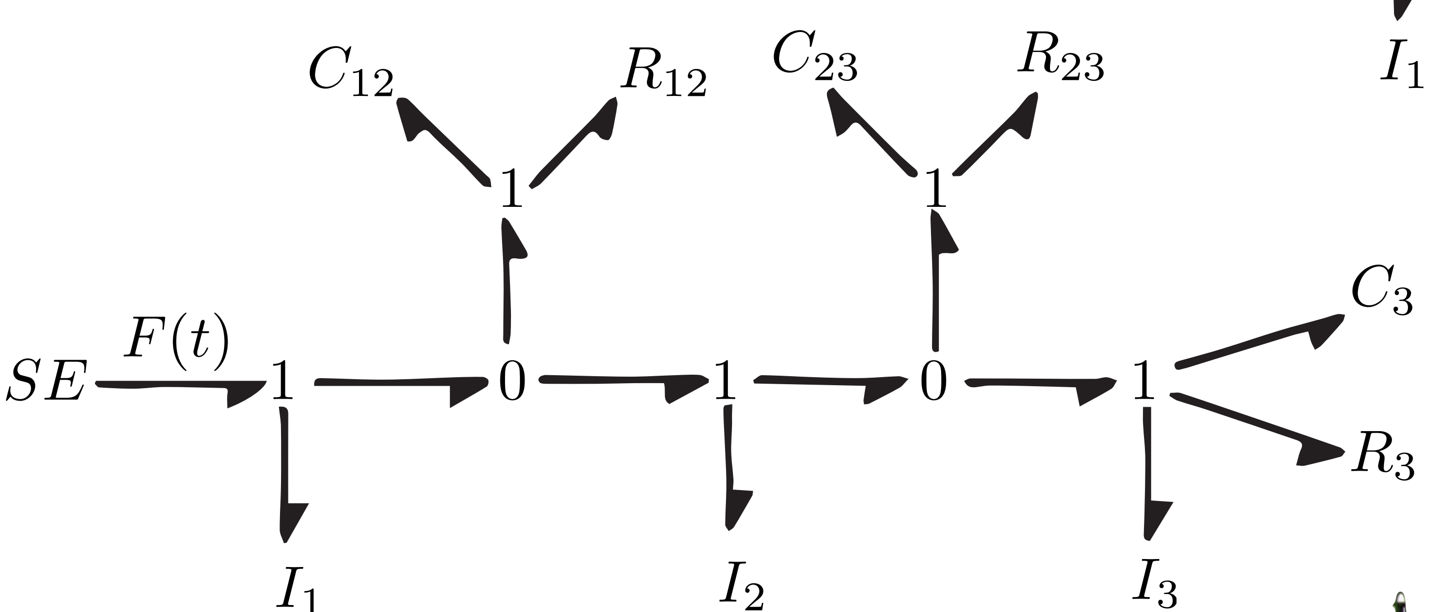
Mechanics of translation

☑ Model of *impacting train*

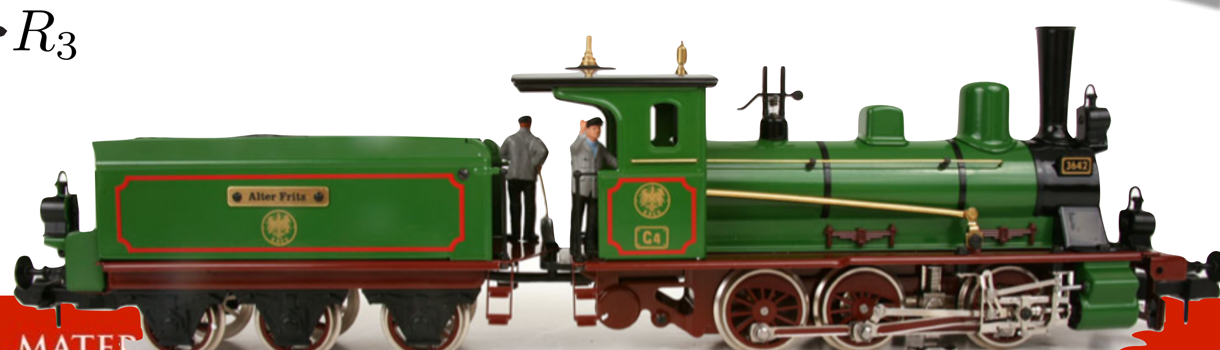






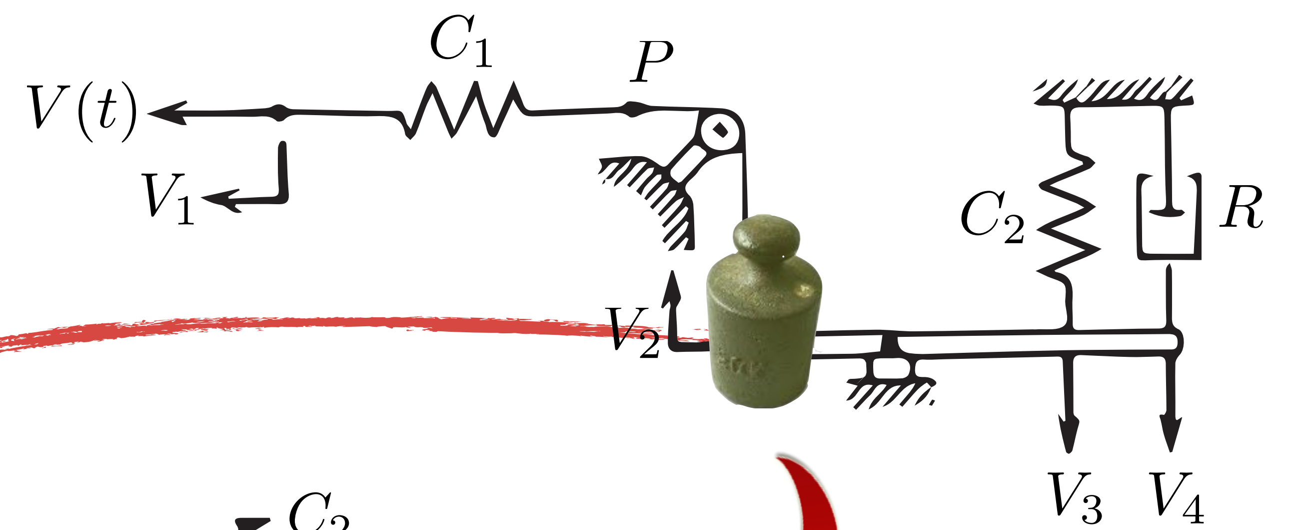


A *zero source* flow has been omitted…

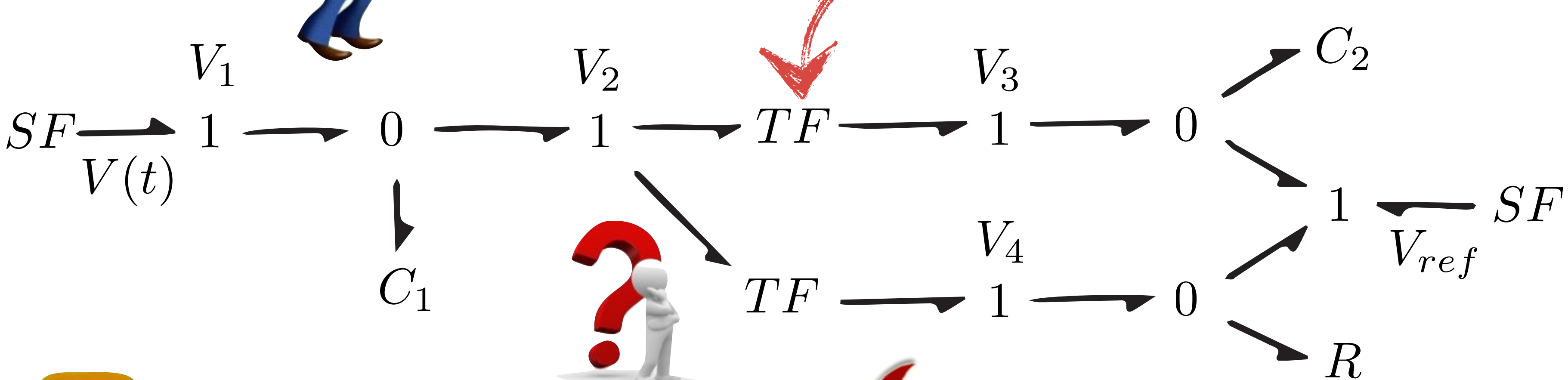


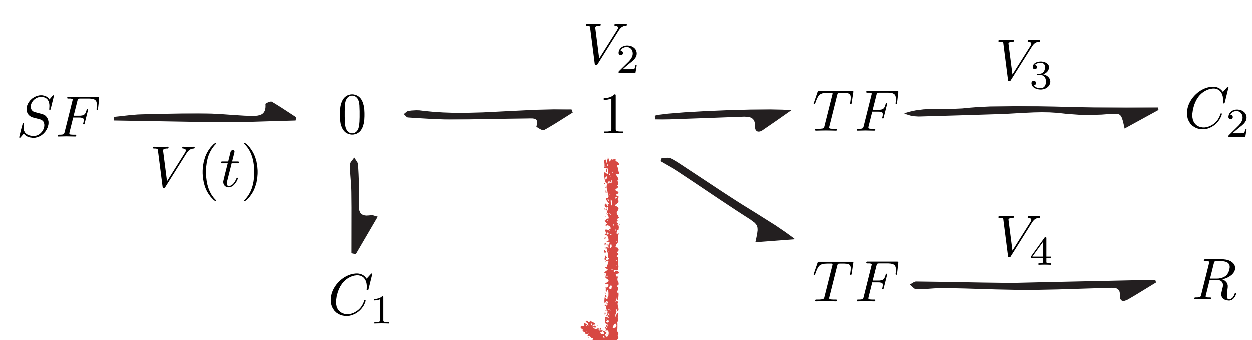
Mechanics of translation

- With minor efforts, it is possible to deal with translational systems containing *levers, pulleys,* and other simple *motion–force transforming devices*



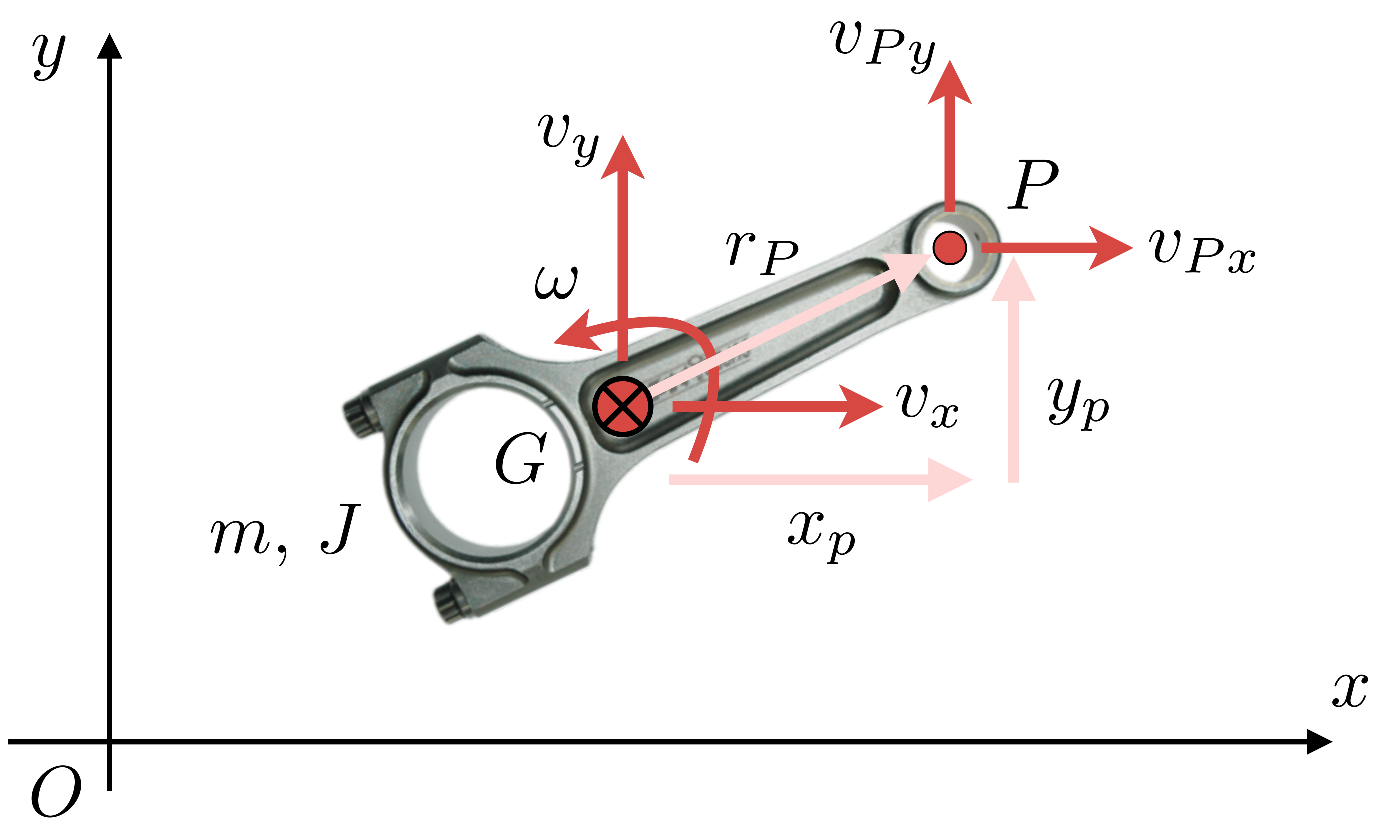
It is easy too add a mass…

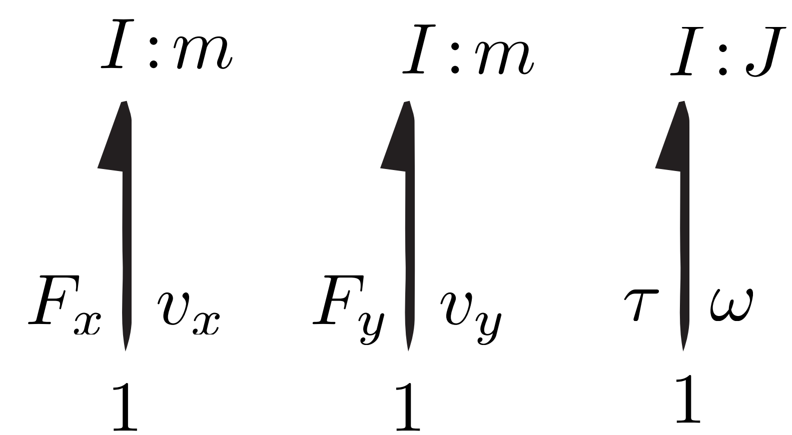




Plane motion

- Mechanical bodies with finite mass both translate and rotate in real applications, i.e. the dynamics are governed by a simultaneous combination of *translation and rotation*
  - 3-D motion is not trivial, *plane motion* is much simpler
  - Plane motion results when the inertial bodies of a physical system are constrained to *translate in two dimensions* and to *rotate* only about an axis perpendicular to the plane of motion





$$T = \frac{1}{2}\left(\frac{p_x^2}{m} + \frac{p_y^2}{m} + \frac{p_\theta^2}{J}\right)$$
$$= \frac{1}{2}\left(m v_x^2 + m v_y^2 + J\omega^2\right)$$

- Devices are *attached* to rigid bodies at different places on the body

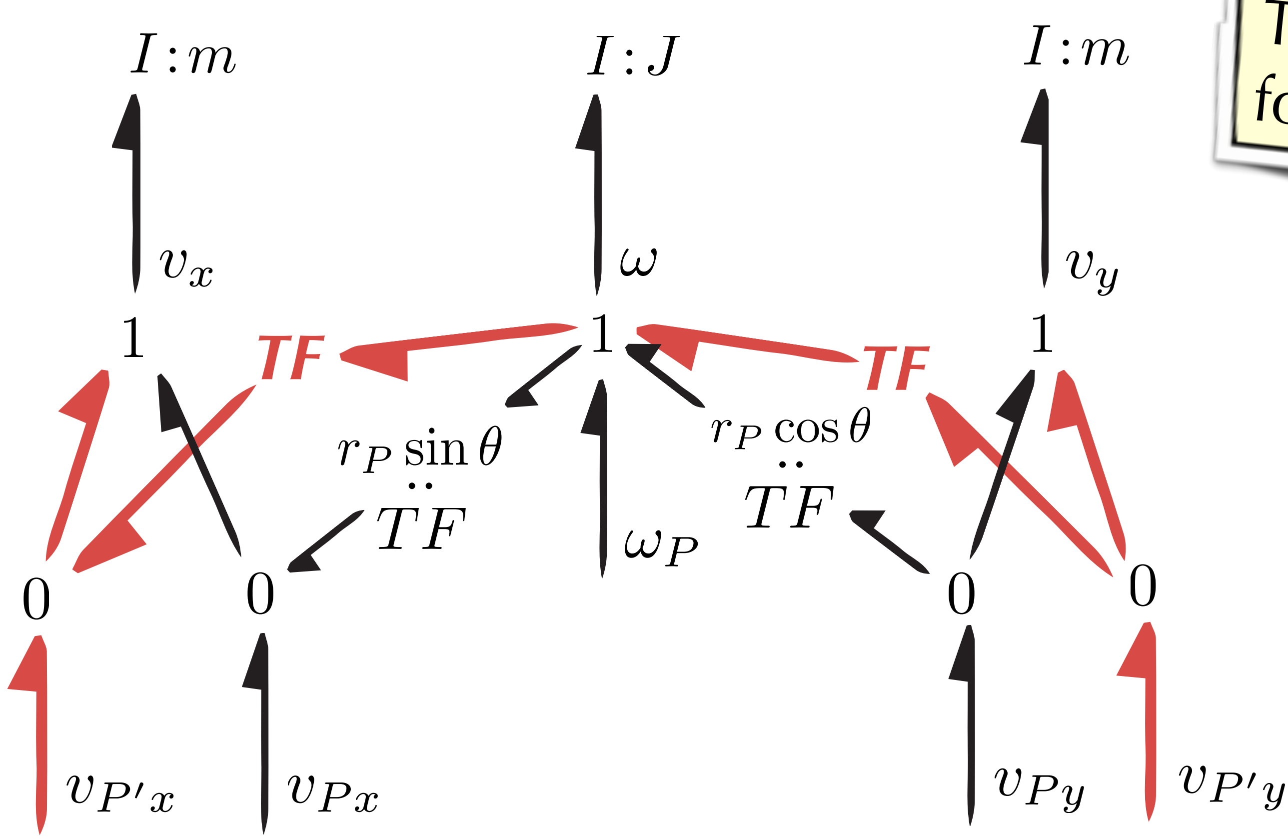
$$v_P = v_G + \omega \times r_P$$

$$\dot{\theta} = \omega$$

$$v_{Px} = v_x - y_p \omega \qquad v_{Py} = v_y + x_p \omega$$

$$v_{Px} = v_x - (r_p \sin\theta)\omega \qquad v_{Py} = v_y + (r_p \cos\theta)\omega$$

- Associated *bond graph*



The procedure is *scalable* for *multiple points*

…but, what about the *forces / torques??*

- *The forces / torques are transformed "dually"* by construction

$$F_x v_x + F_y v_y + \tau\omega = F_{Px} v_{Px} + F_{Py} v_{Py} + \tau_P \omega_P$$

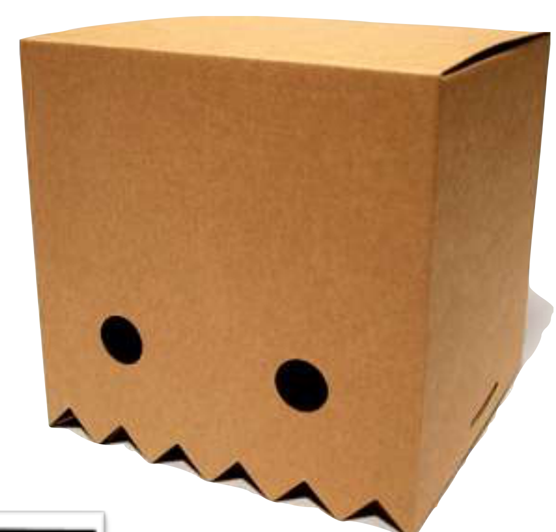
power conservation

$$v_{Px} = v_x - (r_P \sin\theta)\omega$$
$$v_{Py} = v_y + (r_P \cos\theta)\omega$$
$$\omega_P = \omega$$

$$F_x = F_{Px}$$
$$F_y = F_{Py}$$
$$\tau = \tau_P - (r_P \sin\theta) F_{Px} + (r_P \cos\theta) F_{Py}$$

- The result is an "object" with a *force / velocity interface* towards the environment, i.e. what is outside the system

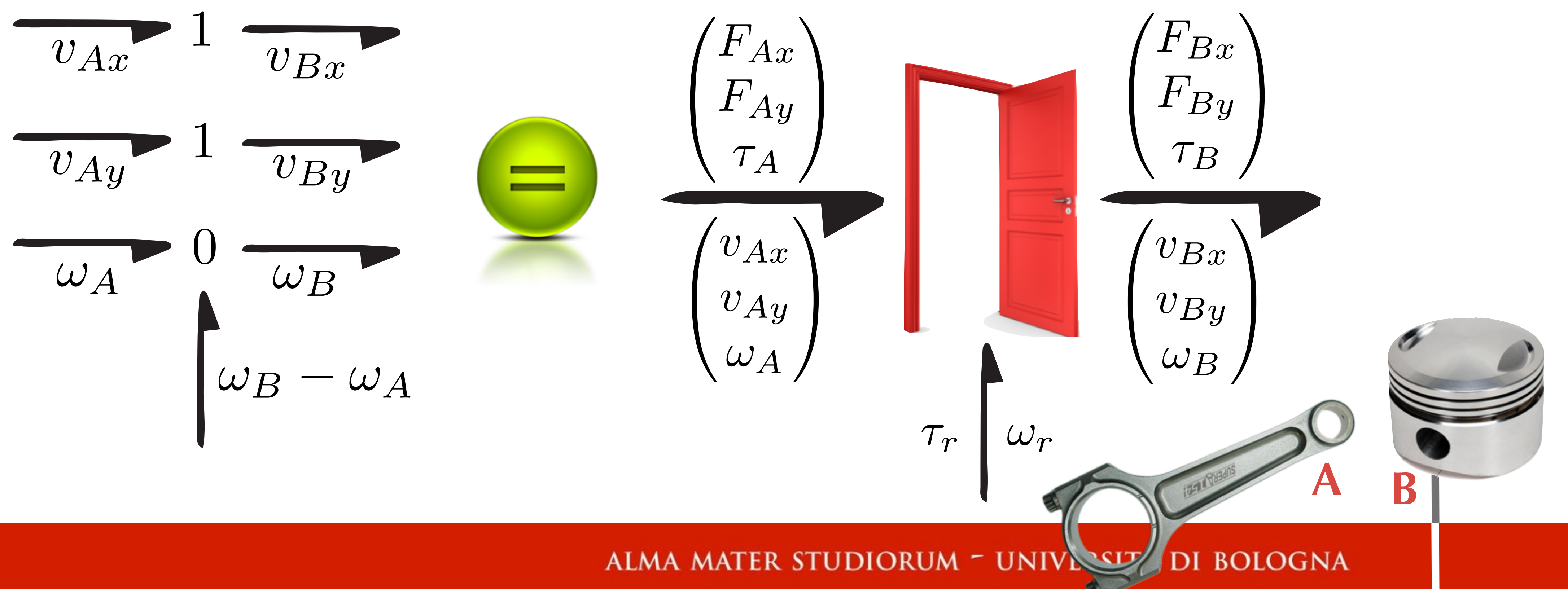
$$\begin{pmatrix} F_{Px} \\ F_{Py} \\ \tau_P \end{pmatrix} \qquad \begin{pmatrix} F_{P'x} \\ F_{P'y} \\ \tau_{P'} \end{pmatrix}$$



$$\begin{pmatrix} v_{Px} \\ v_{Py} \\ \omega_P \end{pmatrix} \qquad \begin{pmatrix} v_{P'x} \\ v_{P'y} \\ \omega_{P'} \end{pmatrix}$$

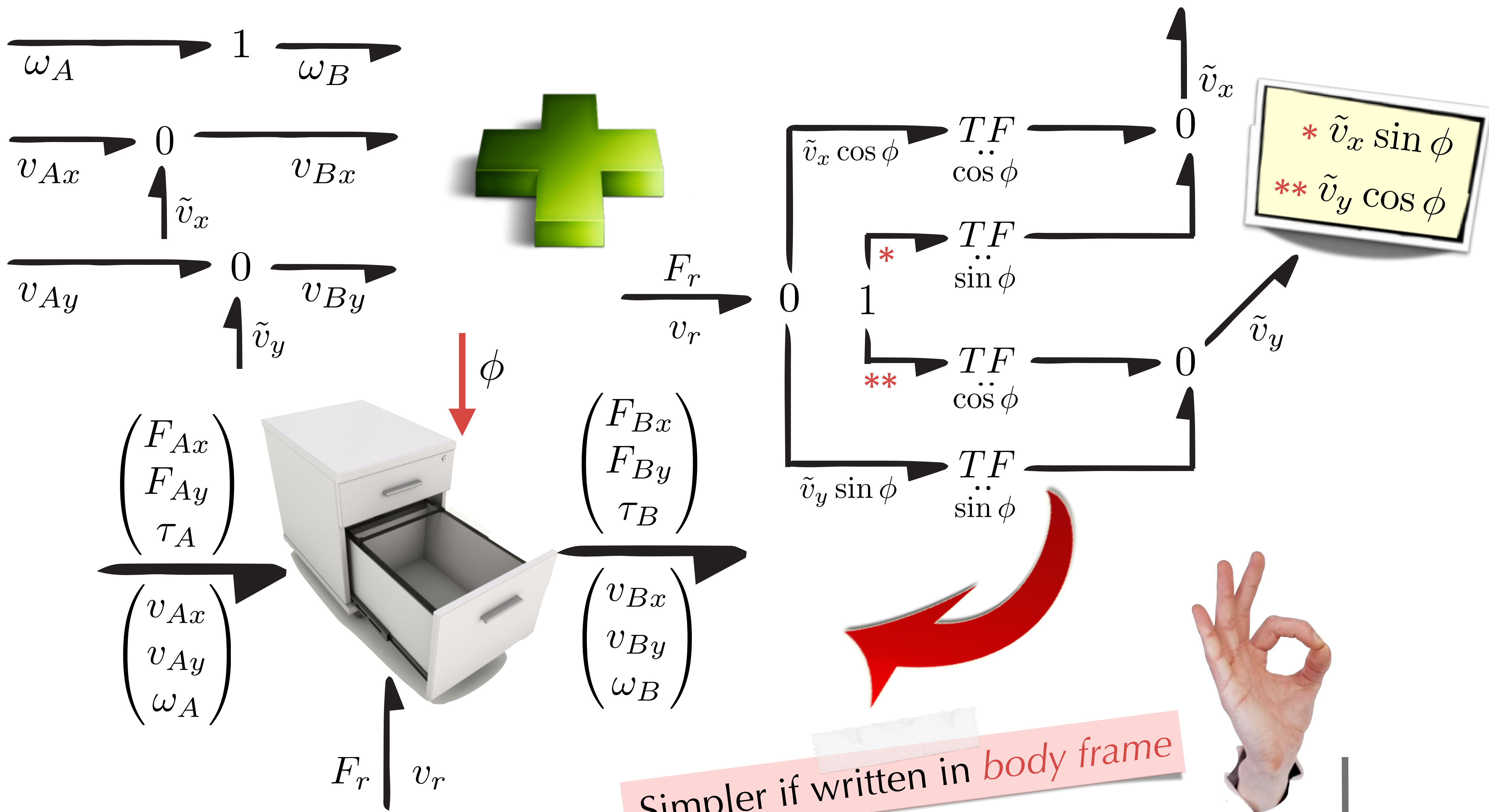
All the port variables are expressed with respect to the *inertial frame*

- **Question:** how can we *interconnect rigid bodies* to obtain complex mechanisms?
- It is necessary to define a *(set of) joints,* i.e. element that constraint the motion in only in some directions
  - *Rotational* joint
  - *Translational* joint
- For a *rotational joint,* the relative linear velocity is set equal to zero, while the angular one is free

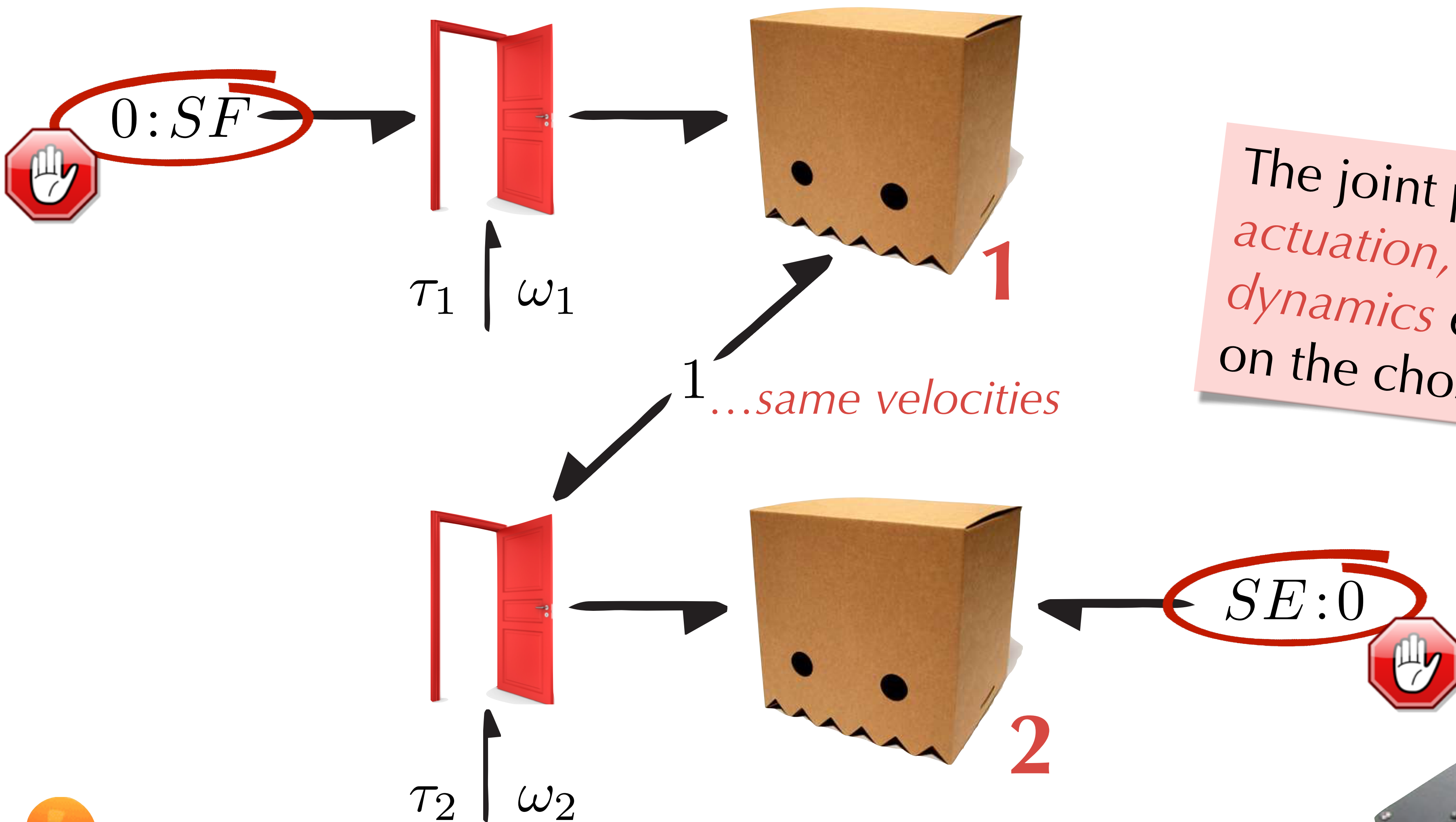


Plane motion

- A *translational joint* allows a relative motion only along a specified linear direction
  - Since velocities are in the inertial frame, this direction is *not* fixed



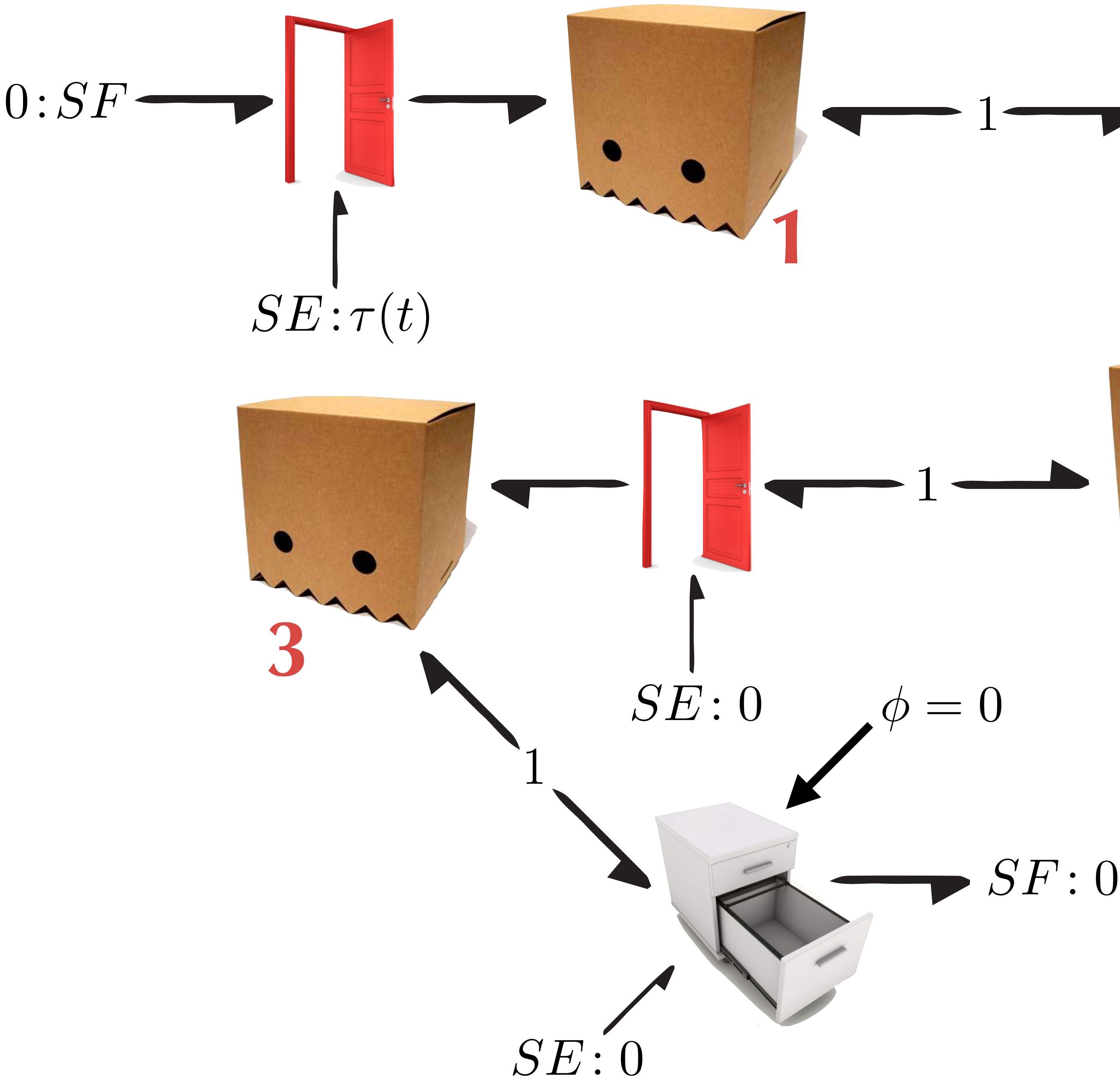
☑ *2-dof planar manipulator*

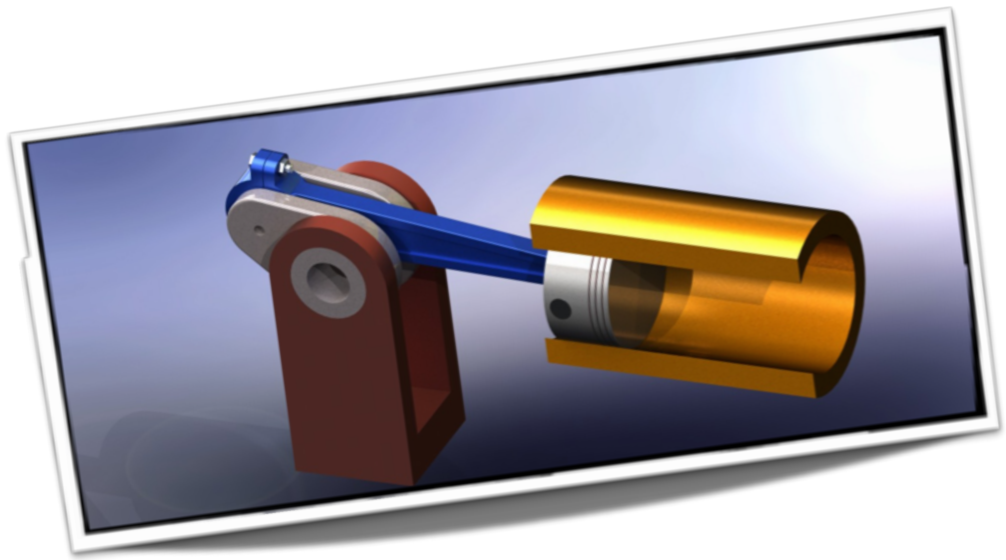


The joint ports can be used to attach *actuation*, or to perform *inverse dynamics* experiments. This depends on the chosen *causality*

Plane motion

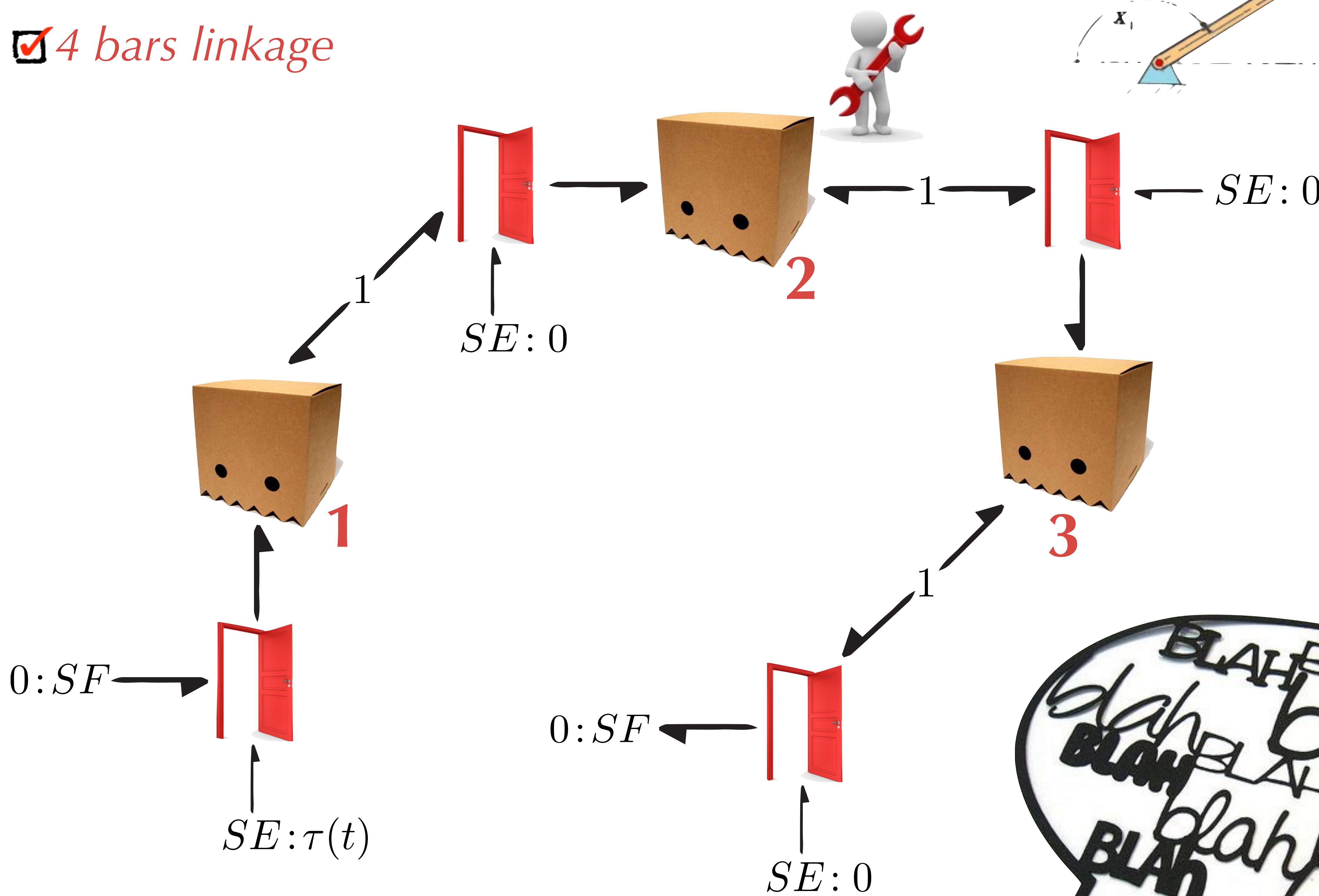
Slider-crank mechanism

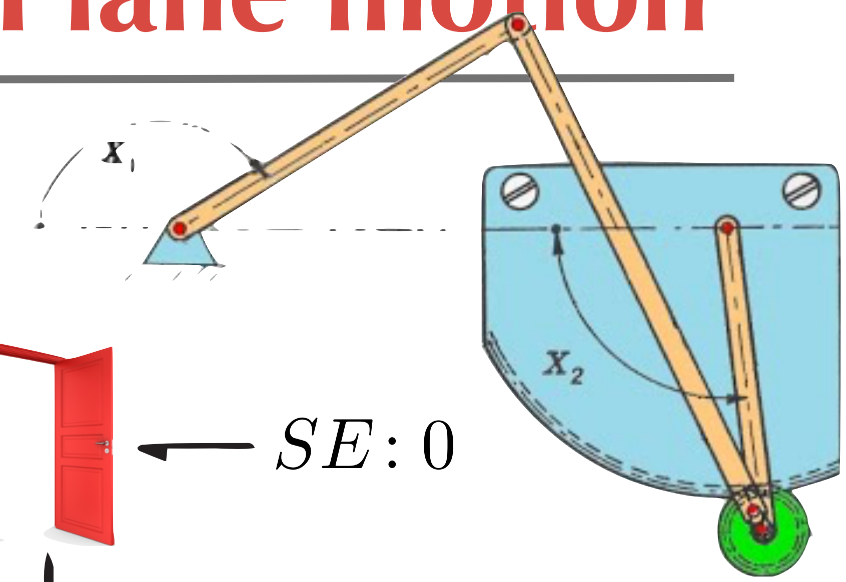




Plane motion

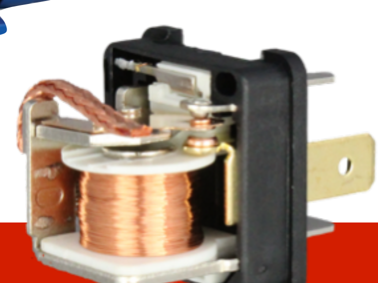
- *4 bars linkage*



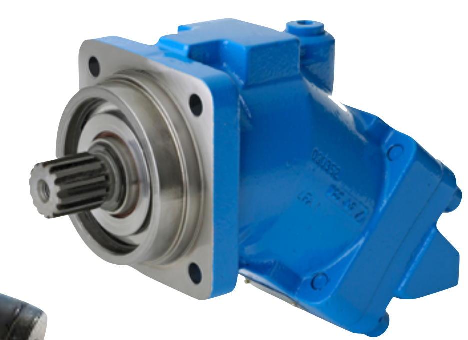


Transducers & multi-energy domain models

- *Transducers* are devices that *couple subsystems of distinct energy domains*
  - Electromechanical devices: motors, generators, and relays
  - Hydraulic-mechanical devices: pumps, motors, and rams
  - Electro-hydraulic valves
- We start with some relatively *simple transducers* based on *2-port transformers and gyrators* that allow the modelling of multi-energy-domain systems using bond graphs
- A major reason for studying bond graphs is that they provide a *uniform and precise way to represent systems* when several forms of power and energy are involved

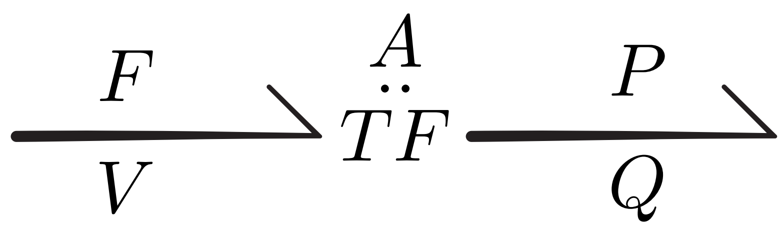


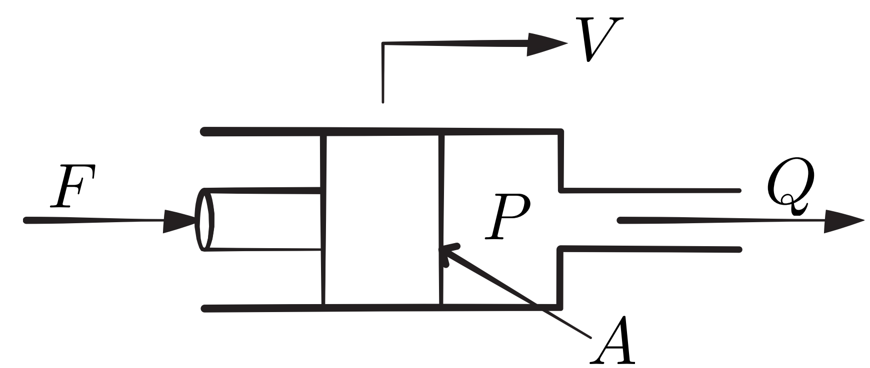



Transformer transducers

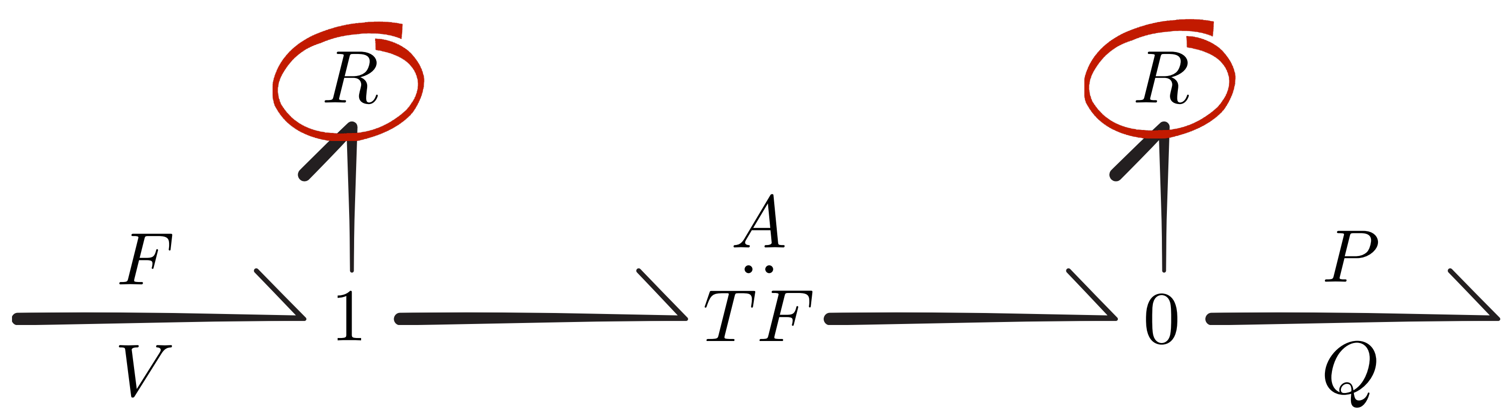
- A simple example, the *hydraulic ram*

$$F = AP \qquad AV = Q \qquad FV = PQ$$



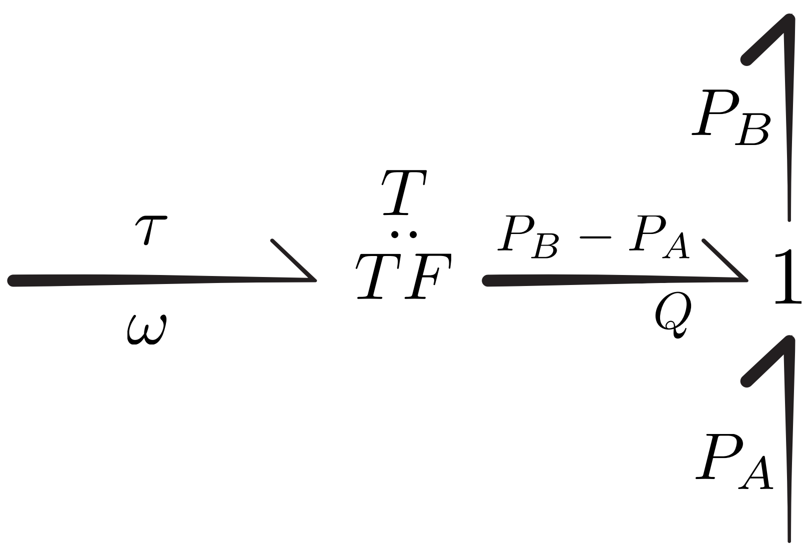


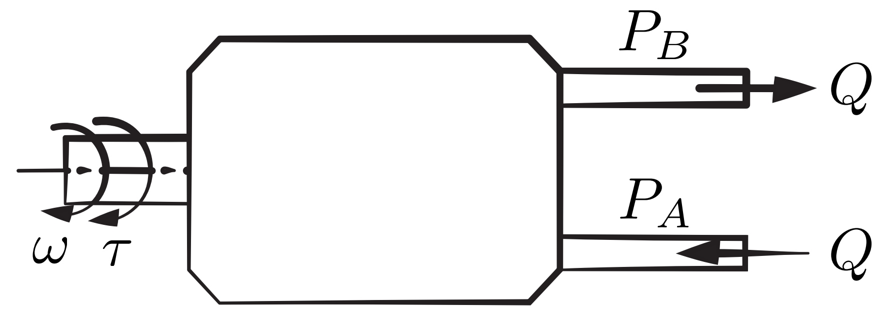
- *Ideal transformer transducer* applies only to the effects at the face of the piston, but all *real devices* contradict the idea that mechanical power can be converted *with no loss* to hydraulic power
  - On the mechanical side, the added 1-junction and resistor are used to include *mechanical friction effects*
  - On the hydraulic side, the combination of the 0-junction and resistance represent a possible *leakage* past the piston



Transformer transducers

- *Positive displacement hydraulic pumps* and *motors* consist of a number of pistons and a mechanism for moving them back and forth as a function of the angular position of a shaft
  - With a large number of pistons (7 to 9), the angular speed of the shaft is related to the volume flow rate by a nearly constant coefficient





$$(P_B - P_A)Q = \tau\omega$$

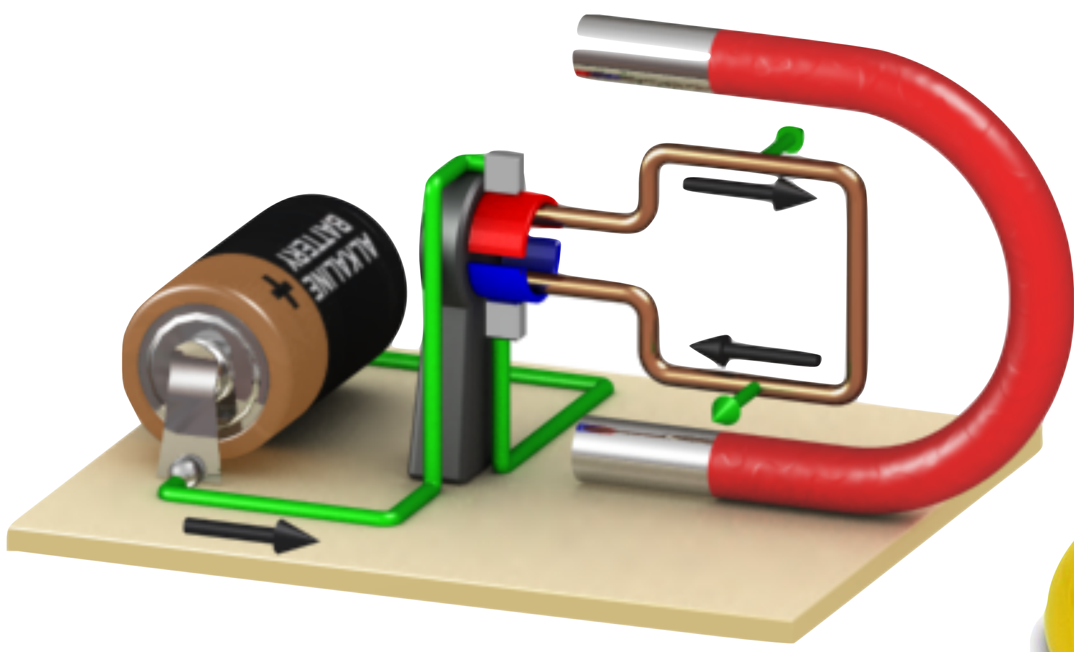
$$TQ = \omega$$

$$P_B - P_A = T\tau$$

  - Mechanical and hydraulic powers could both be *negative,* indicating that the model could as well represent a *motor*
  - The bond graph is *ideal,* but the model could be augmented by adding *resistors to model losses,* or an *I-element* on a 1-junction on the mechanical side to represent the *moment of inertia* of the rotating parts

Gyrator transducers

- The importance of *electromechanical transducers,* such as rotary and linear motors or voice coils in electrodynamic loudspeakers, justifies a short discussion of why *gyrators* are used to describe a number of useful devices
- The basis of the gyrator models is a *current-carrying conductor moving in a magnetic field* under the action of an applied *force* that is equal but opposite to a magnetic force
  - Because of the motion, a *voltage* is induced in the conductor



$$e = BlV \qquad + \qquad Bli = F$$

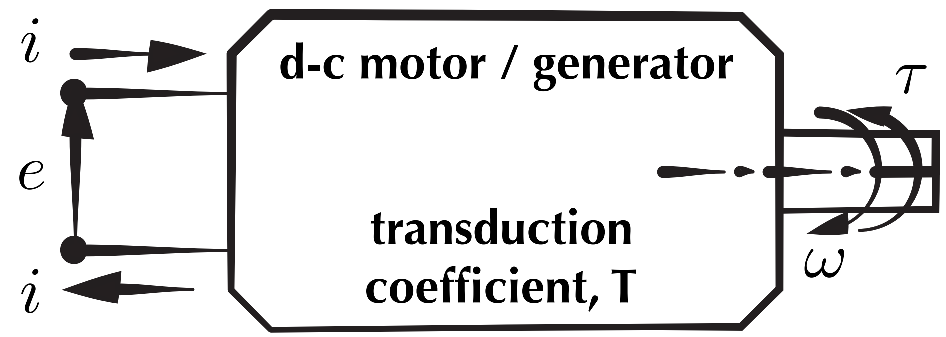
Faraday's law

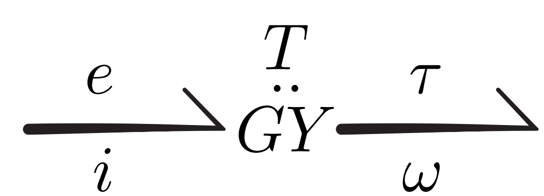
Lorentz force

$$\xrightarrow[V]{\quad F \quad} \overset{Bl}{\ddot{GY}} \xrightarrow[i]{\quad e \quad}$$

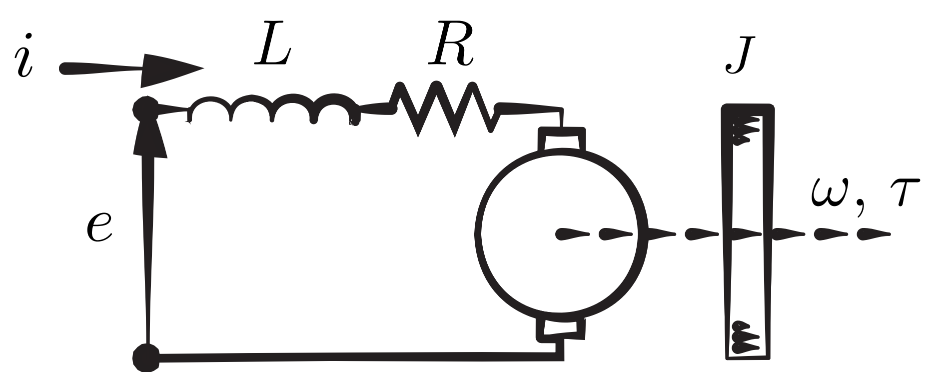
Gyrator transducers

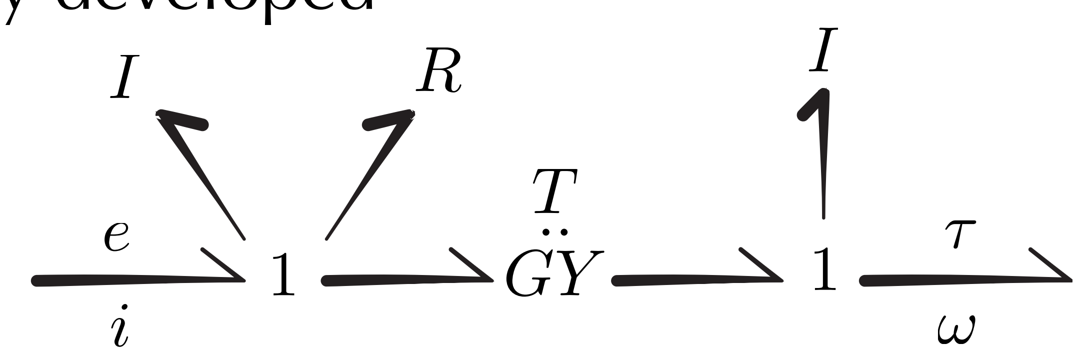
- The case of a conductor in a magnetic field lead to the representation of *electric motors* and *generators* as *gyrators*
  - It is assumed that the field is due to permanent magnets and a commutator switches the coils so that the *voltage* at the terminals induced by the rotary motion is *proportional* to the *angular speed* of the rotor
  - The *torque* produced is *proportional* to the *current* at the terminals





  - A *more detailed model* can be easily developed



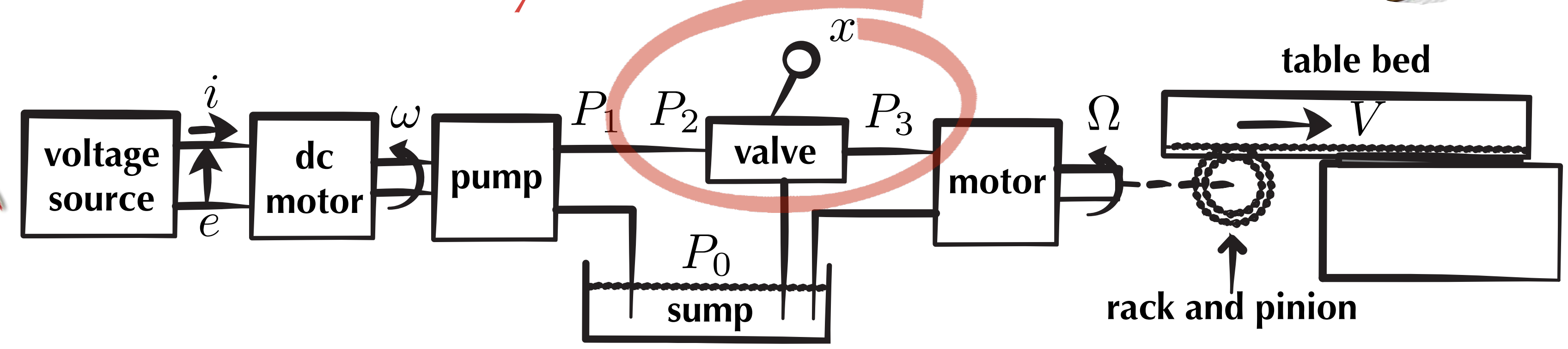


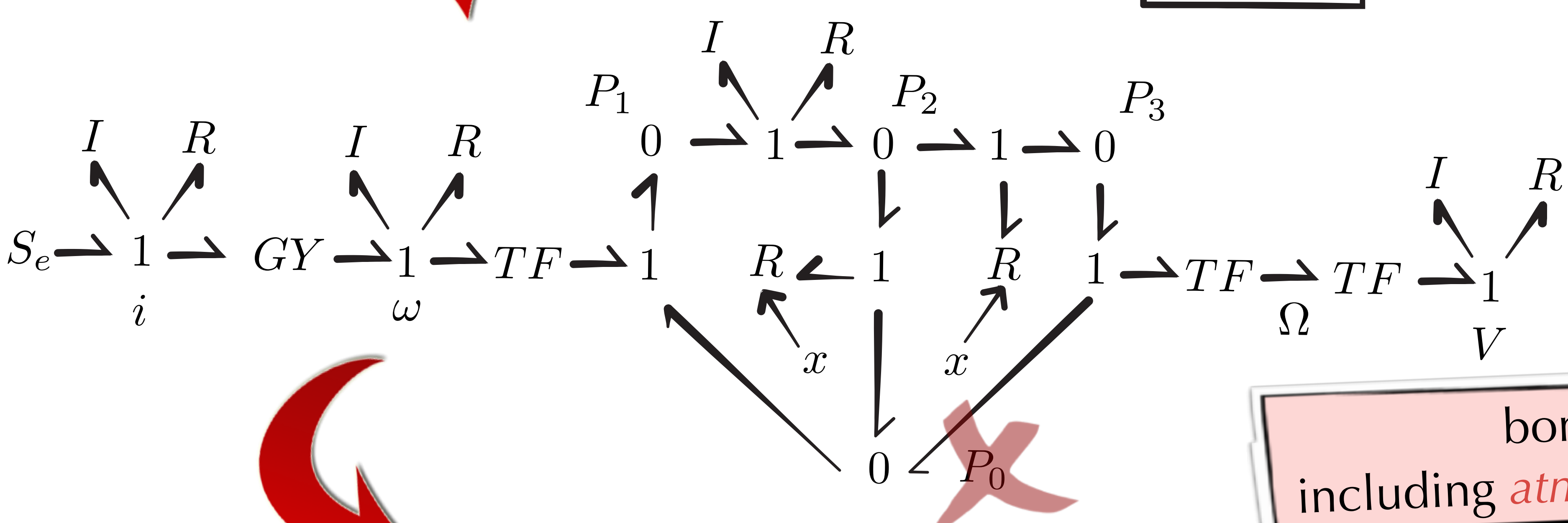
The idea of starting with *ideal* transducers and *adding realistic effects* is developed further

Multi-energy-domain models

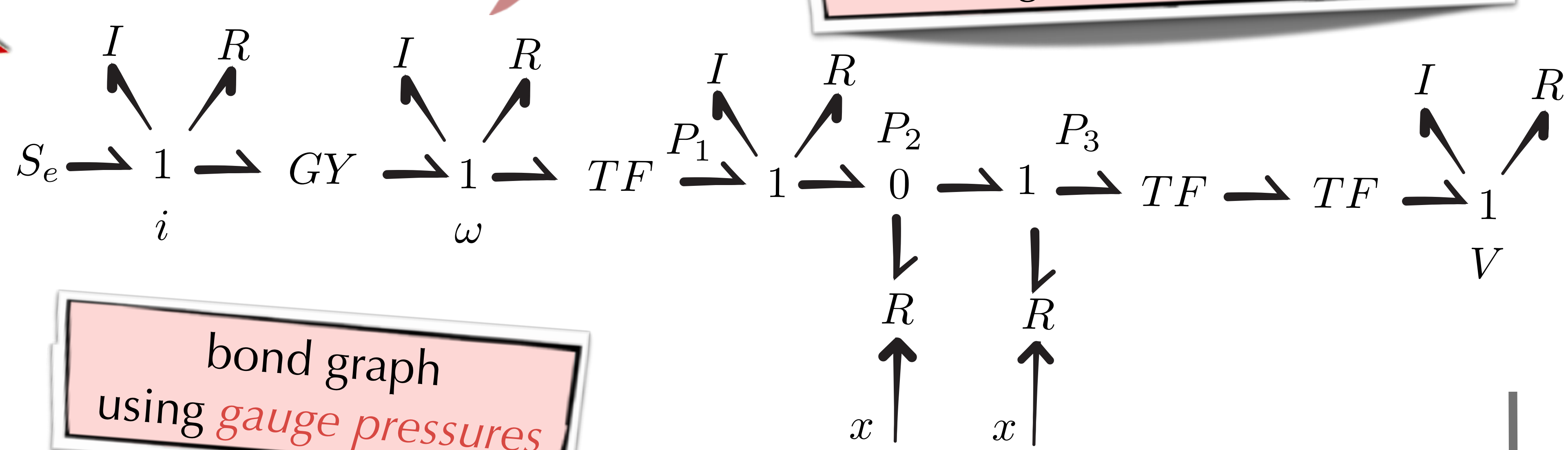
☑ An *electrical-hydraulic-mechanical system*

combination valve





bond graph
including atmospheric pressure



bond graph
using gauge pressures

Multi-energy-domain models

- ☑ There is an *art* to making a *useful mathematical model*
  - ✱ There is *a scientific basis for the ideal models* that involve power or energy conservation, and these are elegantly incorporated in bond graph elements
  - ✱ There are *a number of effects that can be added* to the ideal elements to account for effects that occur in real systems
  - ✱ These extra loss and dynamic elements must be added with restraint, since a complicated model that is hard to understand is often just as bad as an oversimplified model

- ☑ The *best model* is the simplest one capable of demonstrating those aspects of the behaviour of the system that need to be understood
- ☑ Good modellers are always ready to *modify* a preliminary model…

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