Formal Methods for the Control of Large-scale Networked Nonlinear Systems with Logic Specifications


Basilica di Santa Maria di Collemaggio, L'Aquila (Italy), 1287

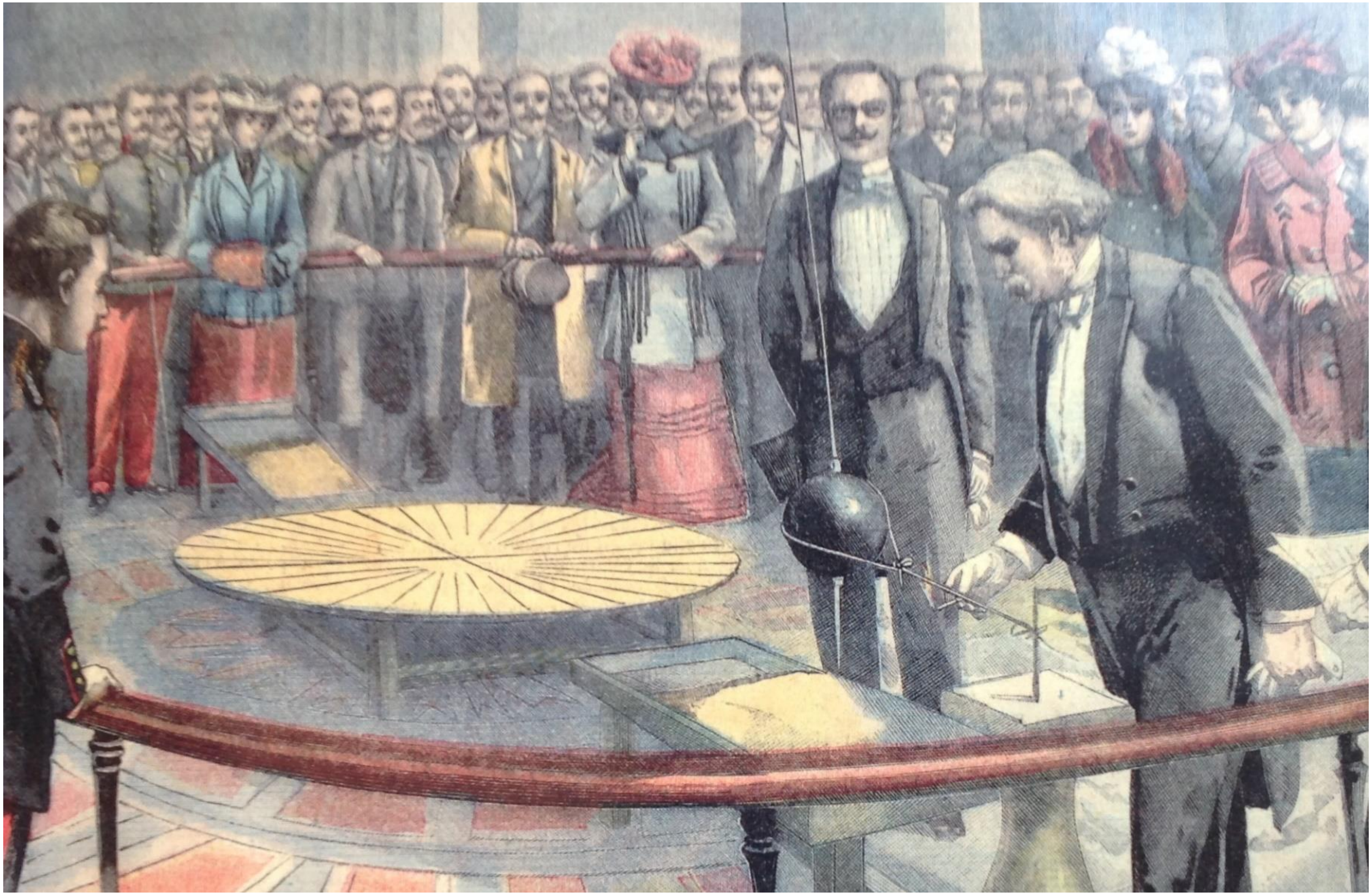
Lecture L6b:

Symbolic models for stable nonlinear control systems:

Example

Speaker: **Giordano Pola**

Enforcing logic specifications on a pendulum



Reconstruction of Foucault's demonstration in 1902 (illustration taken from the cover of William Tobin's book)

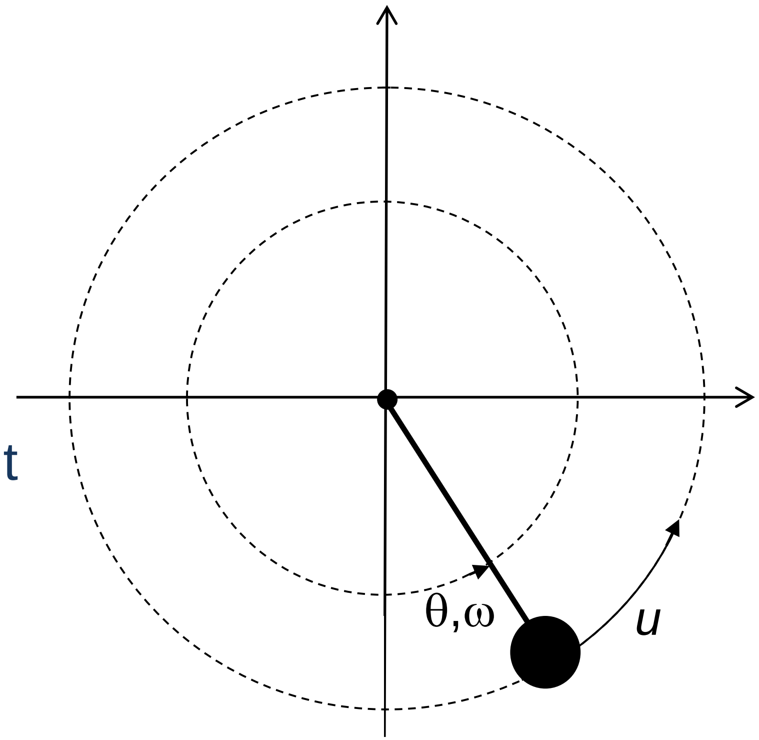
Enforcing logic specifications on a pendulum

One of the simplest mechanical systems studied in the literature is the pendulum:

$$\Sigma: \begin{cases} \dot{\theta} = \omega \\ \dot{\omega} = -\frac{g}{l}\sin\theta - \frac{k}{m}\omega + \frac{1}{ml^2}u \end{cases}$$

We suppose:

- $g = 9.8, l = 5, m = 0.5, k = 3$
- $u \in U \subset [-1.5,1.5]$
- Control signals are piecewise constant
- $(\theta, \omega) \in X = [-1,1] \times [-1,1]$



Taken from:

[Pola et al., 2008] Pola, G., Girard A., Tabuada, P., Approximately bisimilar symbolic models for nonlinear control systems, Automatica, 44(10):2508-2516, October 2008

Enforcing logic specifications on a pendulum

Consider a specification given by the concatenation of tasks P_1 and P_2 as follows:

$$P_1, P_1, P_2, P_1, P_1$$

where:

- P_1 requires a periodic orbit from $(\theta, \omega) = (-0.4, 0)$ to $(0, 0)$
- P_2 requires a periodic orbit from $(\theta, \omega) = (-0.4, 0)$ to $(0.4, 0)$

This type of specification is a paradigm for illustrating more complex controller synthesis problems where a task is given by the coordination of smaller tasks

Assume desired accuracy $\mu = 0.25$

Construction of the symbolic model

- **Check the δ-GAS property**

  It can be shown that function

$$V(\mathrm{x}, x') = \frac{1}{2}(x - x')^T \begin{bmatrix} \frac{1}{2}\left(\frac{k}{m}\right)^2 & \frac{1}{2}\frac{k}{m} \\ \frac{1}{2}\frac{k}{m} & 1 \end{bmatrix} (x - x')$$

  is a (local) δ-GAS Lyapunov function for the pendulum with

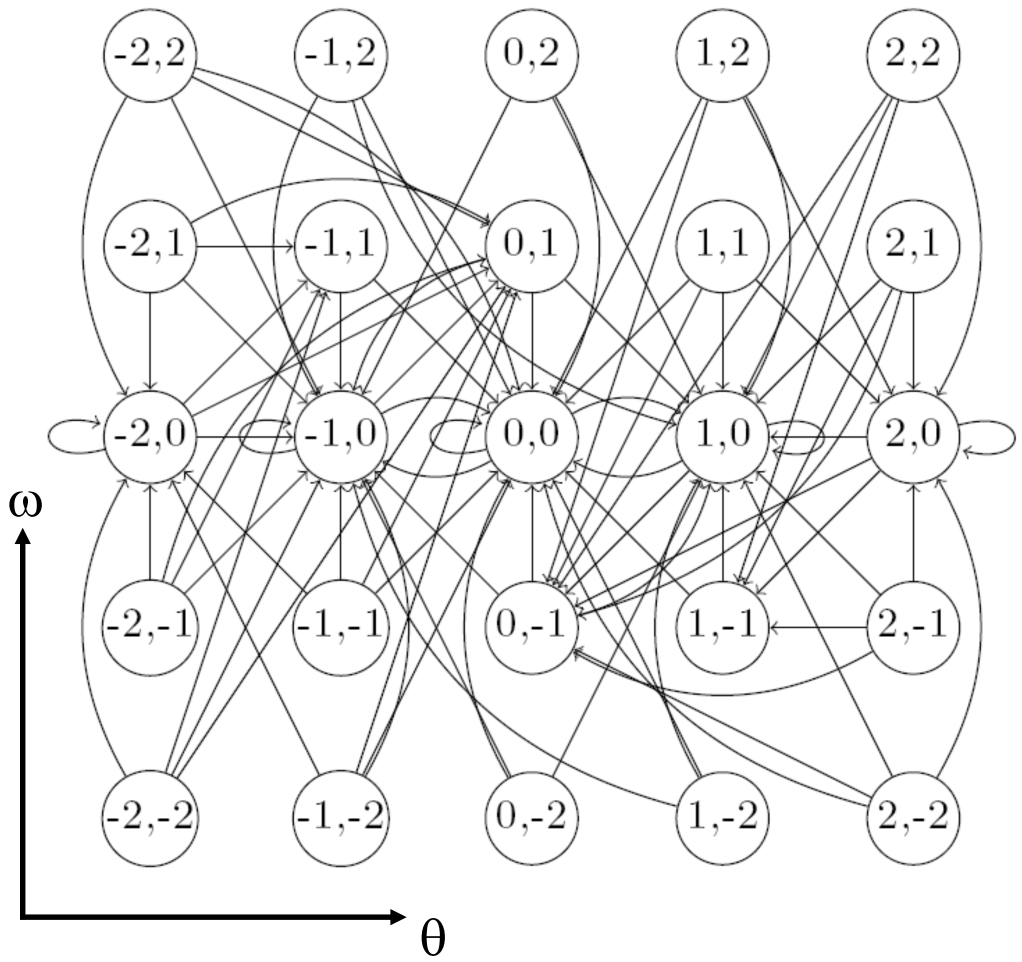
$$\alpha_1(s) = 0.49\, s^2$$
$$\alpha_2(s) = 18.51 s^2$$
$$\kappa = 4.12$$

- **Design of quantization parameters**

  For $\mu = 0.25$ and $\tau = 2$ one can pick $\eta = 0.4$

Construction of the symbolic model

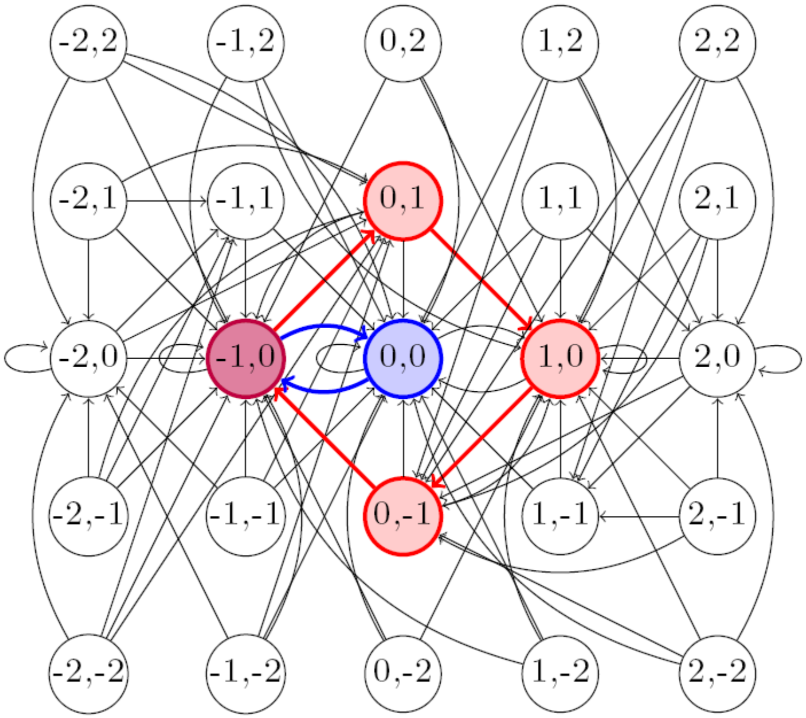
Symbolic model obtained:



where discrete states $(\eta i, \eta j)$ have been labelled by (i, j)

Design of the controller

By exploiting the symbolic model, one obtains:



- Specification P_1:

$(-1,0) \xrightarrow{1.38} (0,0) \xrightarrow{-1.5} (-1,0)$

- Specification P_2:

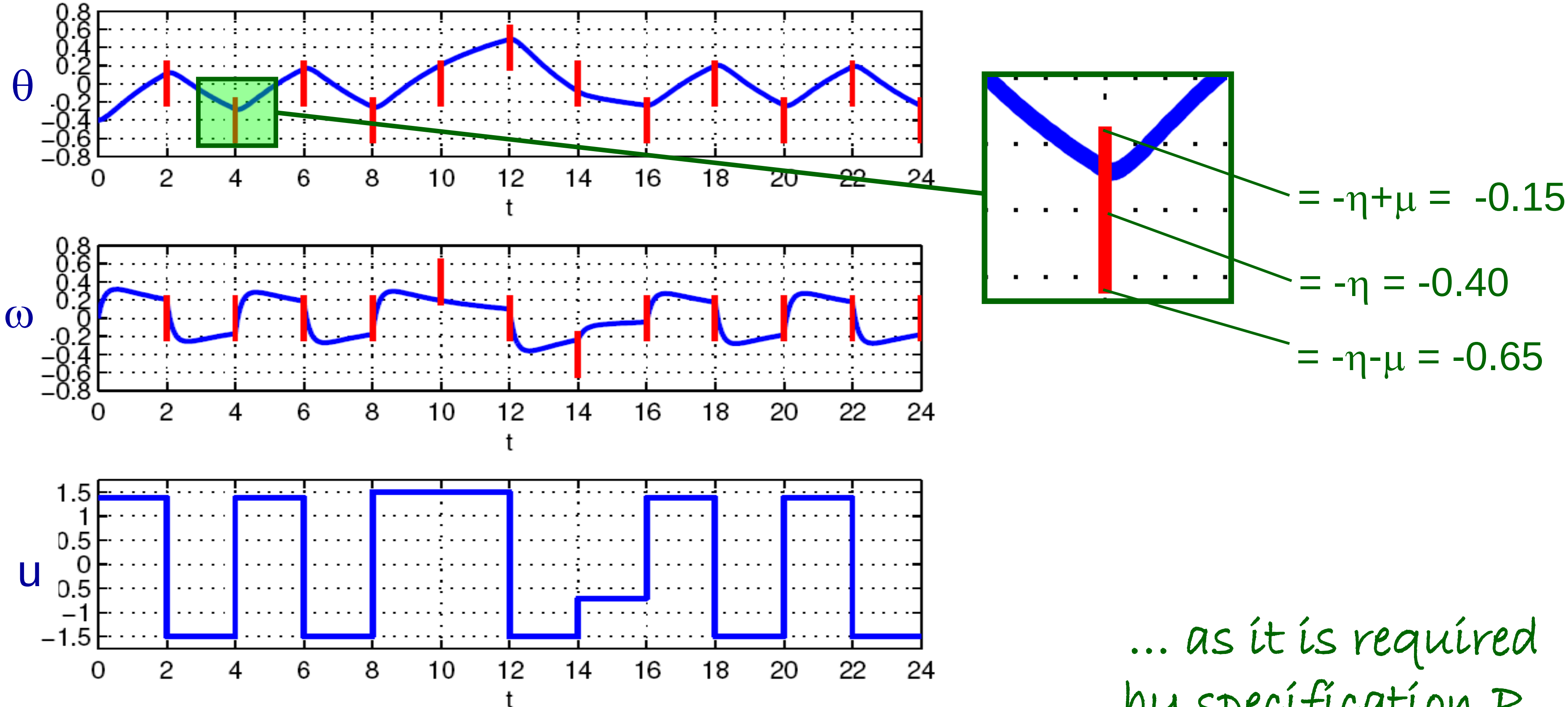
$(-1,0) \xrightarrow{1.5} (0,1) \xrightarrow{1.5} (1,0) \xrightarrow{-1.5} (0,-1) \xrightarrow{-0.71} (-1,0)$

- Overall specification:

$(-1,0) \xrightarrow{1.38} (0,0) \xrightarrow{-1.5} (-1,0) \xrightarrow{1.38} (0,0) \xrightarrow{-1.5} (-1,0) \xrightarrow{1.5} (0,1) \xrightarrow{1.5} (1,0) \xrightarrow{-1.5} (0,-1)$
$\xrightarrow{-0.71} (-1,0) \xrightarrow{1.38} (0,0) \xrightarrow{-1.5} (-1,0) \xrightarrow{1.38} (0,0) \xrightarrow{-1.5} (-1,0)$

Validation

Let us apply the control strategy synthesized on the symbolic model to the original continuous system:



... as it is required by specification P_1 and precision μ!