Formal Methods for the Control of Large-scale Networked Nonlinear Systems with Logic Specifications

Basilica di Santa Maria di Collemaggio, 1287, L'Aquila

Lecture L12: Symbolic models and control for networked control systems

Speaker: Alessandro Borri

What's new?

In this lecture we will remove the ideality assumption placed on the communication infrastructure conveying information between the plant and the controller

Tools:

- δ-GAS, δ-FC
- Alternating approximate (bi)simulation

Lecture based on:

[Borri et al., HSCC12] Borri, A., Pola, G., Di Benedetto, M.D., A symbolic approach to the design of nonlinear networked control systems, Hybrid Systems: Computation and Control 2012, Bejing, China, April 2012, pp. 255-264, I. Mitchell and T. Dang, Eds.

[Borri et al., CDC12] Borri, A., Pola, G., Di Benedetto, M.D., Integrated Symbolic Design of Unstable Nonlinear Networked Control Systems, 51st IEEE Conference on Decision and Control, Maui, Hawaii, USA, December 2012, pp. 1374-1379

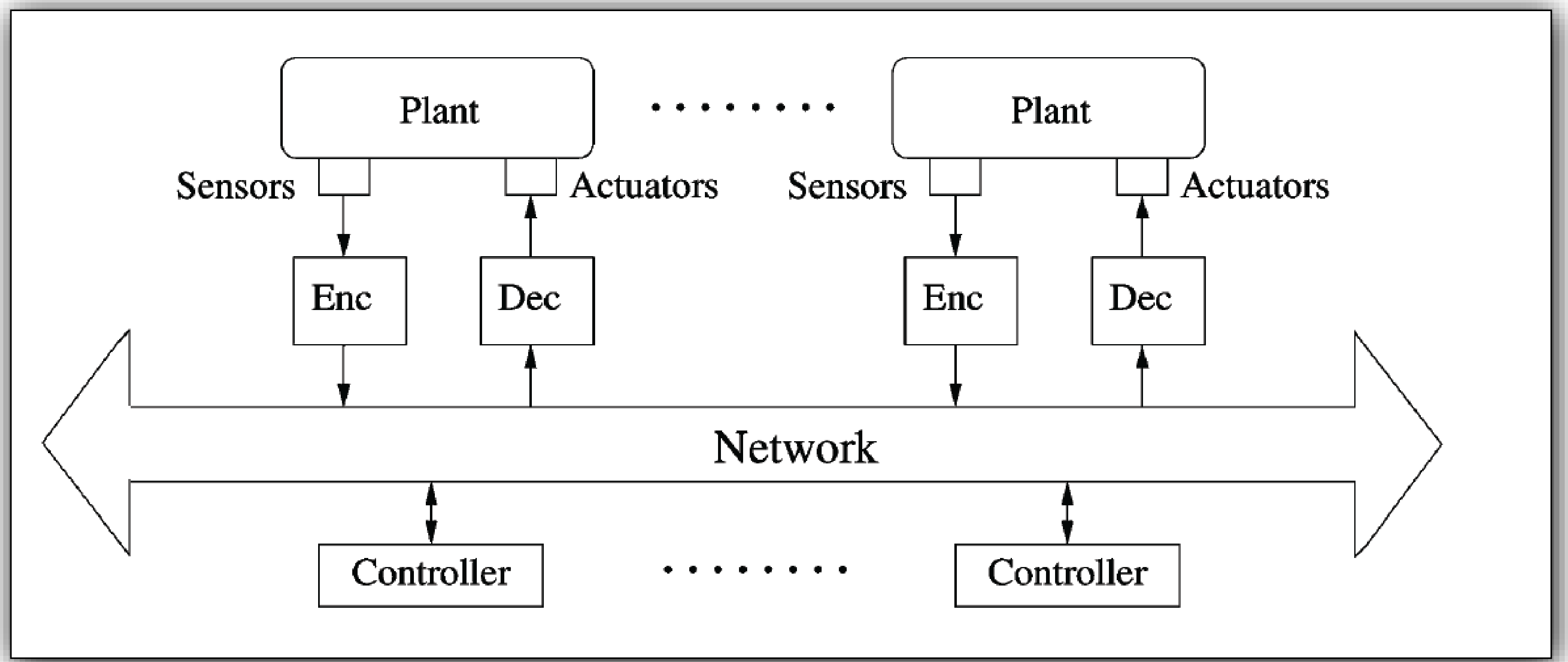
Networked control systems

- Networked Control Systems (NCS) are spatially distributed systems where the communication among plants, sensors, actuators and controllers occurs in a shared communication network
- At present, most of the results concerning NCS focus on stability and stabilizability problems
- Results available in the literature vary depending on the class of systems considered (linear vs. nonlinear), controllers synthesized (continuous vs. digital), and assumptions on the network non-idealities

Symbolic Control Design of Nonlinear Networked Control Systems

- Mathematical model of nonlinear NCS
- Symbolic models for stable and unstable NCS
- Symbolic control design of NCS
- Efficient control design algorithms

Networked control systems: Our model

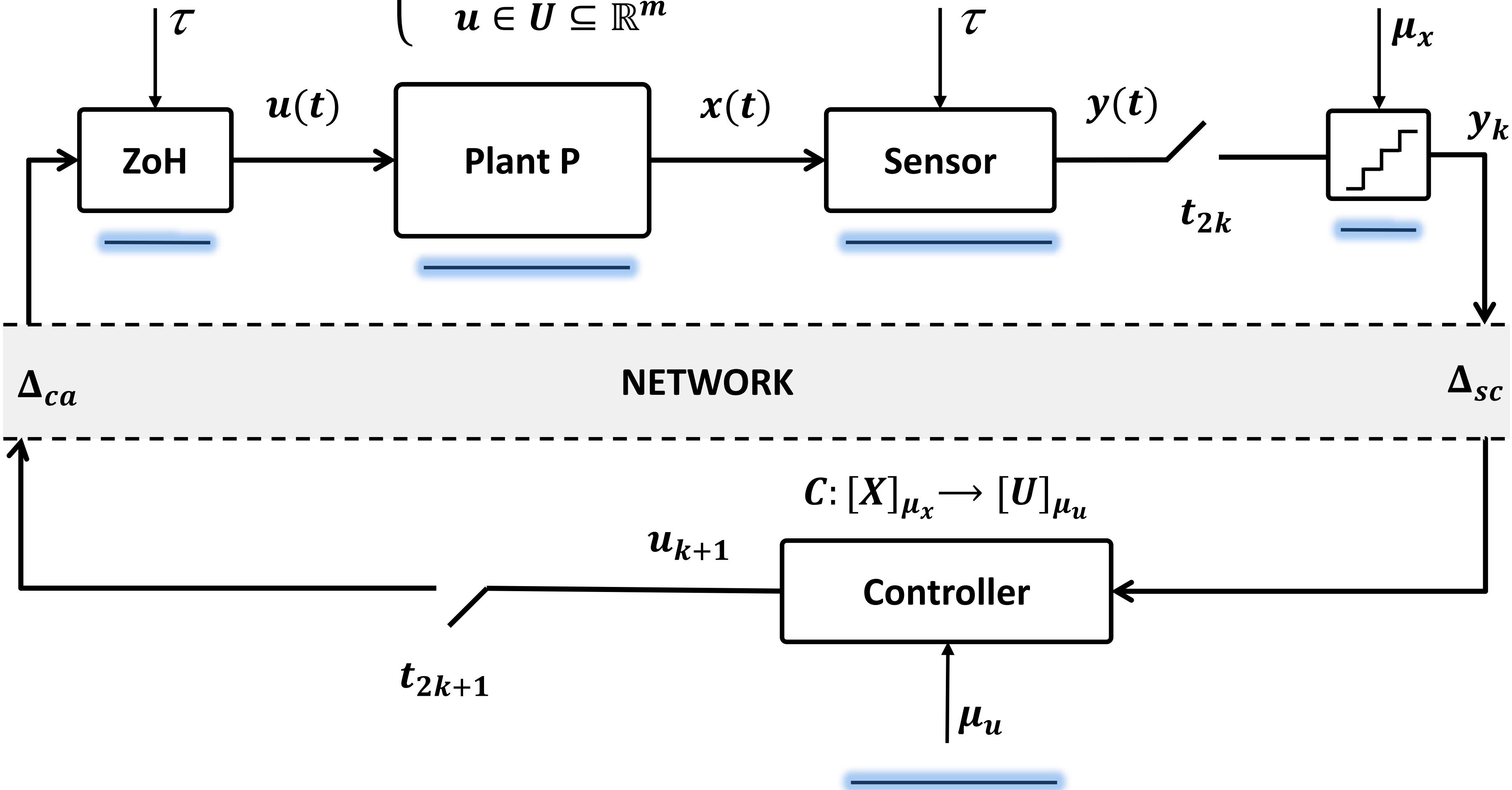


Networked control systems: Our model

$$u(s\tau + t) = u(s\tau),\quad t \in [0,\tau[,\ s \in \mathbb{N}_0$$

$$\begin{cases} \dot{x} = f(x(t), u(t)) \\ x \in X \subseteq \mathbb{R}^n \\ x(0) \in X_0 \subseteq X \\ u \in U \subseteq \mathbb{R}^m \end{cases}$$

$$y(s\tau + t) = y(s\tau) = x(s\tau),\quad t \in [0,\tau[,\ s \in \mathbb{N}_0$$



Networked control systems: Our model

Network non-idealities: quantization, packet drops, variable delays
(e.g. [Andersson, IEEE-CDC-05], [Antsaklis, IEEE-TAC-04],
[Heemels, IEEE-TAC-10], [Hespanha, Proc. IEEE-07], [Murray, SMTNS-06])

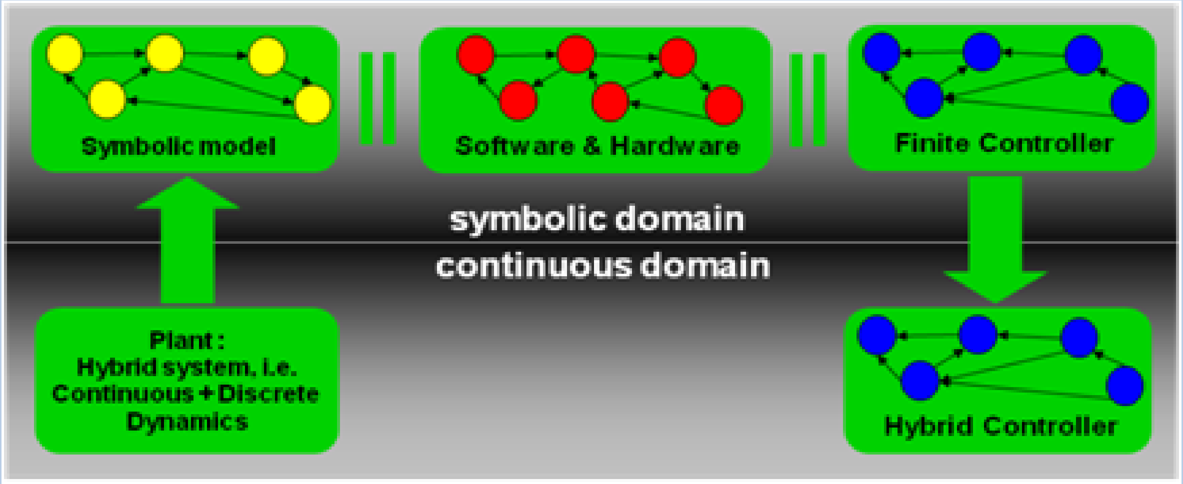
Network and computing non-idealities in our model:

- Quantization errors
- Bounded time-varying network access times
- Bounded time-varying communication delays induced by the network
- Bounded time-varying computation time of computing units
- Limited bandwidth
- Bounded packet losses

Correct-by-design controller synthesis

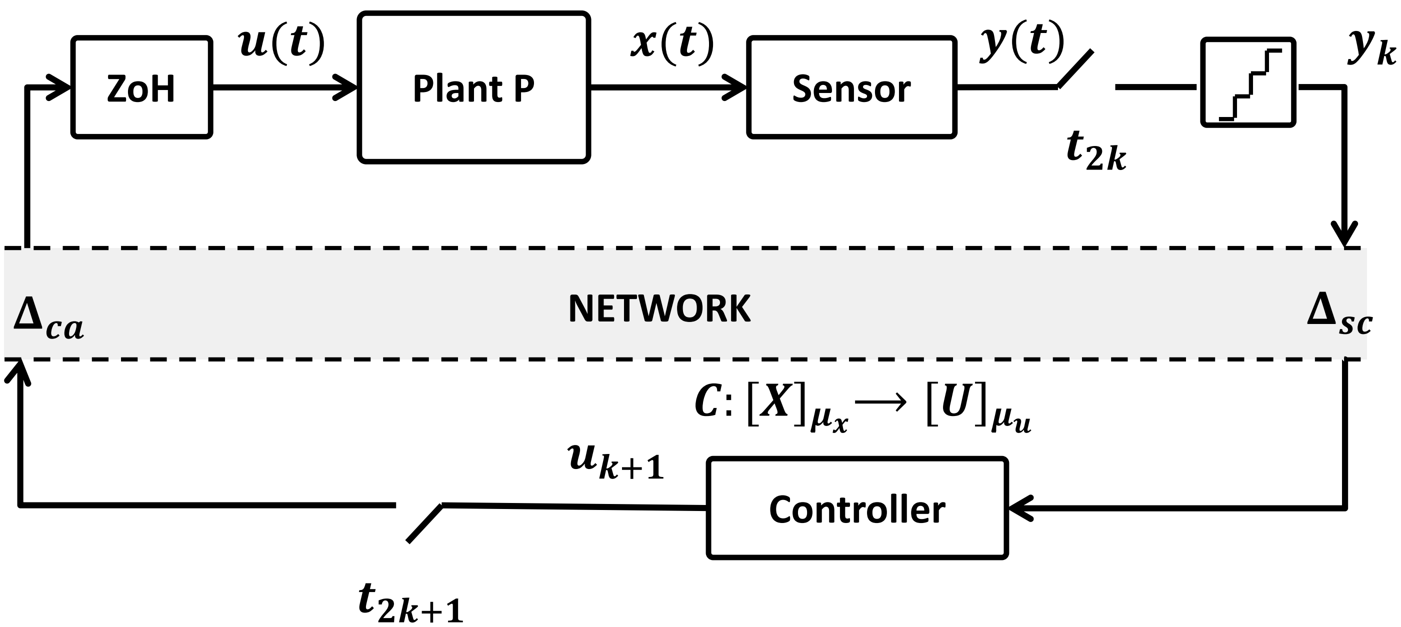
… a three-step process:

1. Construct the finite/symbolic model T of the plant system Σ
2. Design a finite/symbolic controller C that solves the specification S for T
3. Refine the controller C to obtain controller C' for Σ



Dealing with heterogeneity

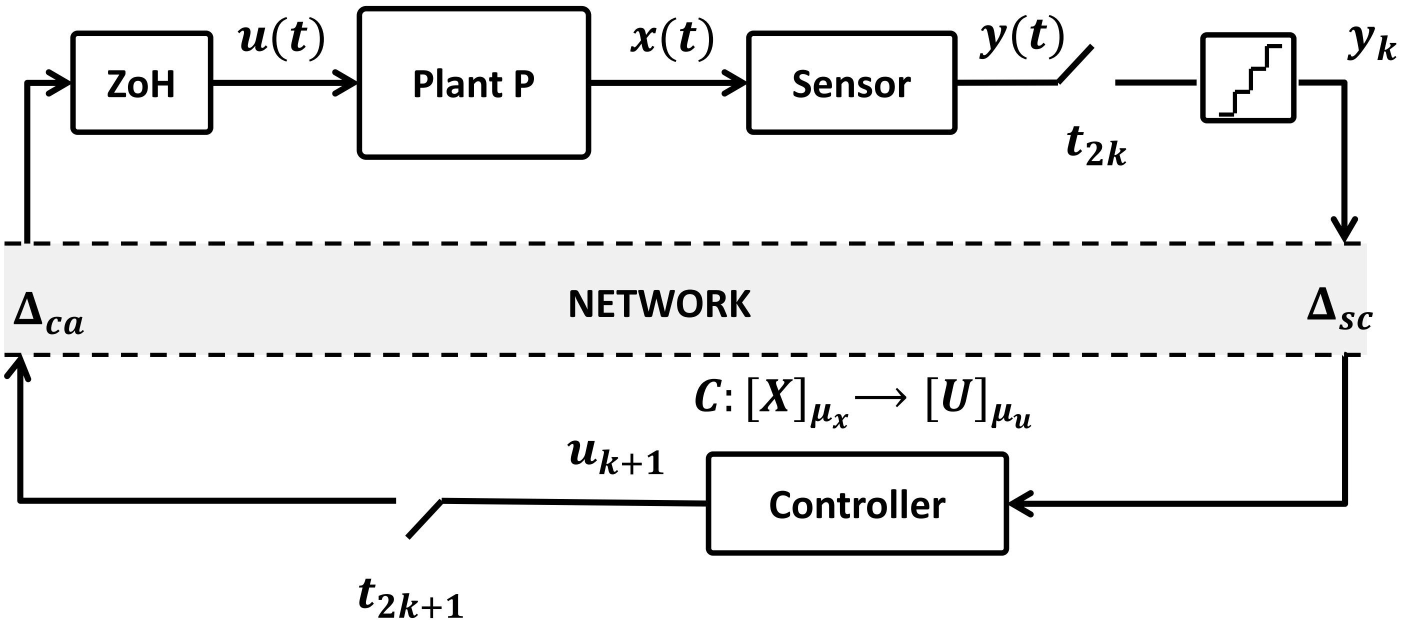
Nonlinear Networked control systems as TSs



t	0	τ	2τ	3τ	4τ	5τ	6τ	7τ	8τ	9τ	…
u	0	0	0	u_1	u_1	u_1	u_1	u_1	u_1	u_2	…
x	x(0)	$x(\tau)$	$x(2\tau)$	$x(3\tau)$	$x(4\tau)$	$x(5\tau)$	$x(6\tau)$	$x(7\tau)$	$x(8\tau)$	$x(9\tau)$	…
	$N_1 = 4$				$N_2 = 6$						…

Dealing with heterogeneity

Nonlinear Networked control systems as TSs



t	0	τ	2τ	3τ	4τ	5τ	6τ	7τ	8τ	9τ	…
u	0	0	0	u_1	u_1	u_1	u_1	u_1	u_1	u_2	…
x	x(0)	x(τ)	x(2τ)	x(3τ)	x(4τ)	x(5τ)	x(6τ)	x(7τ)	x(8τ)	x(9τ)	…
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Dealing with heterogeneity

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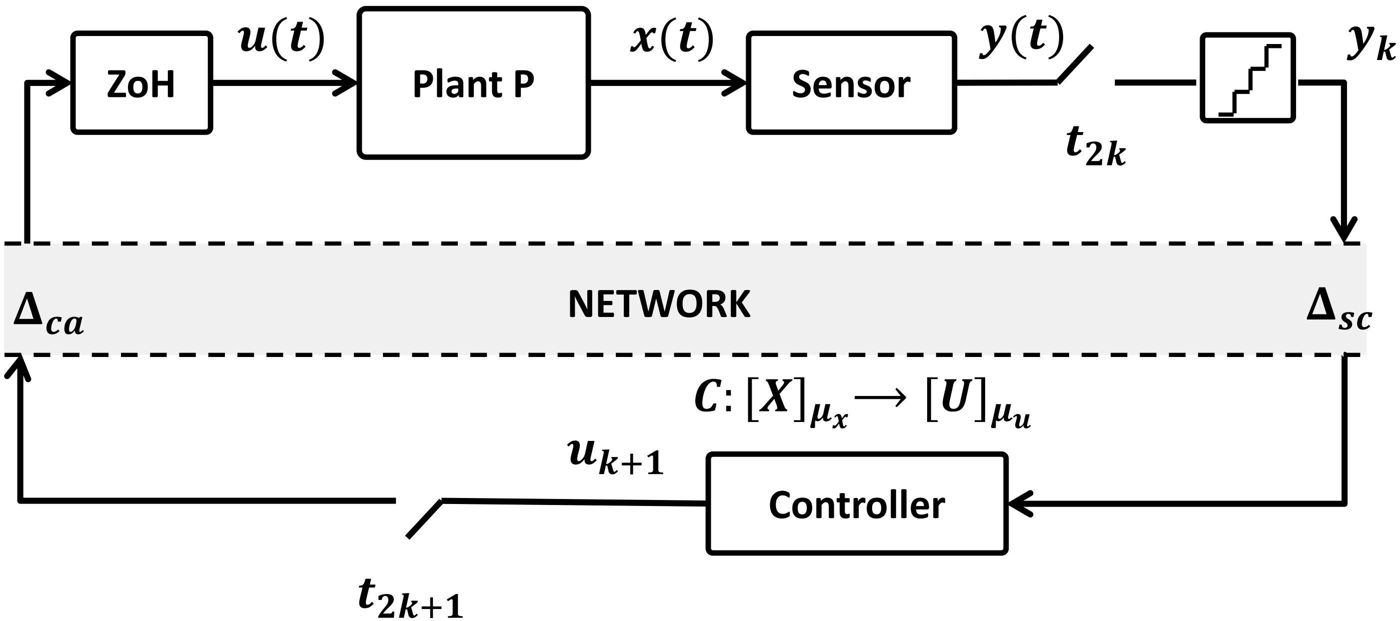

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Dealing with heterogeneity

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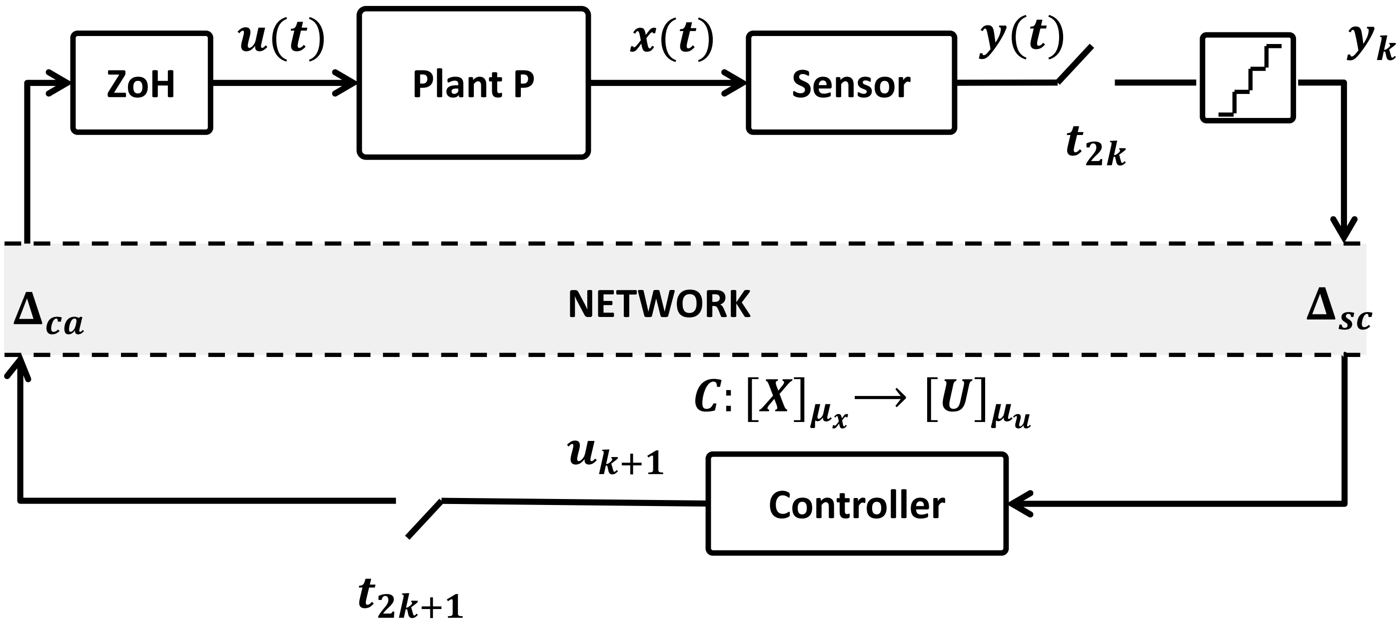

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Dealing with heterogeneity

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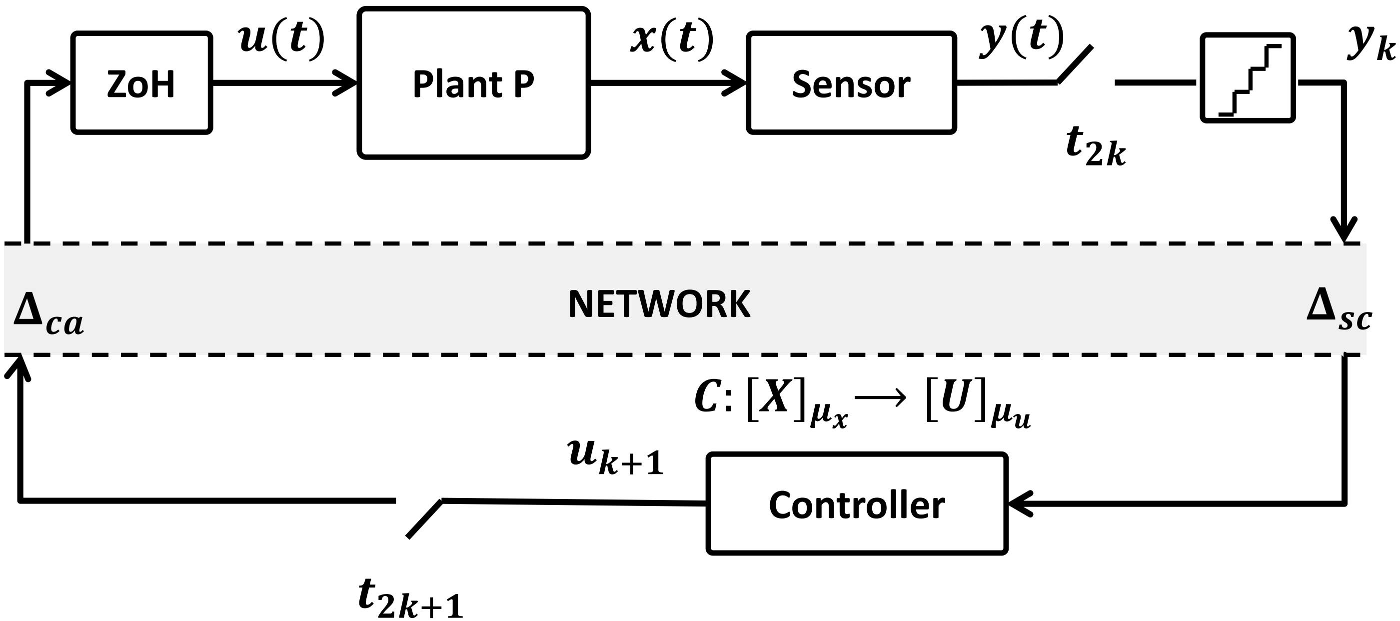

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Dealing with heterogeneity

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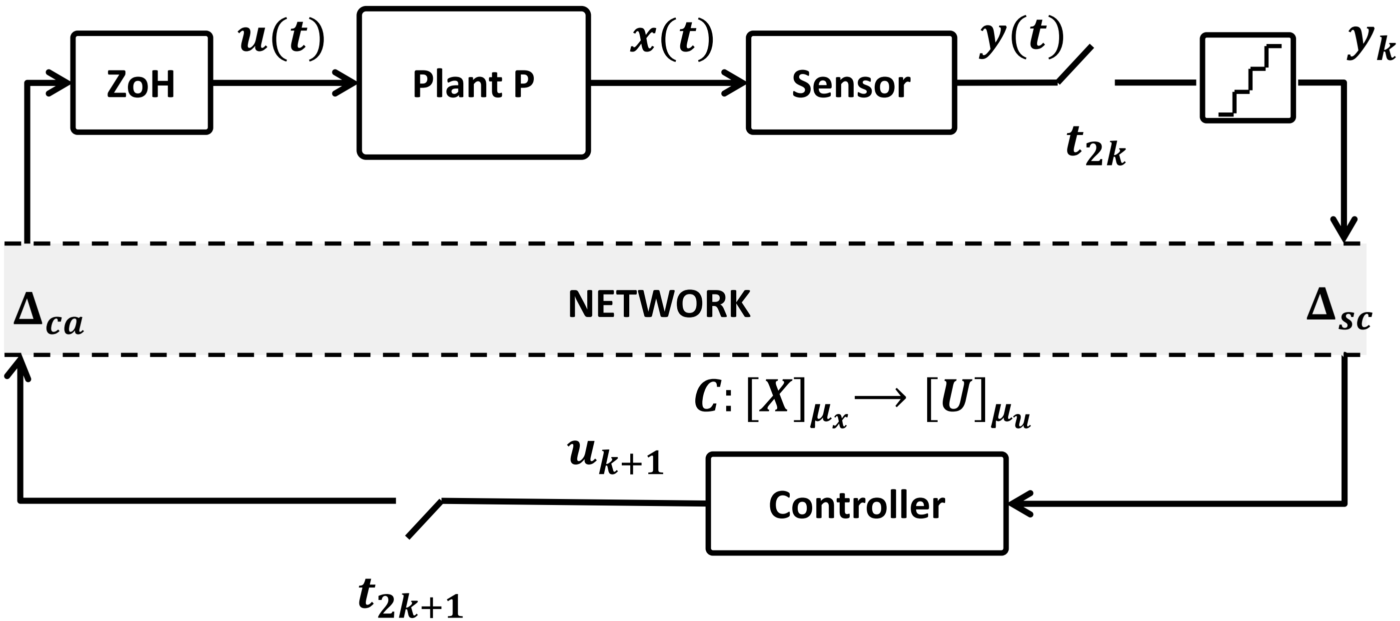

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Dealing with heterogeneity

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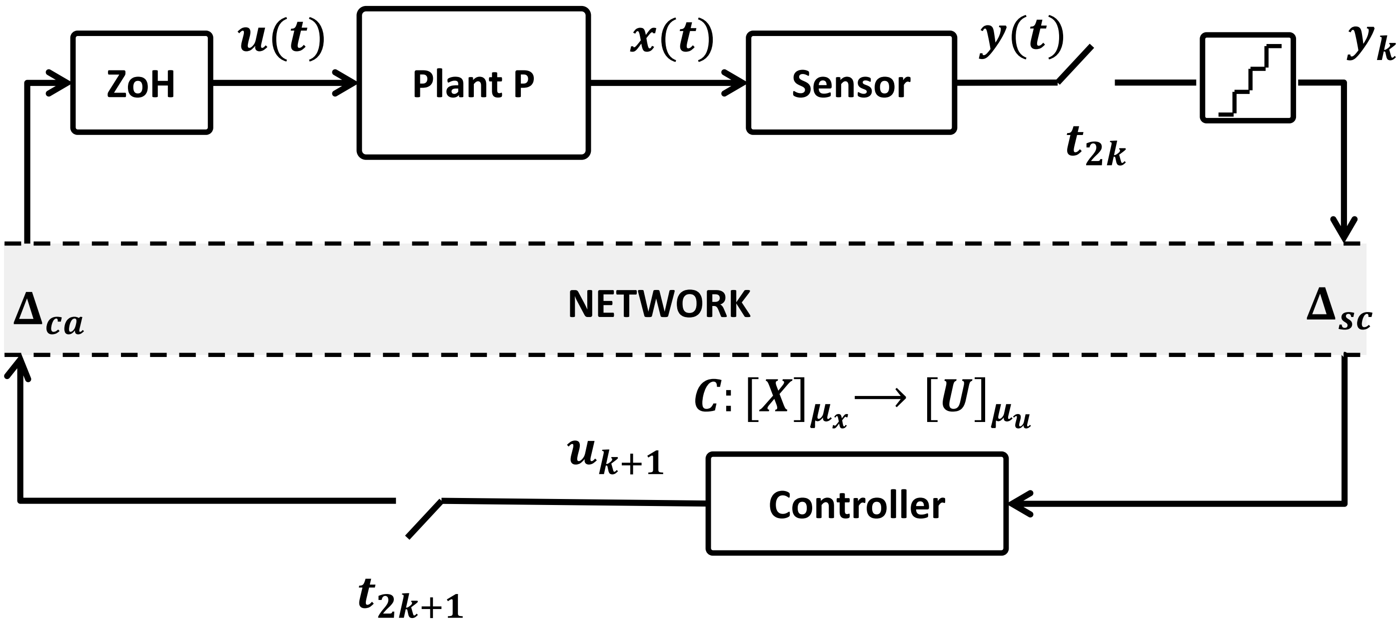

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Dealing with heterogeneity

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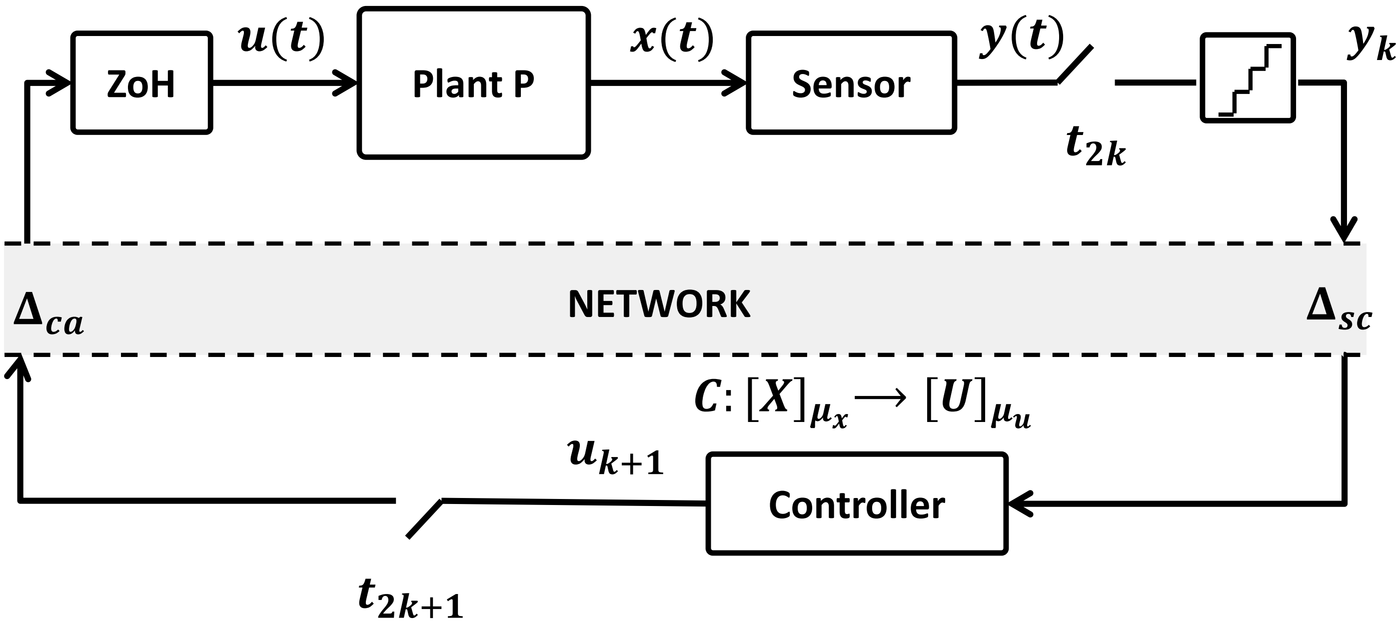

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Dealing with heterogeneity

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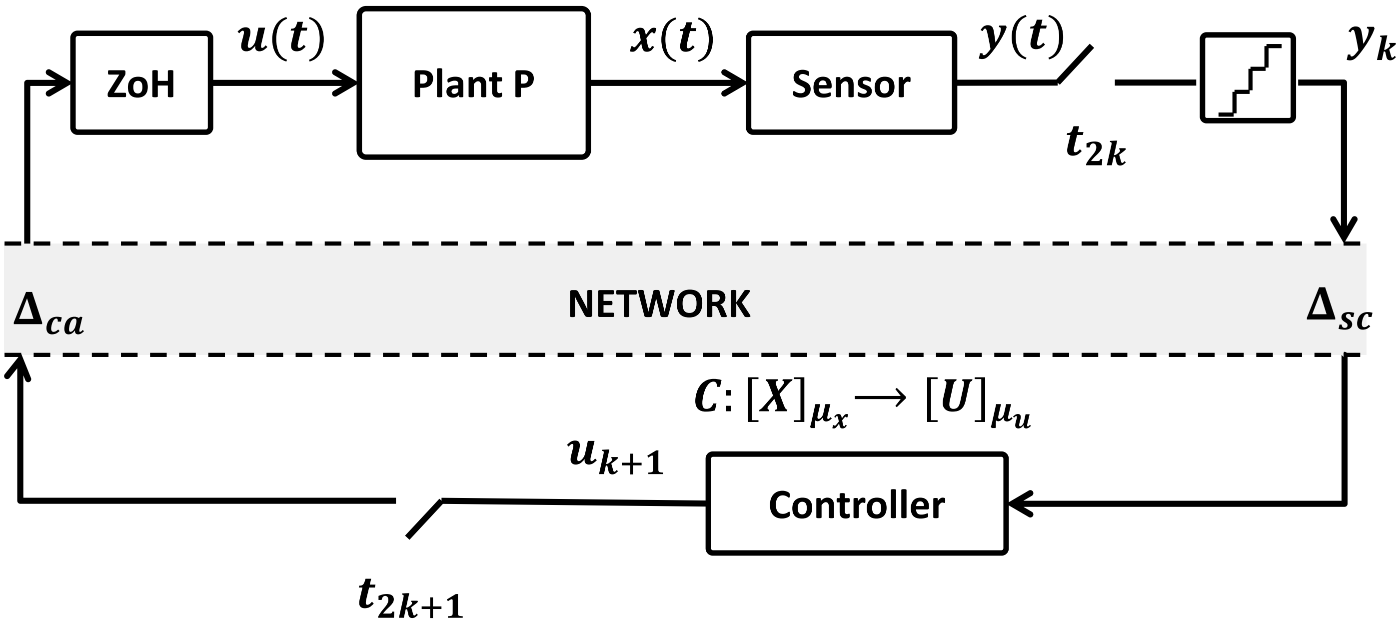

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Dealing with heterogeneity

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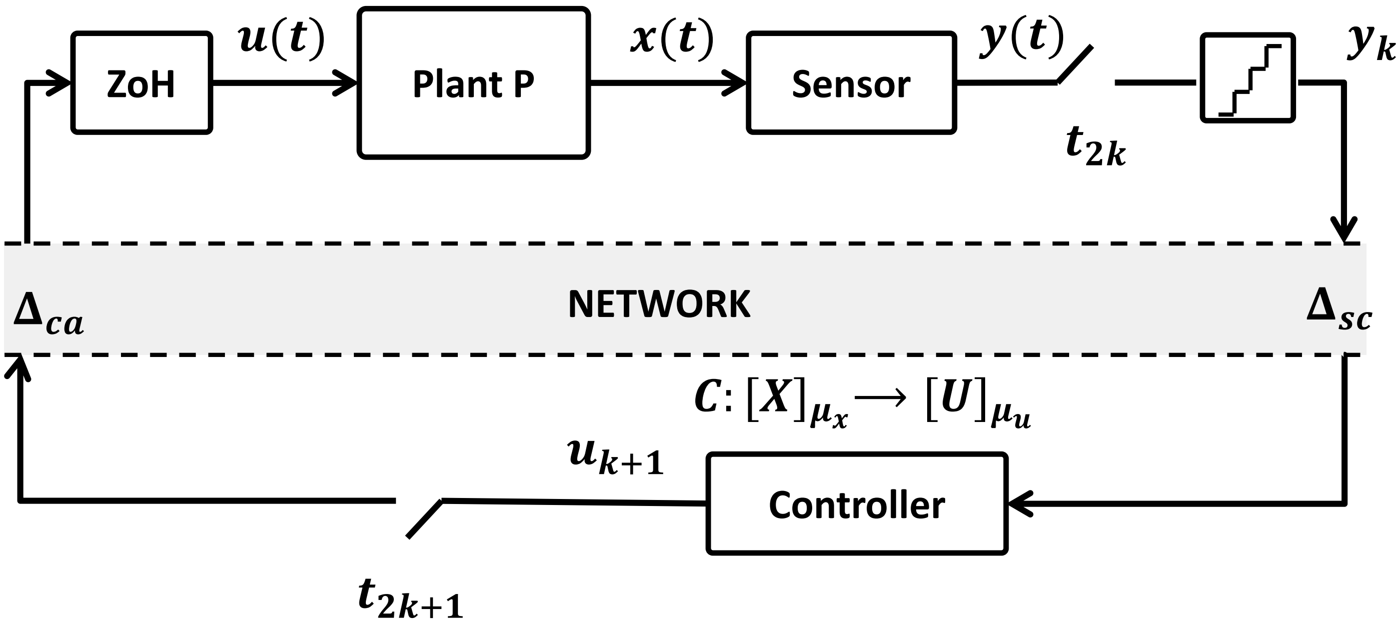

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u	0	0	0	u_1	u_1	u_1	u_1	u_1	u_1	u_2	…
x	x(0)	x(τ)	x(2τ)	x(3τ)	x(4τ)	x(5τ)	x(6τ)	x(7τ)	x(8τ)	x(9τ)	…
	$N_1 = 4$				$N_2 = 6$						…

Dealing with heterogeneity

Nonlinear Networked control systems as TSs

$$(x(0),x(\tau),x(2\tau),x(3\tau)) \xrightarrow{u_1} (x(4\tau),x(5\tau),x(6\tau),x(7\tau),x(8\tau),x(9\tau))$$

t	0	τ	2τ	3τ	4τ	5τ	6τ	7τ	8τ	9τ	...
u	0	0	0	u_1	u_1	u_1	u_1	u_1	u_1	u_2	...
x	$x(0)$	$x(\tau)$	$x(2\tau)$	$x(3\tau)$	$x(4\tau)$	$x(5\tau)$	$x(6\tau)$	$x(7\tau)$	$x(8\tau)$	$x(9\tau)$	...
	$N_1 = 4$				$N_2 = 6$						...

Dealing with heterogeneity

Nonlinear Networked control systems as TSs

$(x(0),x(\tau),x(2\tau),x(3\tau)) \xrightarrow{u_2} (x(4\tau),x(5\tau),x(6\tau),x(7\tau),x(8\tau),x(9\tau))$ $(N_2=6)$

$(x(0),x(\tau),x(2\tau),x(3\tau)) \xrightarrow{u_2} (x(4\tau),x(5\tau),x(6\tau),x(7\tau))$ $(N_2=4)$

$(x(0),x(\tau),x(2\tau),x(3\tau)) \xrightarrow{u_2} (x(4\tau),x(5\tau),x(6\tau),x(7\tau),x(8\tau))$ $(N_2=5)$

t	0	τ	2τ	3τ	4τ	5τ	6τ	7τ	8τ	9τ	…
u	0	0	0	u_1	u_1	u_1	u_1	u_1	u_1	u_2	…
x	x(0)	$x(\tau)$	$x(2\tau)$	$x(3\tau)$	$x(4\tau)$	$x(5\tau)$	$x(6\tau)$	$x(7\tau)$	$x(8\tau)$	$x(9\tau)$	…
	$N_1 = 4$				$N_2 = 6$						…

Dealing with heterogeneity

Given a NCS Σ define the TS

$T(\Sigma) = (Q_\tau, Q_{0,\tau}, L_\tau, \longrightarrow_\tau, Q_{m,\tau}, O_\tau, H_\tau)$ where:

- $Q_\tau \subseteq Q_0 \cup Q_e$ where $Q_e := \bigcup_{N=N_{min}}^{N_{max}} Q^N$ and for any $q = (x_1, x_2, \ldots, x_N) \in Q^N, x_{i+1} = \boldsymbol{x}(\tau, x_i, u^-)$, $i \in [1; N-2]$, and $x_N = \boldsymbol{x}(\tau, x_{N-1}, u^+)$ for some control inputs u^-, u^+

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- $Q_{0,\tau} = Q_0$

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- $Q_{0,\tau} = Q_0$
- $L_\tau = [U]_{\mu_U}$

Dealing with heterogeneity

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- $Q_{0,\tau} = Q_0$
- $L_\tau = [U]_{\mu_U}$
- $q^1 \xrightarrow{u}_\tau q^2$ where, for some $N_1, N_2 \in [N_{min}; N_{max}]$

Dealing with heterogeneity

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- $Q_{0,\tau} = Q_0$
- $L_\tau = [U]_{\mu_U}$
- $q^1 \xrightarrow{\mathrm{u}}_\tau q^2$ where, for some $N_1, N_2 \in [N_{min}; N_{max}]$
  $x_{i+1}^1 = \boldsymbol{x}(\tau, x_i^1, u_1^-)$, $i \in [1; N_1 - 2]$
  $x_N^1 = \boldsymbol{x}(\tau, x_{N_1-1}^1, u_1^+)$

Dealing with heterogeneity

Given a NCS Σ define the TS

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  $x_{i+1}^2 = \boldsymbol{x}(\tau, x_i^2, u_2^-)$, $i \in [1; N_2 - 2]$
  $x_N^2 = \boldsymbol{x}(\tau, x_{N_2-1}^2, u_2^+)$

Dealing with heterogeneity

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- $Q_{0,\tau} = Q_0$
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- $q^1 \xrightarrow{\mathrm{u}}_\tau q^2$ where, for some $N_1, N_2 \in [N_{min}; N_{max}]$

$$x_{i+1}^1 = \boldsymbol{x}(\tau, x_i^1, u_1^-), i \in [1; N_1 - 2]$$
$$x_N^1 = \boldsymbol{x}(\tau, x_{N_1-1}^1, u_1^+)$$
$$x_{i+1}^2 = \boldsymbol{x}(\tau, x_i^2, u_2^-), i \in [1; N_2 - 2]$$
$$x_N^2 = \boldsymbol{x}\left(\tau, x_{N_2-1}^2, u_2^+\right)$$
$$u_2^- = u_1^+$$
$$u_2^+ = u$$
$$x_1^2 = \boldsymbol{x}(\tau, x_{N_1}^1, u_2^-)$$

Dealing with heterogeneity

Given a NCS Σ define the TS

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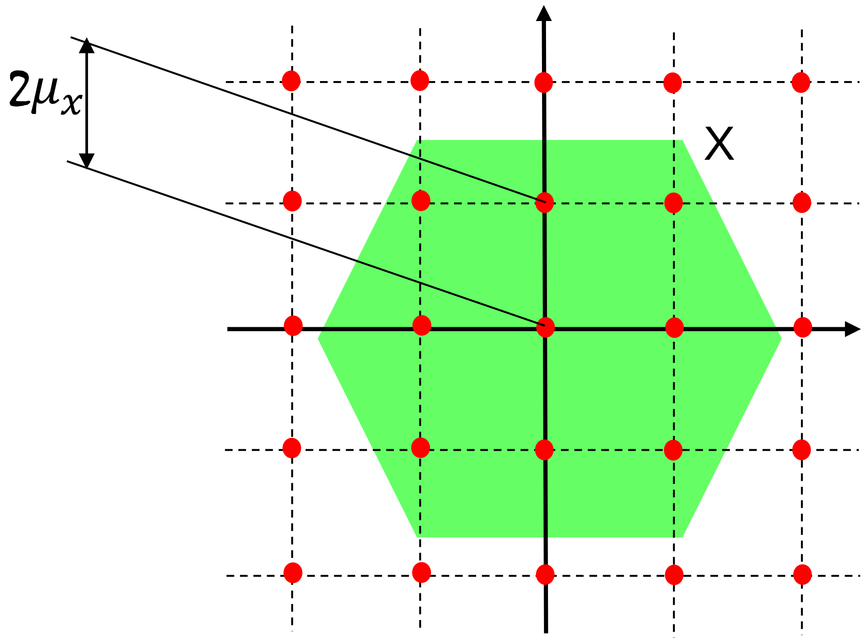
- $Q_{0,\tau} = Q_0$
- $L_\tau = [U]_{\mu_U}$
- $q^1 \xrightarrow{u}_\tau q^2$ where, for some $N_1, N_2 \in [N_{min}; N_{max}]$
  $x^1_{i+1} = \boldsymbol{x}(\tau, x^1_i, u^-_1), i \in [1; N_1 - 2]$
  $x^1_N = \boldsymbol{x}(\tau, x^1_{N_1-1}, u^+_1)$
  $x^2_{i+1} = \boldsymbol{x}(\tau, x^2_i, u^-_2), i \in [1; N_2 - 2]$
  $x^2_N = \boldsymbol{x}(\tau, x^2_{N_2-1}, u^+_2)$
  $u^-_2 = u^+_1$
  $u^+_2 = u$
  $x^2_1 = \boldsymbol{x}(\tau, x^1_{N_1}, u^-_2)$
- $Q_{m,\tau} = Q_\tau$
- $O_\tau = X_\tau$
- H_τ is the identity function

Symbolic models for NCS

T(Σ) collects all the information of the NCS Σ available at the sensor, but it is not a symbolic model. We therefore propose a symbolic model by quantizing the state space X of the plant P



Given x ∈ X let $[x]_{\mu_X} \in [X]_{\mu_X}$ be such that $|x - [x]_{\mu_X}| \leq \mu_X$

Symbolic models for NCS

Define the transition system $T^*(\Sigma) = (Q_*, Q_{0,*}, L_*, \longrightarrow_*, Q_{m,*}, O_*, H_*)$ where:

- $Q_* \subseteq [Q_0 \cup Q_e]_{\mu_x}$ s.t. for any $q^* = (x_1^*, x_2^*, \dots, x_N^*) \in Q_*$, $x_{i+1}^* = [\boldsymbol{x}(\tau, x_i^*, u_*^-)]_{\mu_x}$, $i \in [1; N-2]$, and $x_N^* = [\boldsymbol{x}(\tau, x_{N-1}^*, u_*^+)]_{\mu_x}$ for some u_*^-, u_*^+
- $Q_{0,*} = [X_0]_{\mu_x}$
- $L_* = [U]_{\mu_u}$
- $q^1 \longrightarrow_*^{u_*} q^2$ where, for some N_1, N_2

  $x_{i+1}^1 = [\boldsymbol{x}(\tau, x_i^1, u_1^-)]_{\mu_x}$, $i \in [1; N-2]$

  $x_N^1 = [\boldsymbol{x}(\tau, x_{N_1-1}^1, u_1^+)]_{\mu_x}$

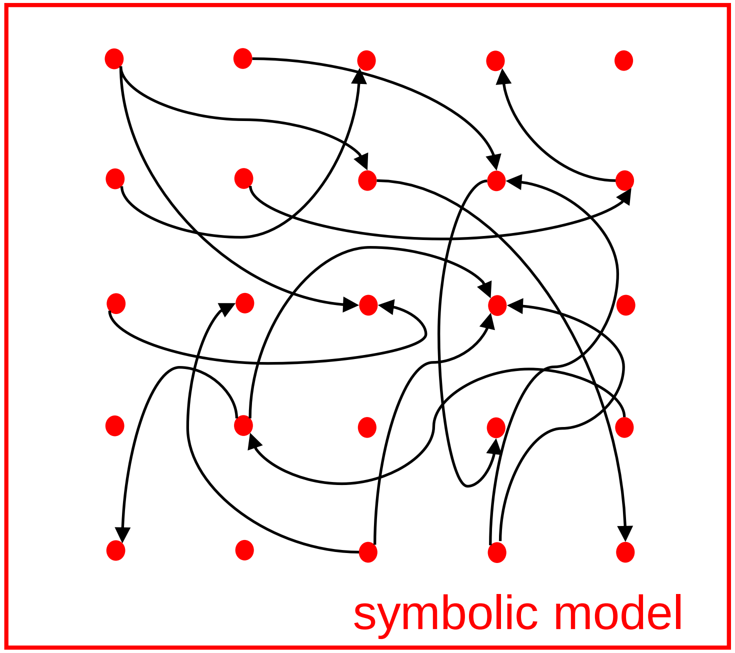
  $x_{i+1}^2 = [\boldsymbol{x}(\tau, x_i^2, u_2^-)]_{\mu_x}$, $i \in [1; N-2]$

  $x_N^2 = [\boldsymbol{x}(\tau, x_{N_2-1}^2, u_2^+)]_{\mu_x}$

  $u_2^- = u_1^+$

  $u_2^+ = u_*$

  $x_1^2 = [\boldsymbol{x}(\tau, x_{N_1}^1, u_2^-)]_{\mu_x}$
- $Q_{m,*} = Q_*$
- $O_* = X_\tau$
- H_* is the identity function



Symbolic models for NCS

Theorem 1 [HSCC-2012]

Consider the NCS Σ and suppose that the plant nonlinear control system P enjoys the following properties:

1. There exists a δ-GAS Lyapunov function for Σ, hence there exists $\lambda \in \mathbb{R}^+$ s.t. for any $x_1, x_2 \in X$, and any $u \in U$

$$\frac{\partial V}{\partial x_1} f(x_1, u) + \frac{\partial V}{\partial x_2} f(x_2, u) \leq -\lambda V(x_1, x_2).$$

2. There exists a K_∞ function γ such that $V(x, x') \leq V(x, x'') + \gamma(+|x' - x''|)$ for every $x, x', x'' \in X$.

Then for any desired precision $\varepsilon > 0$, any sampling time $\tau > 0$, and any state quantization $\mu_x > 0$ such that

$$\mu_x \leq min\left\{\gamma^{-1}\left(\left(1 - e^{-\lambda\tau}\right)\underline{\alpha}(\varepsilon)\right), \bar{\alpha}^{-1}(\underline{\alpha}(\varepsilon)), \hat{\mu}_X\right\}$$

transition systems $T(\Sigma)$ and $T^*(\Sigma)$ are ε-alternatingly bisimilar

Symbolic models

Theorem [HSCC-2012]

For any δ-GAS nonlinear NCS Σ with compact state and input spaces and for any precision ε there exists a symbolic transition system $T^*(\Sigma)$ that is an ε-alternating approximate bisimulation of Σ and that can be effectively computed

Theorem [IEEE-CDC-2012]

For any δ-FC nonlinear NCS Σ with compact state and input spaces, for any precision ε, there exists a symbolic transition system $T^*(\Sigma)$ that is an ε-alternating approximate simulation of Σ and that can be effectively computed

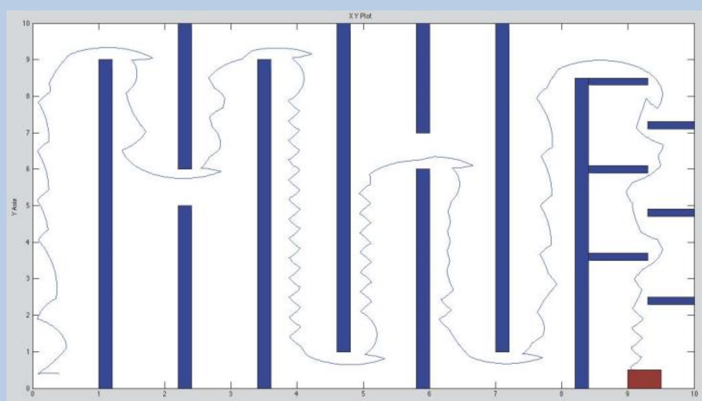
Symbolic control design

<u>Class of specifications</u>

Non-deterministic finite automata on infinite strings

<u>Examples:</u>

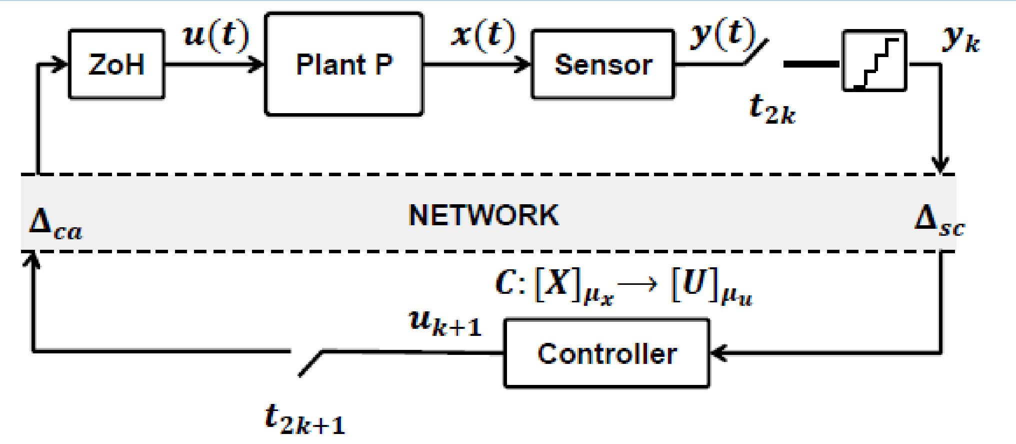
- Language specifications (e.g. robot motion planning)
- Synchronization specifications (e.g. starting from region A reach region B passing through region C in 1s)
- Obstacle avoidance (e.g. starting from region A, reach region B in finite time, while avoiding region C)
- Switching specifications (e.g. rotate clockwise in a certain region of the state space and rotate counter-clockwise in other regions of the state space)
- …



Symbolic control design

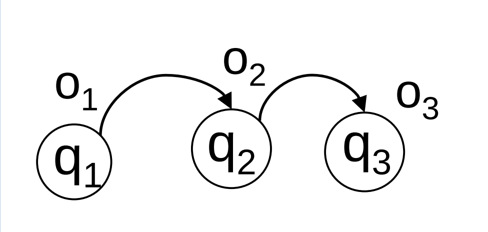
Problem formulation:

Given a NCS Σ, a specification LTS S and a desired precision ε > 0, find a symbolic controller that implements S (up to precision ε) robustly with respect to the non–idealities of the communication network and that is alive when interacting with Σ



Networked Control System Σ

$\preccurlyeq_\varepsilon$

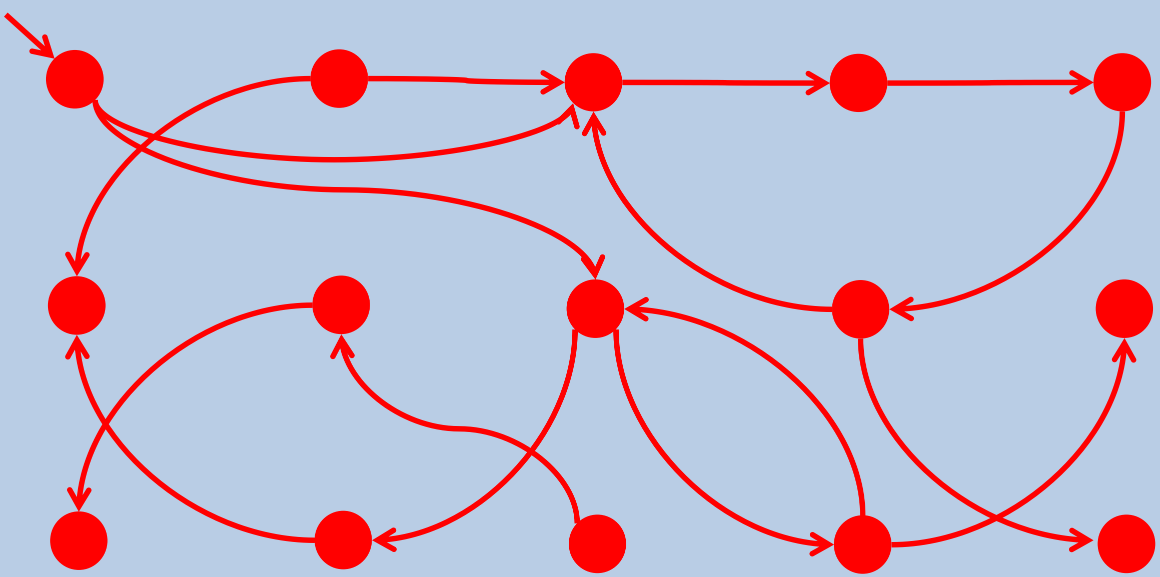


Specification LTS S

Symbolic control design

Synthesis through a three-step process:

1. Compute the symbolic model $T^*(\Sigma)$ of Σ
2. Compute the approximate parallel composition $C^* = T^*(\Sigma) \,||_{\mu x}\, S$
3. Compute the maximal alive part Alive(C^*) of C^*



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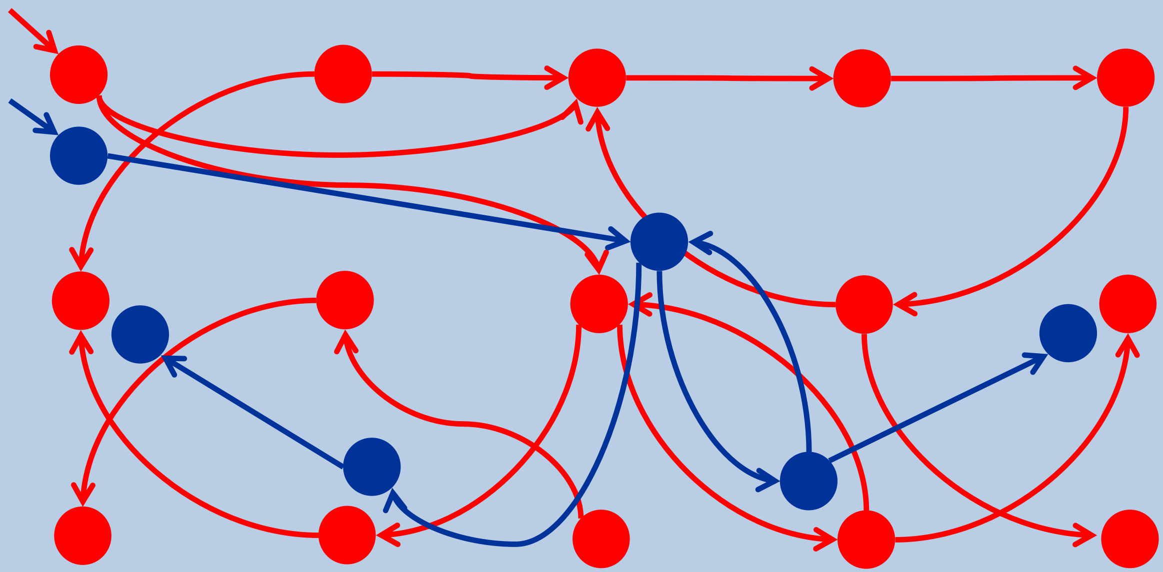
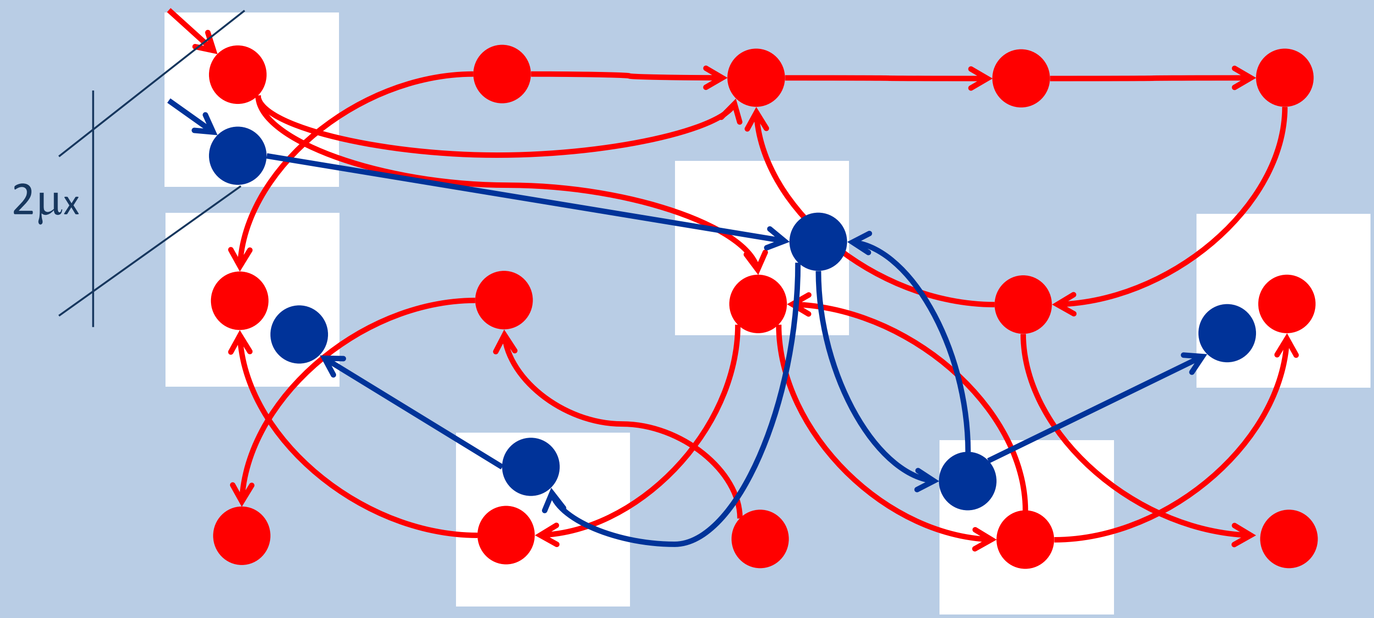


Symbolic control design

<u>Synthesis through a three-step process:</u>

1. Compute the symbolic model T*(Σ) of Σ
2. Compute the approximate parallel composition $C^* = T^*(\Sigma) \,||_{\mu x}\, S$
3. Compute the maximal alive part Alive(C*) of C*



Symbolic control design

Synthesis through a three-step process:

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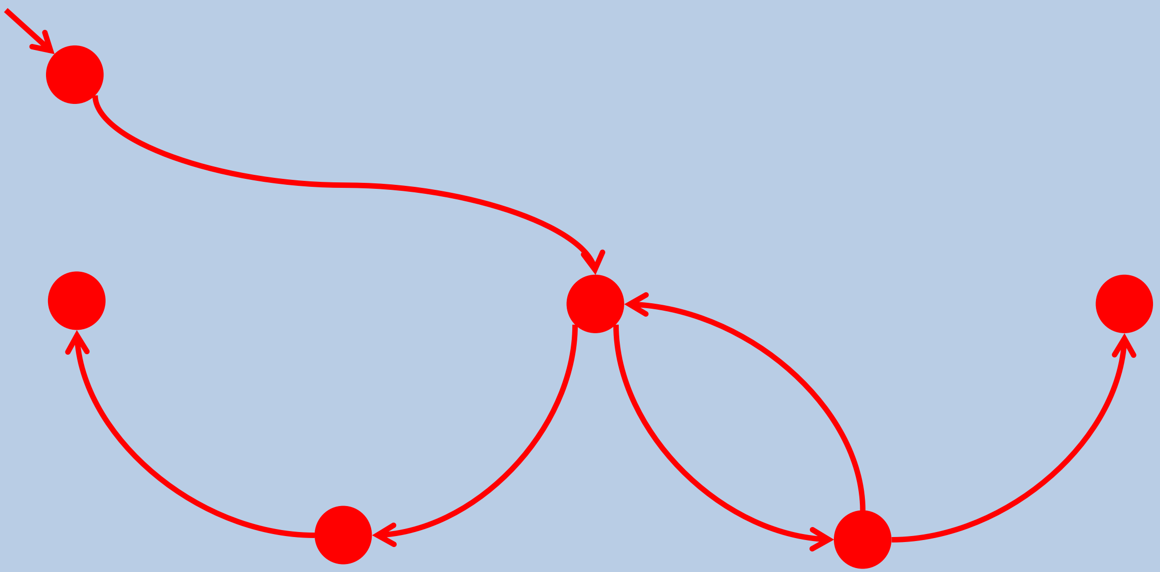

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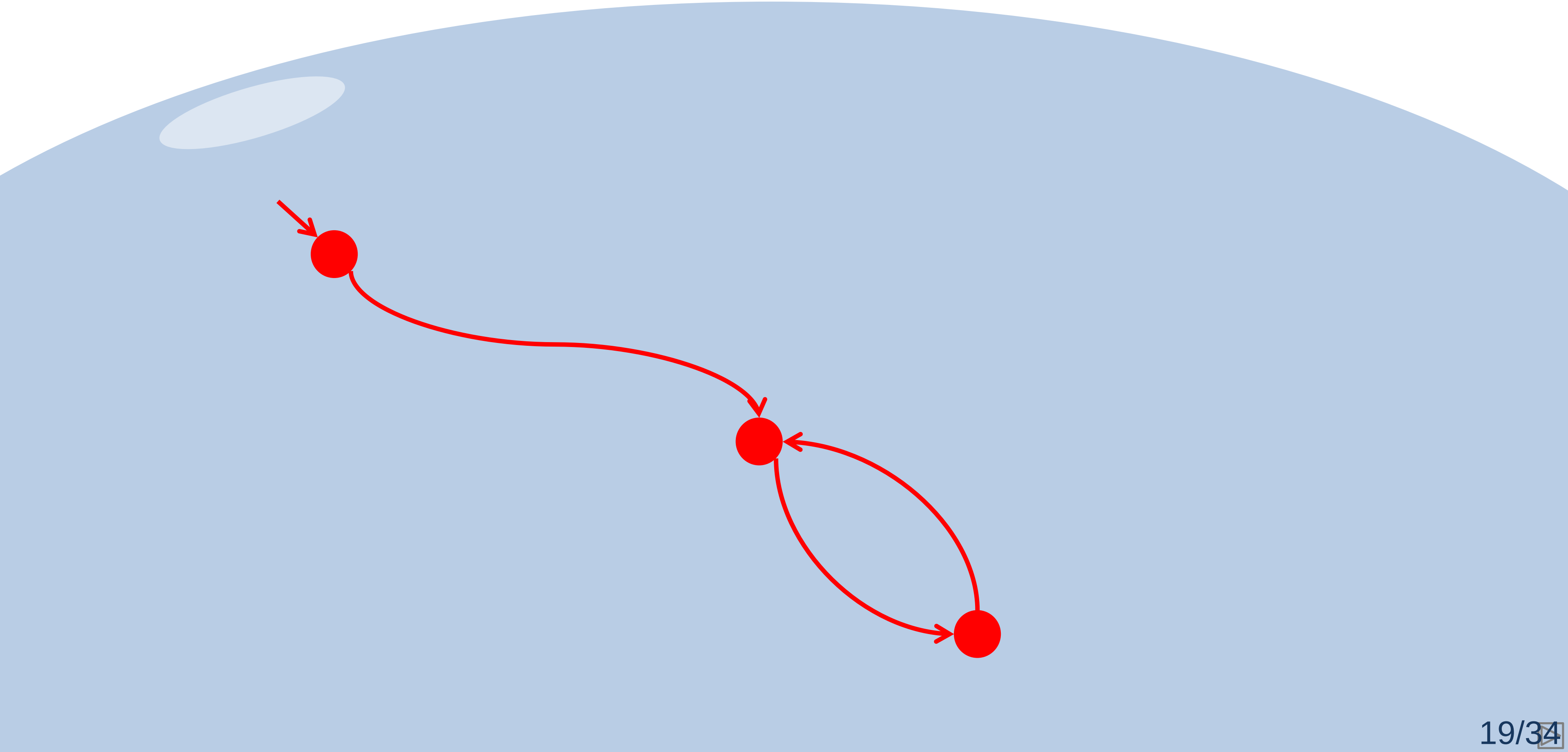

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Drawback

High computational complexity!

Symbolic control design

Synthesis through a three-step process:

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Drawback

High computational complexity!

Efficient on-the-fly (off-line) algorithms that integrate the synthesis of Alive(C^*) with the construction of $T^*(\Sigma)$

[Pola, Borri, Di Benedetto, IEEE-TAC-2012]

Symbolic control design

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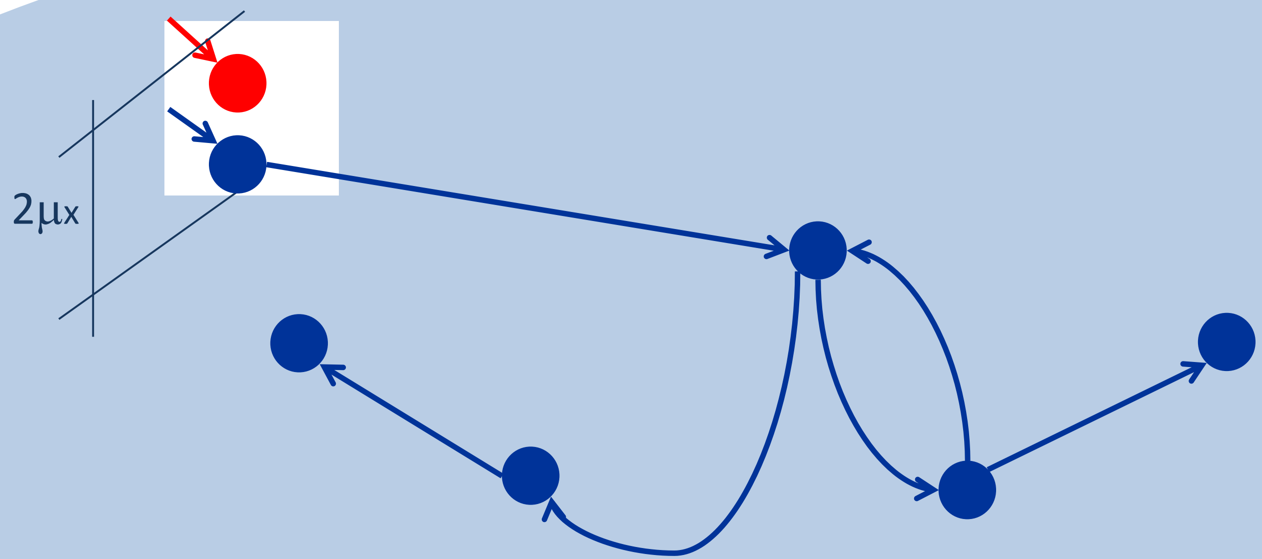

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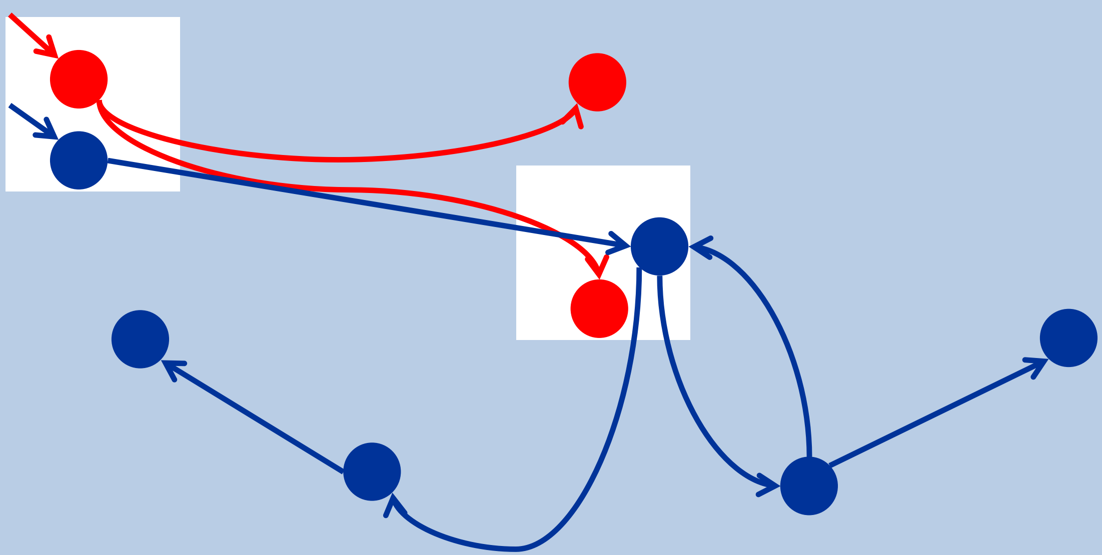

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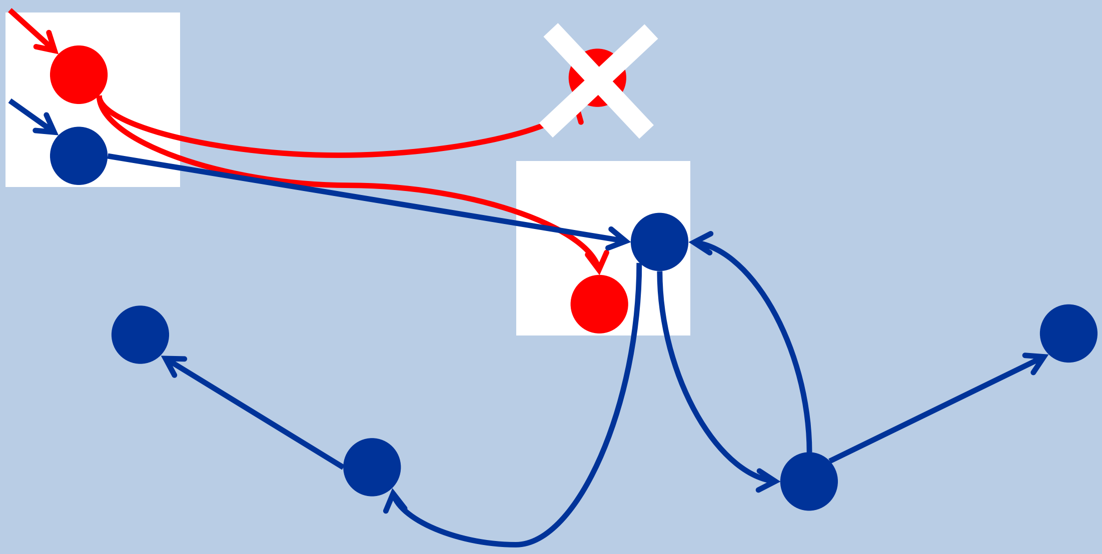

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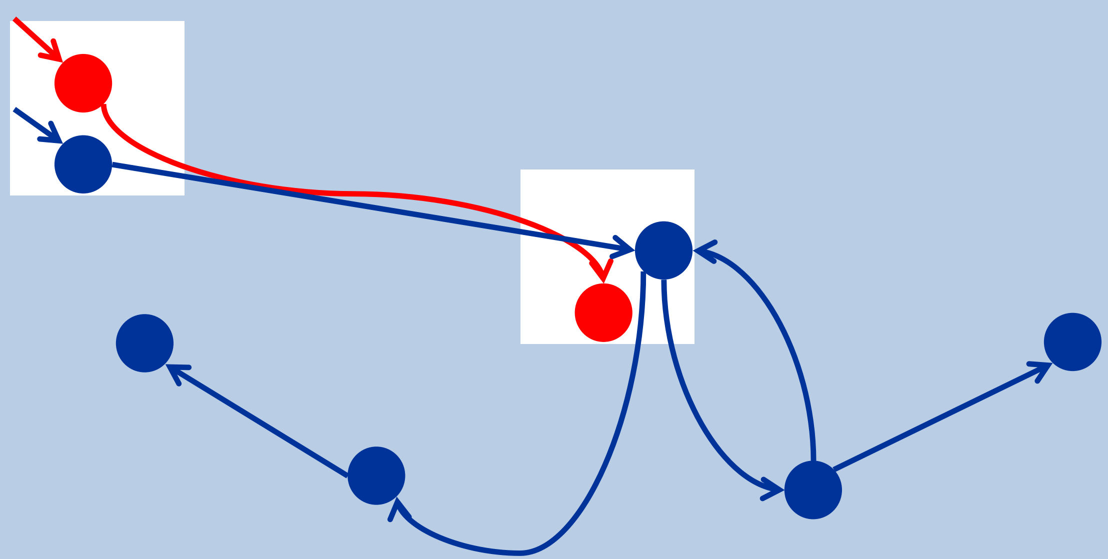

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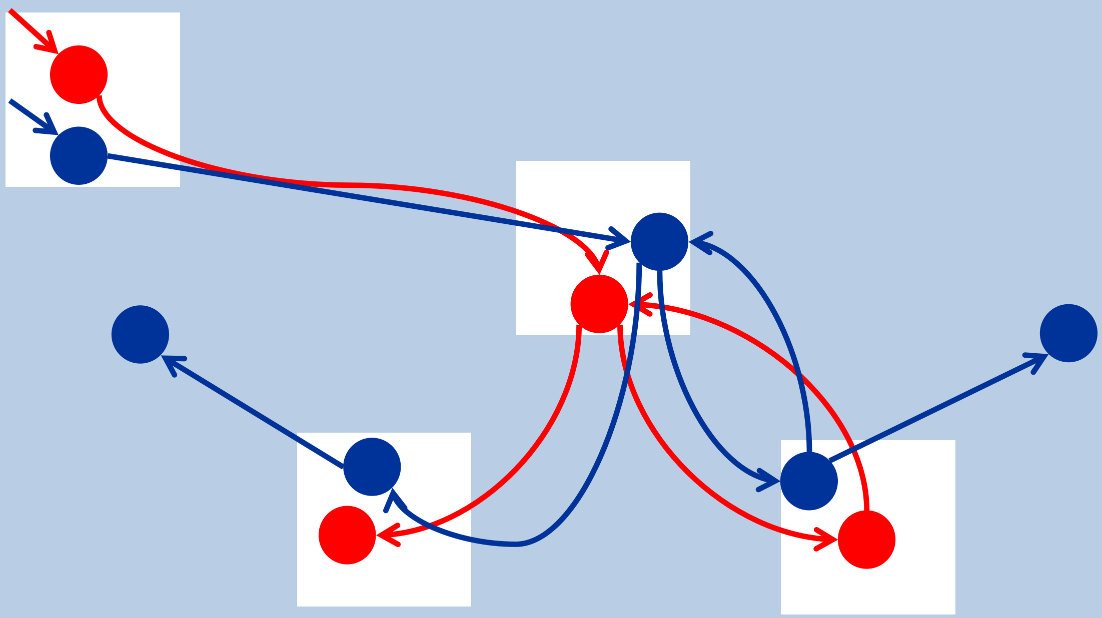

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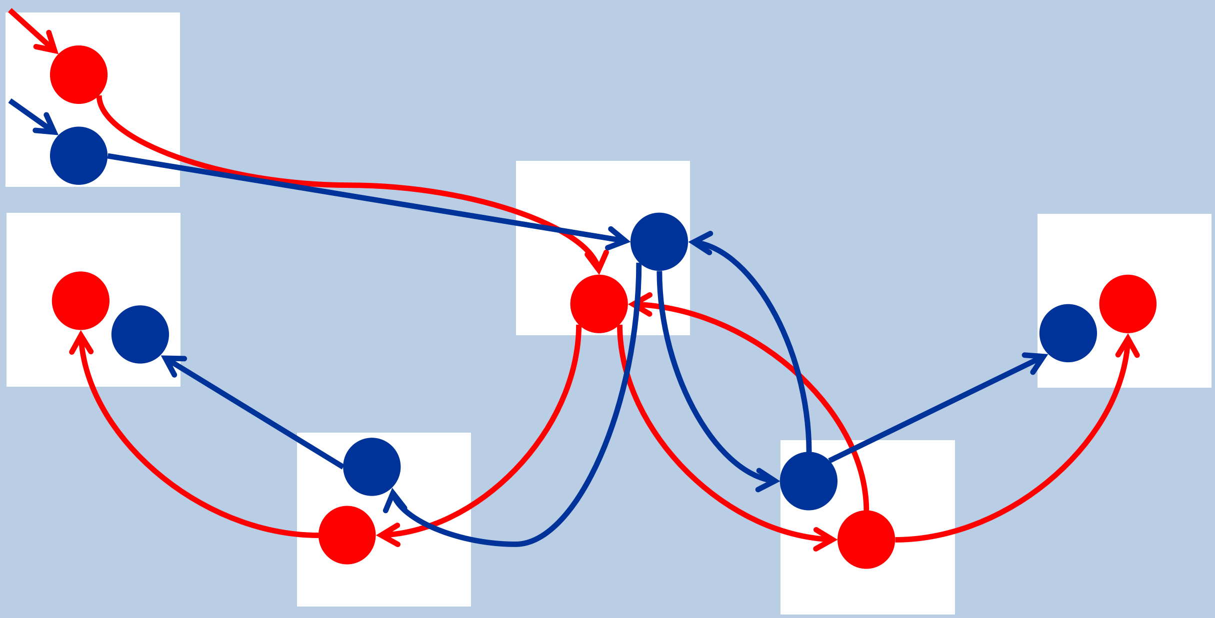

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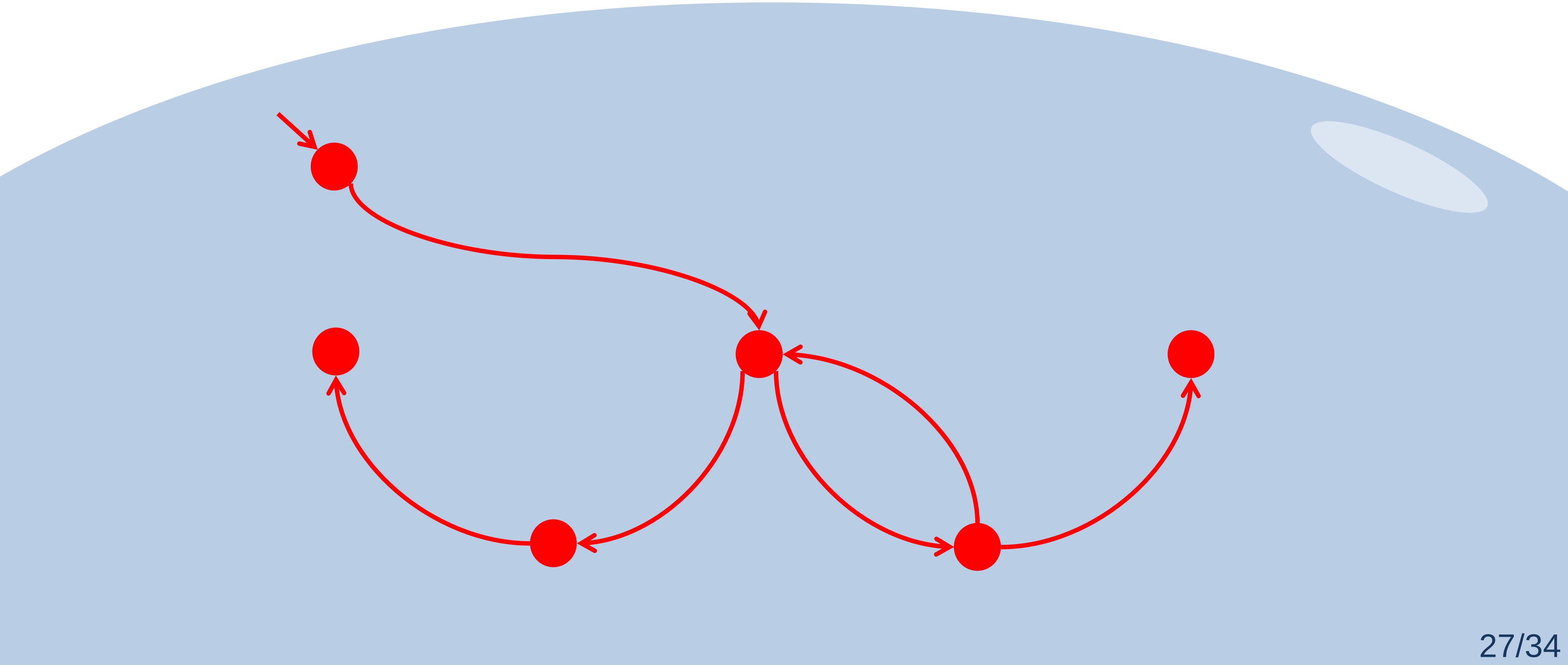

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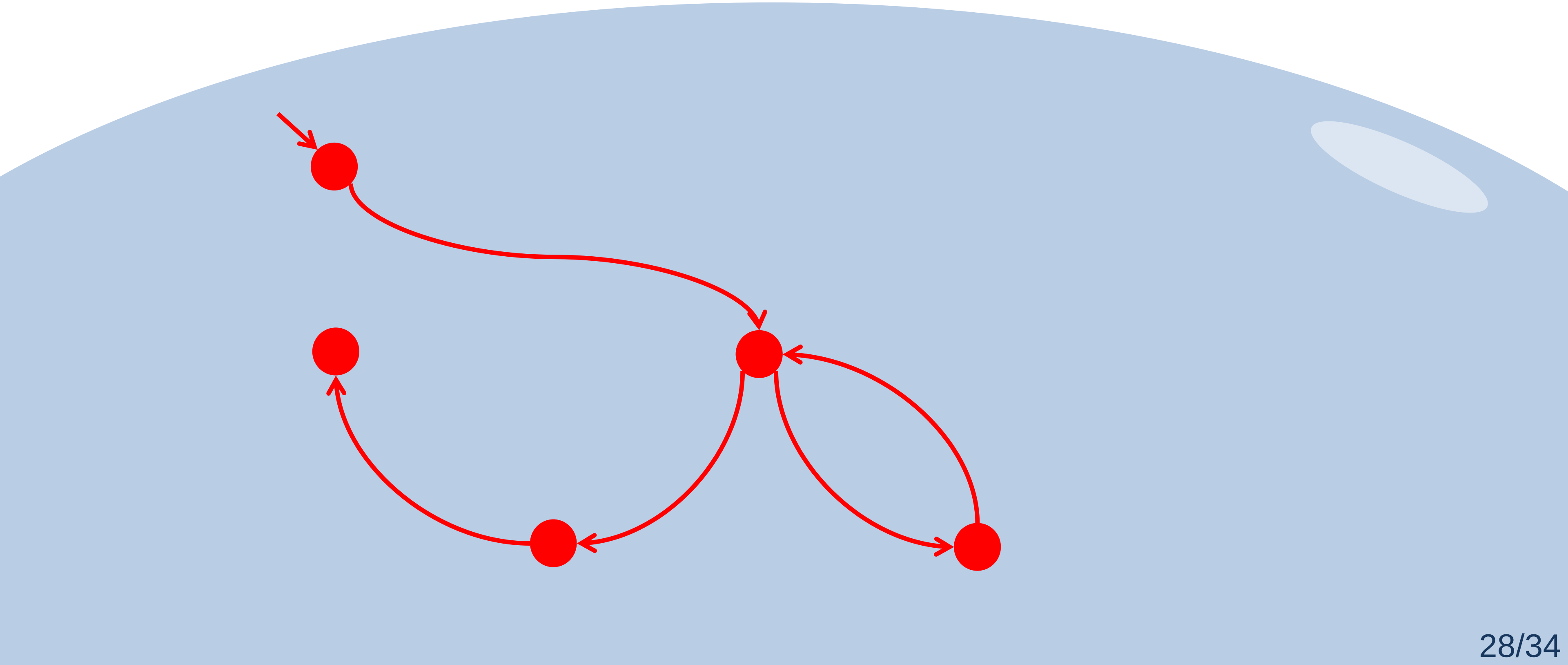

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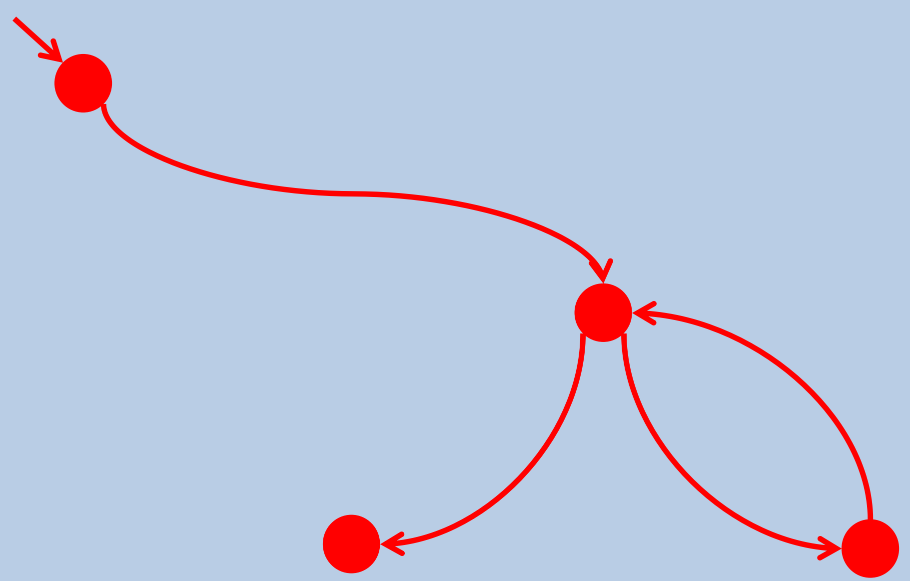

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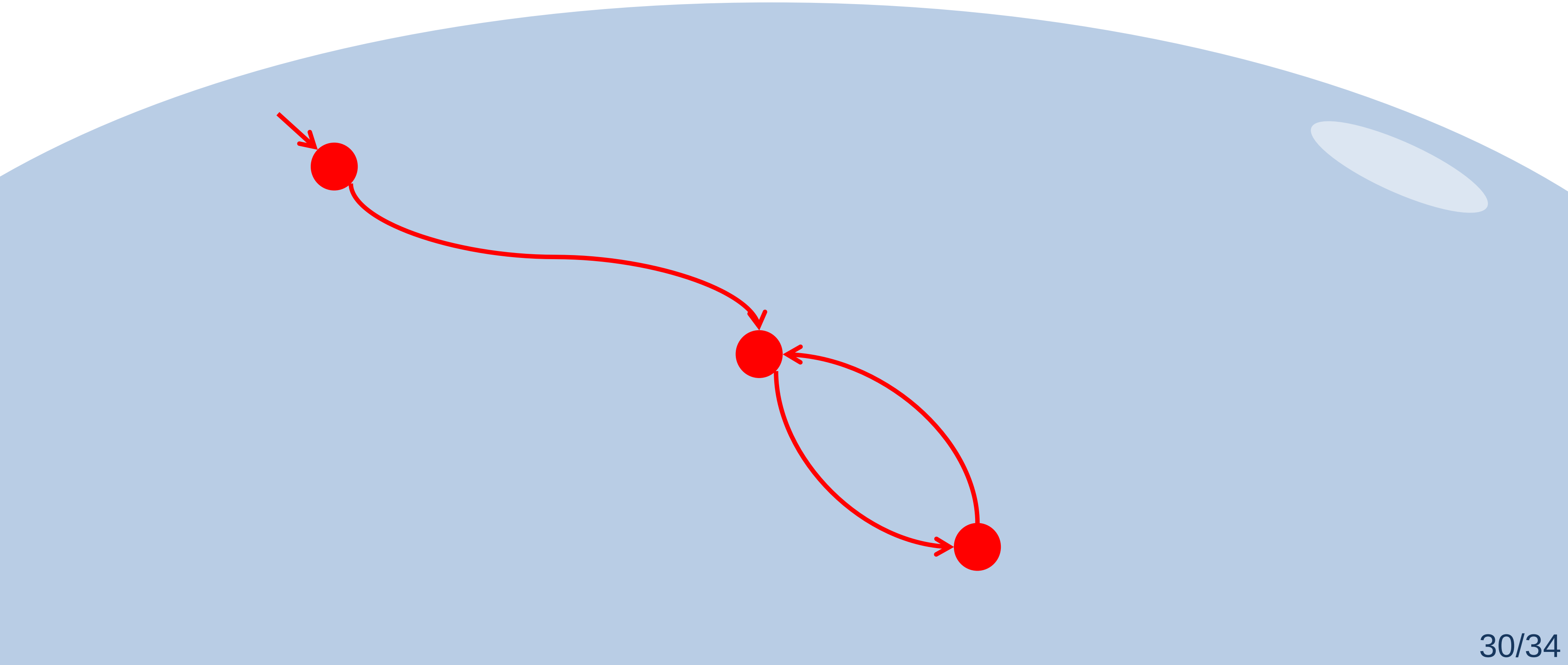
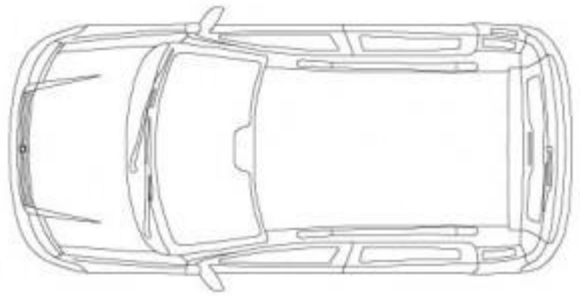


Example

We consider a widely used nonlinear vehicle model, controlled over a non-reliable network:

P: $$\dot{\boldsymbol{x}} = \begin{bmatrix} \dot{\boldsymbol{x}}_1 \\ \dot{\boldsymbol{x}}_2 \\ \dot{\boldsymbol{x}}_3 \end{bmatrix} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) = \begin{bmatrix} \boldsymbol{u}_1 \cos \boldsymbol{x}_3 \\ \boldsymbol{u}_1 \sin \boldsymbol{x}_3 \\ \boldsymbol{u}_2 \end{bmatrix}$$



State space $x \in X = X_0 = [-1,1[\times [-1,1[\times [-\pi, \pi[$

Input space $u \in U = [-1,1[\times [-1,1[$

Sampling/quantization parameters $\tau = 0.2\ s$ $\mu_x = 0.02$ $\mu_u = 0.25$

The plant P is δ-FC with $\lambda = 2$

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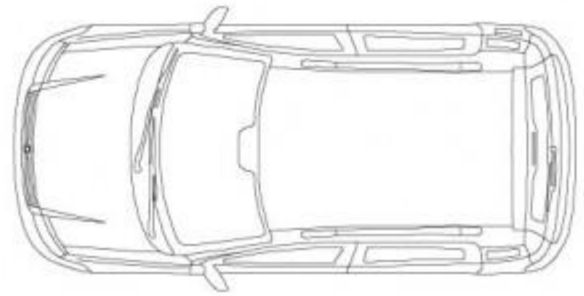
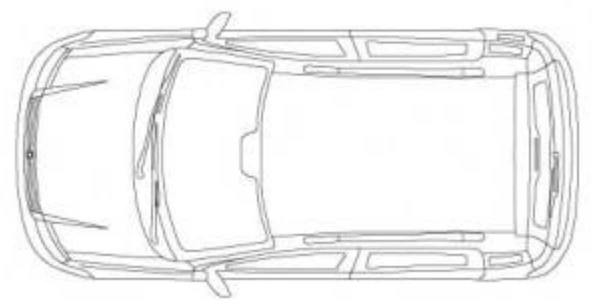
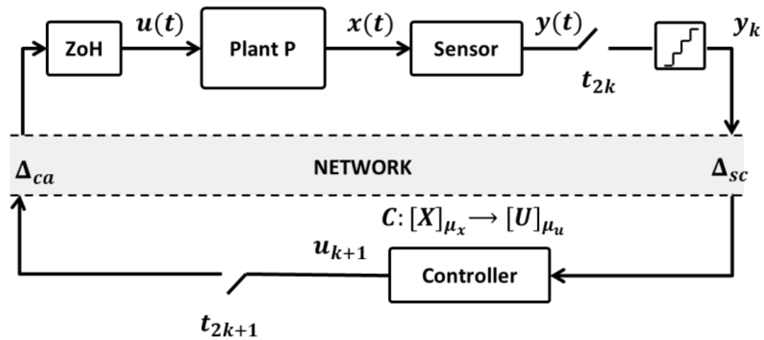


Bandwidth	$B_{max} = 1\ kbit/s$
Maximum control computation time	$\Delta_{max}^{ctrl} = 0.01\ s$
Maximum waiting time to access the network	$\Delta_{max}^{req} = 0.05\ s$
Maximum network delays	$\Delta_{max}^{delay} = 0.1\ s$

Example

Required precision $\varepsilon = 0.15$





$N_{min} = \lceil \Delta_{min} / \tau \rceil = 1$ Δ_{min} = (total) minimum delay allowed

$N_{max} = \lceil \Delta_{max} / \tau \rceil = 2$ Δ_{max} = (total) maximum delay allowed

Robust control design problem: enforce trajectories in the state space independently from the realization of the network uncertainties.

This example shows how formal methods offer a systematic approach to deal with complex specifications, such as obstacle avoidance and path planning problems *in the presence of nonideal communication infrastructure*, which is often the case in concrete applications.

Example

Motion planning with obstacle avoidance