Formal Methods for the Control of Large-scale Networked Nonlinear Systems with Logic Specifications

Basilica di Santa Maria di Collemaggio, L'Aquila (Italy), 1287

Lecture L13:

Decentralized control of networks of nonlinear systems

Speaker: Giordano Pola

What's new?

In this course:

Control of

Large-scale Heterogeneous

Networked Nonlinear Systems

with Logic Specifications

Lecture based on:

[Pola et al., TAC17] Pola, Pepe, Di Benedetto, Decentralized Supervisory Control of Networks of Nonlinear Control Systems, 2016, submitted, http://arxiv.org/abs/1606.04647

[Pola et al., TAC16] Pola, Pepe, Di Benedetto, Symbolic Models for Networks of Control Systems, IEEE Transactions on Automatic Control, 61(11):3663-3668, November 2016

What's new?

To recap:

Control of

Large-scale Heterogeneous

Networked Nonlinear Systems

with Logic Specifications

Lectures

- L3, L5, L6, L8, L10, L11

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- L4, L7

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- L3, L5, L6, L8, L10, L11
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- L12

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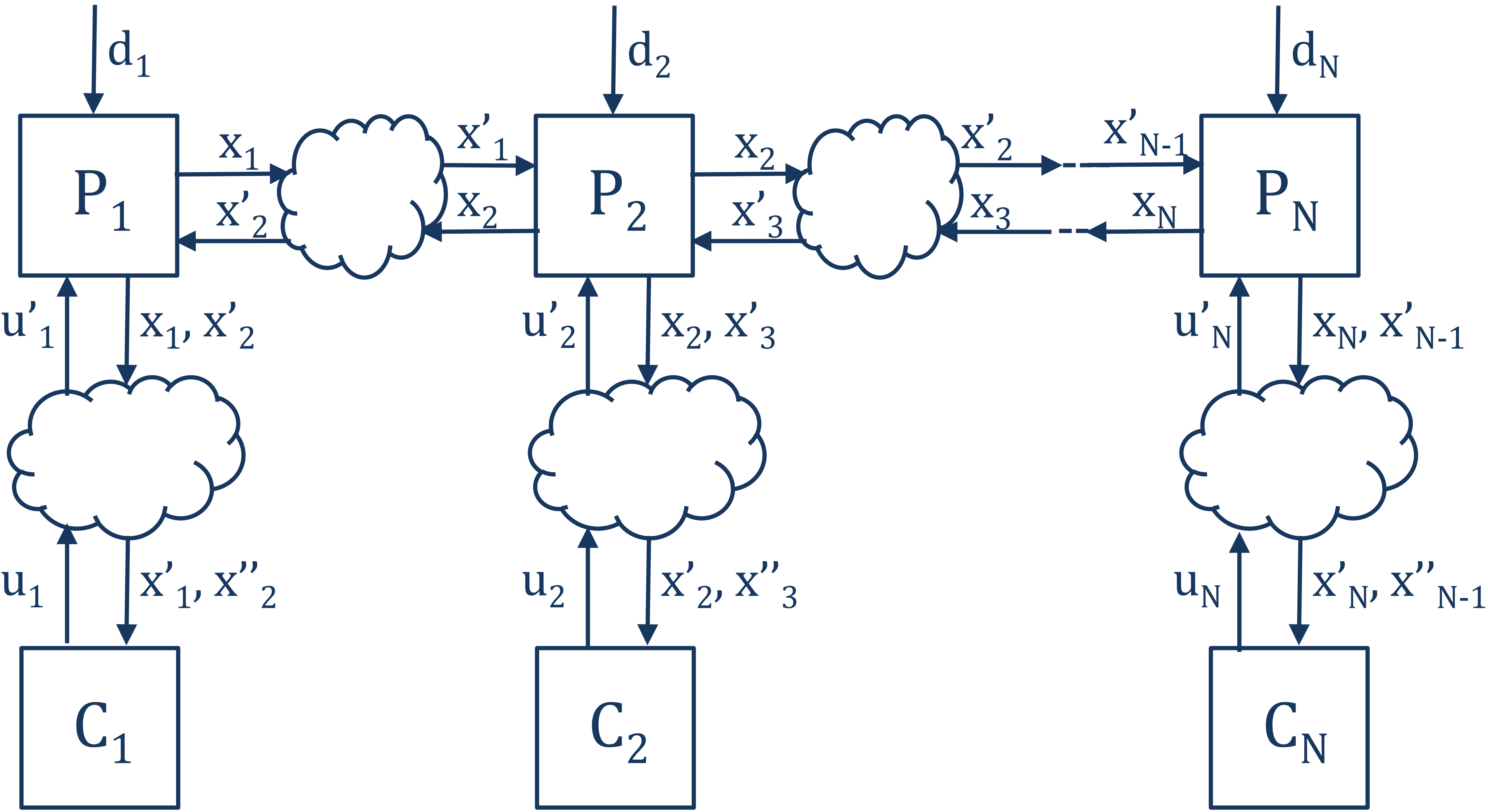
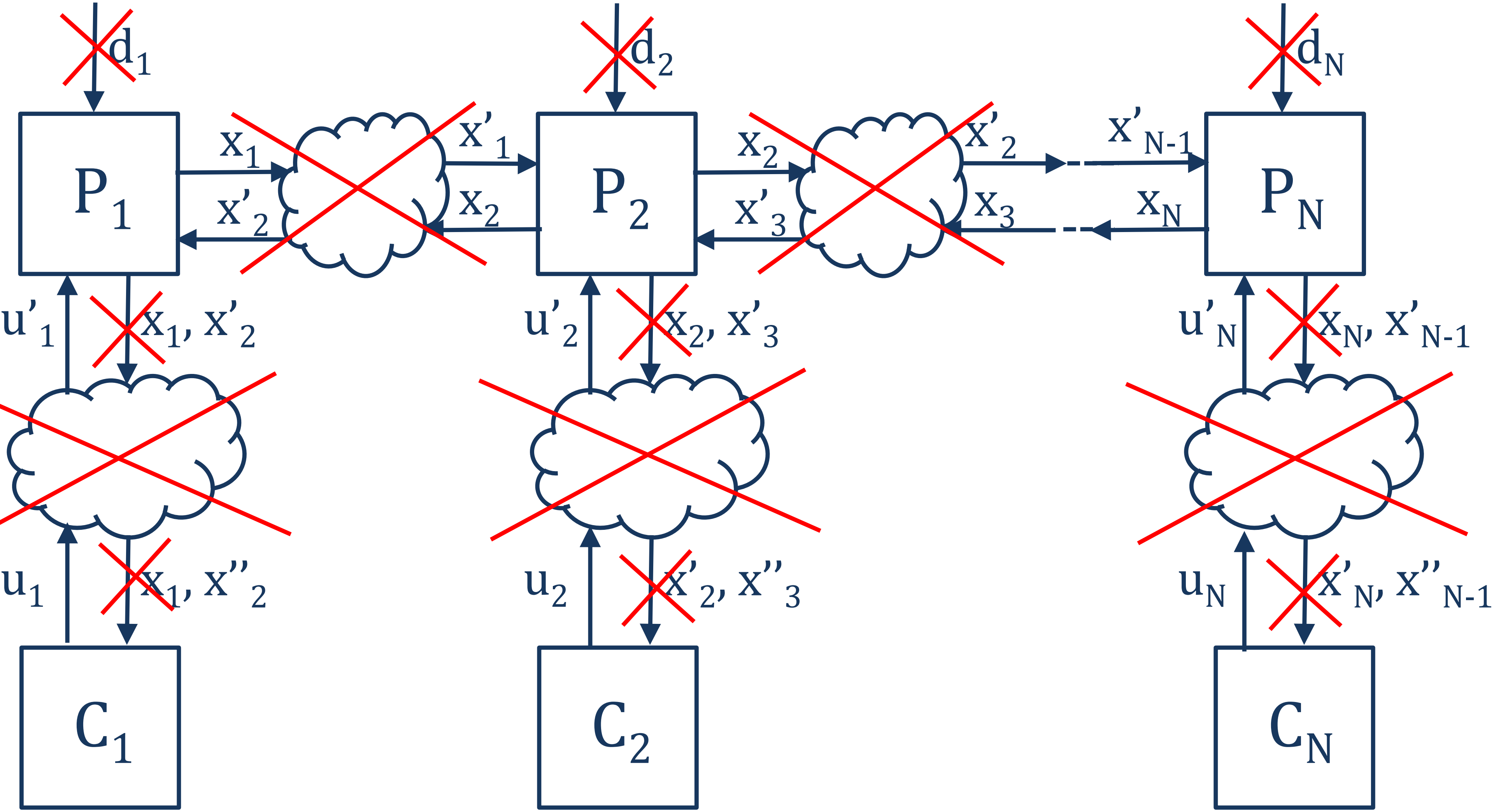
Decentralized control architecture

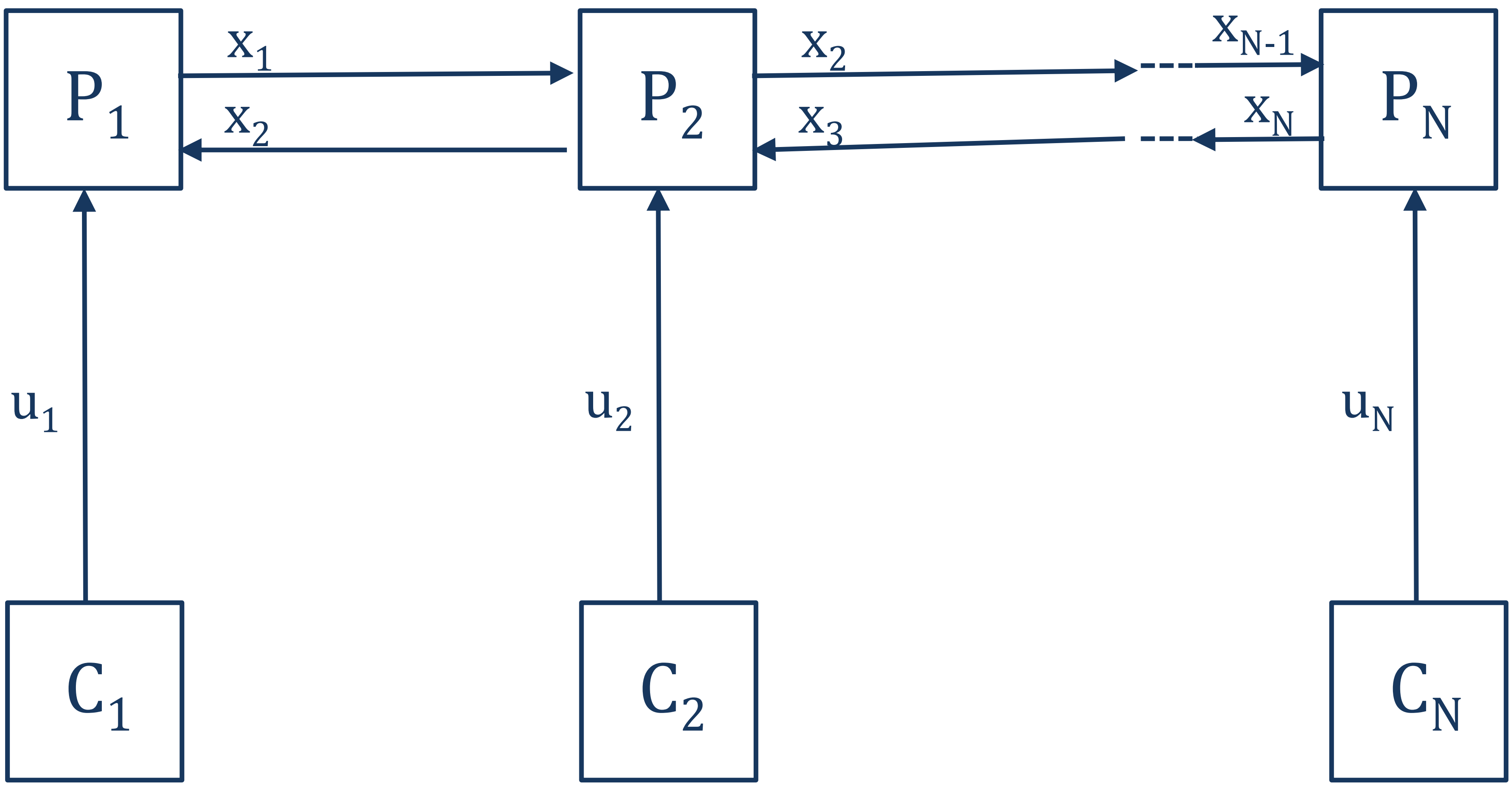


The model we consider here …



Main assumptions: Ideal communication infrastructures, no disturbance inputs, no state delays and systems in discrete-time

The model we consider here …



Main assumptions: Ideal communication infrastructures, no disturbance inputs, no state delays and systems in discrete-time

The model we consider here …

Plant P_i described by the nonlinear discrete-time system

$$P_i: \begin{cases} x_i(t+1) = f_i(x_i(t), x_j(t), \dots, u_i(t)) \\ x_i(t) \in \mathbb{R}^{n_i}, x_j(t) \in \mathbb{R}^{n_j}, u_i(t) \in U_i \end{cases}$$

where:

- $x_i(t)$ is the internal state
- $x_j(t)$ is an external measurable input (corresponding to the internal state of P_j)
- $u_i(t)$ is the control input where set U_i is finite

The model we consider here …

Controller C_i described by the automaton

$$C_i: \begin{cases} z_i(t+1) = g_i\big(z_i(t)\big) \\ u_i(t) \in h_i\big(z_i(k)\big) \\ z_i(t) \in Z_i, z_i(t) \in Z_{i,0} \subseteq Z_i, u_i(t) \in U_i \end{cases}$$

where:

- $z_i(t) \in Z_i$ is the internal state and Z_i is a finite set
- $u_i(t) \in U_i$ is the output and set U_i is a finite set

Features of C_i:

- finite
- dynamic
- open-loop
- state deterministic, but
- output nondeterministic

Control Problem Formulation

Given

- the network of control systems P_i
- a regular language specification L_Q defined over a finite subset Y_Q of $\mathbb{R}^n$
- a desired accuracy $\theta > 0$

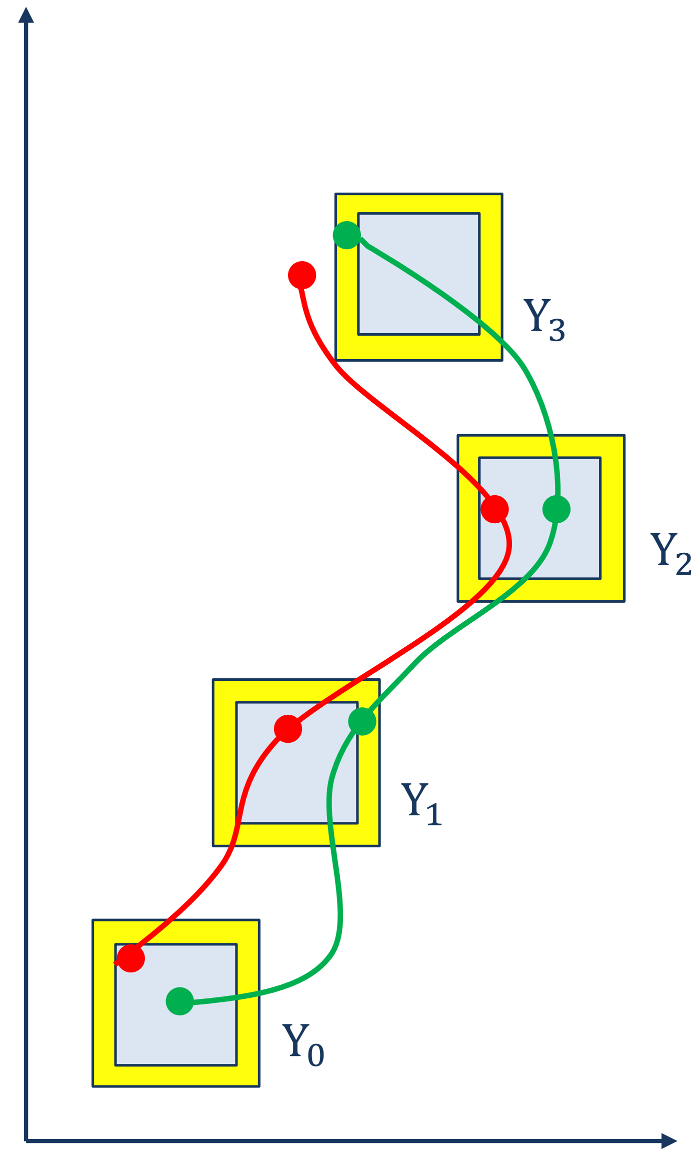
Find

- a set of initial states $X_0 \subseteq \mathbb{R}^n$
- a collection of decentralized controllers C_i

such that the controlled network, denoted P^C, satisfies the specification L_Q up to the accuracy θ, i.e.

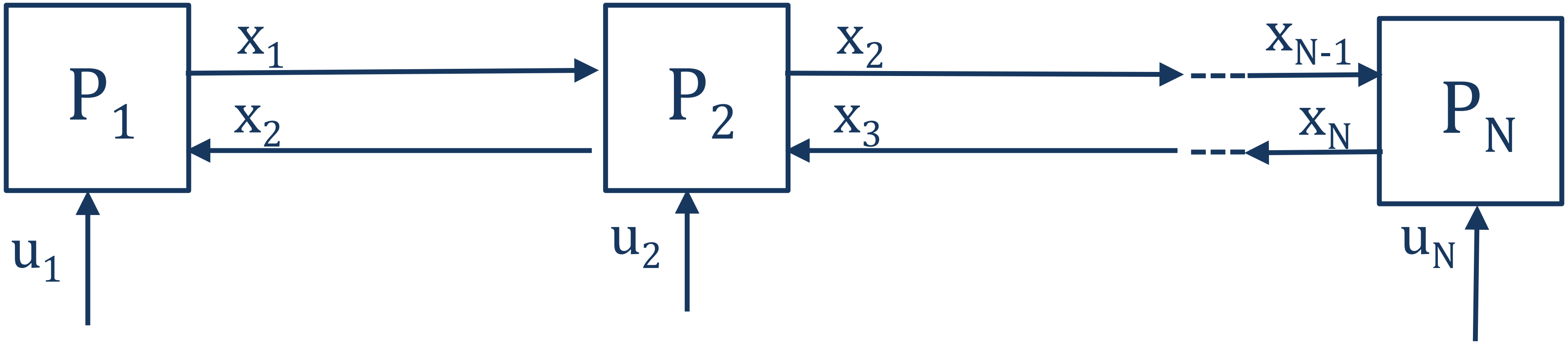
for any trajectory $x(.)$ of P^C with $x(0) \in X_0$, there exists a word $q_0 q_1 \ldots q_{t_f}$ of the specification L_Q such that

$$| x(t) - q_t | \leq \theta, \text{ for all } t \in [0; t_f]$$

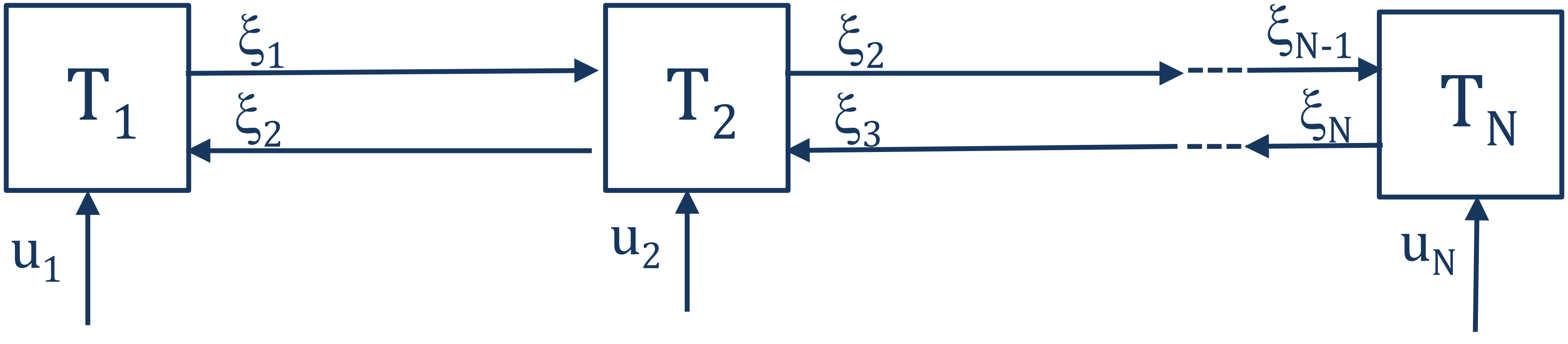


The main idea …

From networks of nonlinear control systems …



… to networks of symbolic models T_i each one approximating P_i



Transition systems' representation of the network

Given

$$P_i : \begin{cases} x_i(t+1) = f_i(x_i(t), x_{i'}(t), \dots, u_i\,(t)) \\ x_i(t) \in \mathbb{R}^{n_i}, x_{i'}(t) \in \mathbb{R}^{n_{i'}}, u_i(t) \in U_i \end{cases}$$

define

$$P : \begin{cases} x(t+1) = f(x(t), u(t)) \\ x(t) \in \mathbb{R}^{n}, u(t) \in U \end{cases}$$

where

$$x(t) = \big(x_1(t), x_2(t), \dots, x_N\,(t)\big)$$
$$u(t) = \big(u_1(t), u_2(t), \dots, u_N\,(t)\big)$$
$$f\big(x(t), u(t)\big) = (f_1(x_1(t), \dots, u_1\,(t)), \dots, f_N(x_N(t), \dots, u_N\,(t)))$$

$$T(P) = (X, X_0, U, \rightarrow, X_m, Y, H)$$

where:

- $X = X_0 = X_m = \mathbb{R}^n$
- $U = U_1 \times U_2 \times \dots \times U_N$
- $x \xrightarrow{u} x'$ if $x' = f(x, u)$
- $Y = \mathbb{R}^n$
- $H(x) = x$

Networks of symbolic models

Given a quantization vector $\eta = (\eta(1), \eta(2), \ldots, \eta(N))$ we first approximate

$$P_i \colon \begin{cases} x_i(t+1) = f_i(x_i(t), x_{i'}(t), \ldots, u_i\,(t)) \\ \quad x_i(t) \in \mathbb{R}^{n_i}, x_{i'}(t) \in \mathbb{R}^{n_{i'}}, u_i(t) \in U_i \end{cases}$$

by

$$T^{\eta}(P_i) = (X_i^{\eta}, X_{0,i}^{\eta}, W_i^{\eta} \times U_i, \longrightarrow_i, X_{m,i}^{\eta}, Y_i^{\eta}, H_i^{\eta})$$

where:

- $X_i^{\eta} = X_{0,i}^{\eta} = X_{m,i}^{\eta} = \eta(i)\, \mathbb{Z}^n$
- $W_i^{\eta} = X_{i_1}^{\eta} \times X_{i_2}^{\eta} \times \ldots$ where indices i_j are those of P_{i_j} that affect dynamics of P_i
- $\xi_i \xrightarrow{(w_i, u_i)}_i \xi'_i$ if $\xi'_i = [f_i(\xi_i, w_i, u_i)]_{\eta(i)}$
- $Y_i^{\eta} = \mathbb{R}^n$
- $H_i^{\eta}(\xi_i) = \xi_i$

Features of $T^{\eta}(P_i)$: deterministic, countable and alive

Networks of symbolic models

Define the network of symbolic models

$$T\left(\{T^{\eta}(P_i)\}_{i\in[1;N]}\right)$$

as the transition system obtained by interconnecting $T^{\eta}(P_i)$, i.e.

$$T\left(\{T^{\eta}(P_i)\}_{i\in[1;N]}\right) = \left(X^{\eta}, X_0^{\eta}, U, \longrightarrow_{\eta}, X_m^{\eta}, Y^{\eta}, H^{\eta}\right)$$

where:

- $X^{\eta} = X_0^{\eta} = X_m^{\eta} = X_1^{\eta} \times X_2^{\eta} \times \ldots \times X_N^{\eta}$
- $U = U_1 \times U_2 \times \ldots U_N$
- $(\xi_1, \xi_2, \ldots, \xi_N) \xrightarrow{(u_1, u_2, \ldots, u_N)}_{\eta} (\xi'_1, \xi'_2, \ldots, \xi'_N)$ if $\xi_i \xrightarrow{(w_i, u_i)}_i \xi'_i$ where $w_i = \left(\xi_{i_1}, \xi_{i_2}, \ldots\right)$
- $Y^{\eta} = Y_1^{\eta} \times Y_2^{\eta} \times \ldots \times Y_N^{\eta}$
- $H^{\eta}(\xi_1, \xi_2, \ldots, \xi_N) = (H_1^{\eta}(\xi_1), H_2^{\eta}(\xi_2), \ldots, H_N^{\eta}(\xi_N))$

Networks of symbolic models

Proposition

Suppose that P admits a locally Lipshitz δ-GAS Lyapunov function V satisfying

$$|V(x,y) - V(x,z)| \leq \sigma(|y-z|)$$

for some K_∞ function σ.

Then, for any desired accuracy $\mu \in \mathbb{R}^+$ and for any quantization vector $\eta \in \mathbb{R}_N^+$ satisfying the following inequality

$$|\eta| \leq \min\left\{(\sigma^{-1} \circ \rho \circ \alpha_1)(\mu), (\alpha_2^{-1} \circ \alpha_1)(\mu)\right\}$$

Relation R_μ specified by

$$(x,\xi) \in R_\mu \quad \Leftrightarrow \quad V(x,\xi) \leq \alpha_1(\mu)$$

is a μ-approximate bisimulation between $T(P)$ and $T\left(\{T^\eta(P_i)\}_{i\in[1;N]}\right)$.

Consequently, transition systems $T(P)$ and $T\left(\{T^\eta(P_i)\}_{i\in[1;N]}\right)$ are μ-bisimilar

How to find V ? Use e.g. small gain theorem

Decentralized supervisory control

Example

- Network of two control systems $P_i\colon x_i(t+1) = 0.5x_i(t) + u_i(t)$ with $U_i = \{-1,0,1\}$
- Specification L_Q collection of words $(0,0)(1,1)$ and $(0,0)(-1,-1)$

Case 1: C_i do not agree in advance on which word to enforce

- Starting from 0, C_1 picks $u_1(0) = 1$
- Starting from 0, C_2 picks $u_2(0) = -1$
- From (0,0) to (1,-1) $\Rightarrow$ Specification violated!

Case 2: C_i do agree in advance on which word to enforce:

- If they want to enforce word $(0,0)(1,1)$ both C_i pick $u_i(0) = 1$
- If they want to enforce word $(0,0)(-1,-1)$ both C_i pick $u_i(0) = -1$

$\Rightarrow$ Specification satisfied!

Decentralized supervisory control

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How to solve problem above ?

1. Restriction of the class of specifications, from L_Q to $L_{Q,1} \times L_{Q,2} \times \ldots \times L_{Q,N}$
2. Online agreement on which word to enforce
   (distributed control architecture needed)
3. Offline agreement on which word to enforce
   (decentralized control architecture enough)

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Decentralized supervisory control: solution

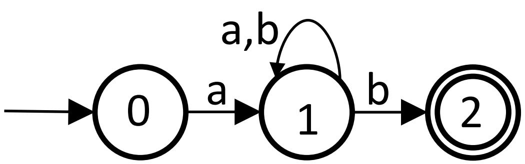
Recap from Lecture L7:

- Given L_Q define the transition system S'_Q such that its input marked language coincides with L_Q, i.e., $L_m^u(S'_Q) = L_Q$
- Construct the dual transition system S_Q of S'_Q, where states of S_Q are transitions of S'_Q and vice versa; we get

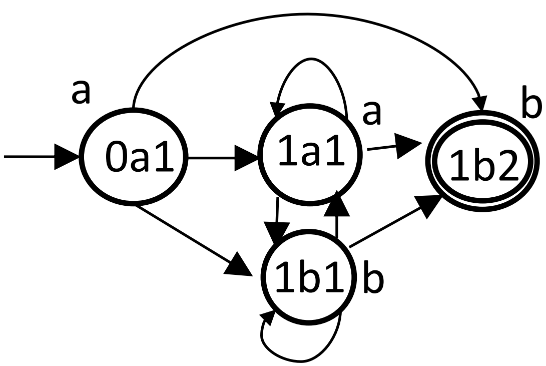
$$L_m^y(S_Q) = L_m^u(S'_Q) = L_Q \text{ and } L^y(S_Q) = L^u(S'_Q)$$

EXAMPLE

- L_Q = all words starting with a and ending with b over $Y_Q = \{a, b\}$
- Regular expression $a(a+b)^*b$



Transition system S'_Q



Transition system S_Q

Decentralized supervisory control: solution

- Let $S_Q = \left(X_Q, X_{Q,0}, U_Q, \underset{Q}{\longrightarrow}, X_{Q,m}, \mathbb{R}^n, H_Q\right)$
- Let $H_{Q,i}$ be the natural projection of H_Q onto $\mathbb{R}^{n_i}$, i.e.

$$H_{Q,i}(x_Q) = q^i \text{ if } H_Q(x_Q) = \left(q^1, q^2, \dots, q^N\right)$$

- Define operator $I_i : (\underset{Q}{\longrightarrow}) \times \mathbb{R}_N^+ \to$ {True, False} such that

$$I_i(x_Q \underset{Q}{\longrightarrow} x'_Q, \eta) = \text{True, if } \exists u_i \text{ s.t. } \left[H_{Q,i}(x_Q)\right]_{\eta(i)} \xrightarrow{(w_i,u_i)}_i \left[H_{Q,i}(x'_Q)\right]_{\eta(i)}$$

$$I_i(x_Q \underset{Q}{\longrightarrow} x'_Q, \eta) = \text{False, otherwise}$$

- Define operator $I : (\underset{Q}{\longrightarrow}) \times \mathbb{R}_N^+ \to$ {True, False} by

$$I(x_Q \underset{Q}{\longrightarrow} x'_Q, \eta) = \bigwedge_{i \in [1;N]} I_i(x_Q \underset{Q}{\longrightarrow} x'_Q, \eta)$$

Decentralized supervisory control: solution

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$$I(x_Q \underset{Q}{\longrightarrow} x'_Q, \eta) = \bigwedge_{i \in [1;N]} I_i(x_Q \underset{Q}{\longrightarrow} x'_Q, \eta)$$

Decentralized supervisory control: solution

Let $S_{Q,\eta}$ be the sub-transition system of S_Q containing all and only transitions $x_Q \underset{Q}{\longrightarrow} x'_Q$ of S_Q for which

$$I(x_Q \underset{Q}{\longrightarrow} x'_Q, \eta) = True$$

Remarks

1. $S_{Q,\eta}$ represents the part of the specification that can be matched by using the decentralized control architecture we consider
2. $S_{Q,\eta}$ is blocking in general

In order to overcome 2 compute $\mathrm{Trim}(S_{Q,\eta})$

Remark

$\mathrm{Trim}(S_{Q,\eta})$ contains all information needed to solve our control problem, i.e. to define the set of initial states and to design local controllers C_i

Decentralized supervisory control: solution

Consider any **q** word marked by $\mathrm{Trim}(S_{Q,\eta})$ and let

$$S_{\boldsymbol{q}} = \left(X_{\boldsymbol{q}}, x_{\boldsymbol{q},0}, U_{\boldsymbol{q}}, \underset{\boldsymbol{q}}{\rightarrow}, \{x_{\boldsymbol{q},m}\}, R^n, H_{\boldsymbol{q}}\right)$$

be a transition system marking **q**

Remark

- Remember that $C_{\boldsymbol{i}}$ need to agree offline on which word to enforce!
- $S_{\boldsymbol{q}}$ can be chosen wlog to be symbolic, accessible and nonblocking

Let $H_{\boldsymbol{q},i}$ be the natural projection of $H_{\boldsymbol{q}}$ onto $\mathbb{R}^{n_i}$

Define

$$X_0 = R_{\mu}^{-1}\left(\prod_{i=1}^{n}\{[H_{\boldsymbol{q},i}(x_{\boldsymbol{q},0})]_{\eta(\boldsymbol{i})}\}\right)$$

Decentralized supervisory control: solution

Define entities of local controllers C_i

$$C_i : \begin{cases} z_i(t+1) = g_i(z_i(t)) \\ u_i(t) \in h_i(z_i(k)) \\ z_i(t) \in Z_i, z_i(t) \in Z_{i,0} \subseteq Z_i, u_i(t) \in U_i \end{cases}$$

as follows:

- $Z_{i,0} = \{x_{\boldsymbol{q},0}\}$
- $Z_i = X_{\boldsymbol{q}}$
- $g_i(z_i) = z'_i$ if $z_i \underset{\boldsymbol{q}}{\rightarrow} z'_i$
- $h_i\big(z_i(t)\big) = \left\{ u_i \in U_i \mid g_i(z_i) = z'_i \text{ and } \left[H_{Q,i}(z_i)\right]_{\eta(i)} \xrightarrow{(w_i, u_i)}_i \left[H_{Q,i}(z'_i)\right]_{\eta(i)} \right\}$

Decentralized supervisory control: solution

Quantization parameters design and formal correctness

Theorem

Suppose that P admits a locally Lipshitz δ-GAS Lyapunov function V satisfying

$$|V(x,y) - V(x,z)| \leq \sigma(|y - z|)$$

for some K_∞ function σ. For any desired accuracy $\theta \in \mathbb{R}^+$ select $\mu \in \mathbb{R}^+$ and $\eta \in \mathbb{R}_N^+$ satisfying

$$|\eta| \leq \min\left\{(\sigma^{-1} \circ \rho \circ \alpha_1)(\mu), (\alpha_2^{-1} \circ \alpha_1)(\mu)\right\}$$

$$\mu + |\eta| \leq \theta$$

Then set X_0 and local controllers C_i solve the decentralized supervisory control problem

Decentralized supervisory control: solution

Remarks

- Comparison with centralized control architectures : achievable behavior in centralized and decentralized control architecture is the same

- Efficient on-the-fly control algorithms allowing also parallel computing architectures to further speed up computations

input:
$S_Q = (X_Q, X_{Q,0}, U_Q, \xrightarrow[Q]{}, X_{Q,m}, \mathbb{R}^n, H_Q)$;
foreach $x_Q \xrightarrow[Q]{} x_Q^+$ **do**
  foreach $i \in [1; N]$ **do**
    set $\mathcal{I}_i(x_Q \xrightarrow[Q]{} x_Q^+, \eta) :=$ `False`;
    compute the set $h_{c,i}(x_Q)$ of all $u_i \in U_i^\eta$ satisfying (20);
    if $h_{c,i}(x_Q) \neq \varnothing$ **then**
      set $\mathcal{I}_i(x_Q \xrightarrow[Q]{} x_Q^+, \eta) :=$ `True`;
    end
  end
  set
  $\mathcal{I}(x_Q \xrightarrow[Q]{} x_Q^+, \eta) := \wedge_{i \in [1;N]} \mathcal{I}_i(x_Q \xrightarrow[Q]{} x_Q^+, \eta)$;
end
compute $S_{Q,\eta}$ in (23);
compute $\mathrm{Trim}(S_{Q,\eta})$;
output: $\mathrm{Trim}(S_{Q,\eta})$ and $h_{c,i}$, $i \in [1; N]$;

Algorithm 1: Decentralized local controllers design.

Decentralized supervisory control: solution

Remarks

- Comparison with centralized control architectures : achievable behavior in centralized and decentralized control architecture is the same
- Efficient on-the-fly control algorithms allowing also parallel computing architectures to further speed up computations
- Computational complexity analysis :

  Decentralized approach : linear growth with the number N of subsystems P_i

  Centralized approach : exponential growth with N

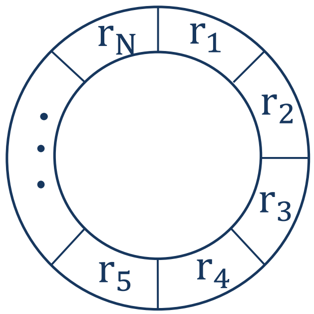
Example

Temperature regulation of a circular building with N rooms r_i

$$\mathbf{T}_i(t+1) = \mathbf{T}_i(t) + \alpha\big(\mathbf{T}_{i+1}(t) + \mathbf{T}_{i-1}(t) - 2\mathbf{T}_i(t)\big) + \beta\big(T_e - \mathbf{T}_i(t)\big) + \gamma\big(T_h - \mathbf{T}_i(t)\big)u_i(t)$$

$$u_i(t) \in U_i = 0.25\,\mathbb{Z} \cap [0,1]$$

$$\theta = 0.5$$



specification :

t mod(12)	$\mathbf{T}_1(t)$	$\mathbf{T}_i(t),\ i \in [2; N]$
0	19	18
1	19	18.5
2	19	19
3	19	19.5
4	19	20
5	19	20
6	19	20
7	19	19.5
8	19	19
9	19	18.5
10	19	18.25
11	19	18

Example

Temperature regulation of a circular building with N rooms r_i

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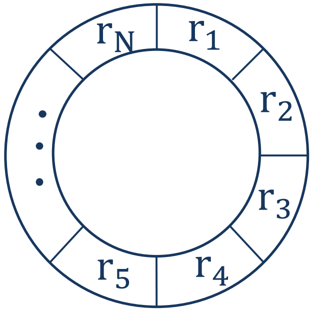

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6	19	20
7	19	19.5
8	19	19
9	19	18.5
10	19	18.25
11	19	18

controller :

t mod(12)	C_1	$C_2 = C_N$	$C_i,\ i \in [3; N-1]$
0	{0.65}	{0.45}	{0.6}
1	{0.475}	{0.55}	{0.625}
2	{0.325}	{0.65}	{0.65}
3	{0.15}	{0.75}	{0.65}
4	{0}	{0.525}	{0.35}
5	{0}	{0.525}	{0.35}
7	{0}	{0.175}	{0.025}
8	{0.15}	{0.1}	{0}
9	{0.325}	{0}	{0}
6	{0.475}	{0.075}	{0.15}
10	{0.55}	{0.025}	{0.15}
11	{0.65}	{0.15}	{0.30}

validation :

t mod(12)	$\mathbf{T}_1(t)$	$\mathbf{T}_2(t) = \mathbf{T}_N(t)$	$\mathbf{T}_i(t),\ i \in [3; N-1]$
0	19.5000	18.5000	17.5000
1	18.9788	18.8462	18.0125
2	18.7329	19.2453	18.5368
3	18.6496	19.6773	19.0709
4	18.6042	20.1282	19.5744
5	18.5992	20.1021	19.5924
6	18.6058	20.0838	19.6098
7	18.6176	19.5475	19.1325
8	18.6200	19.0492	18.6292
9	18.6440	18.5357	18.1385
10	18.6448	18.2824	17.8990
11	18.6431	18.0186	17.9080

$$|20 - 19.5924| = 0.4076 < 0.5 = \theta$$

Computational complexity analysis (ONLY for $N = 4$) :

- Decentralized architecture 0.1563s
- Centralized architecture 163.6304s

Gain: $\frac{163.6304}{0.1563} = 1046$!

Computations on a Lenovo IP YOGA 3 PRO 8GB 512SSD

Conclusions

We proposed decentralized control architectures enforcing regular language specifications on incrementally stable networks of discrete-time nonlinear control systems