



Formal Methods for the Control of Large-scale Networked Nonlinear Systems with Logic Specifications

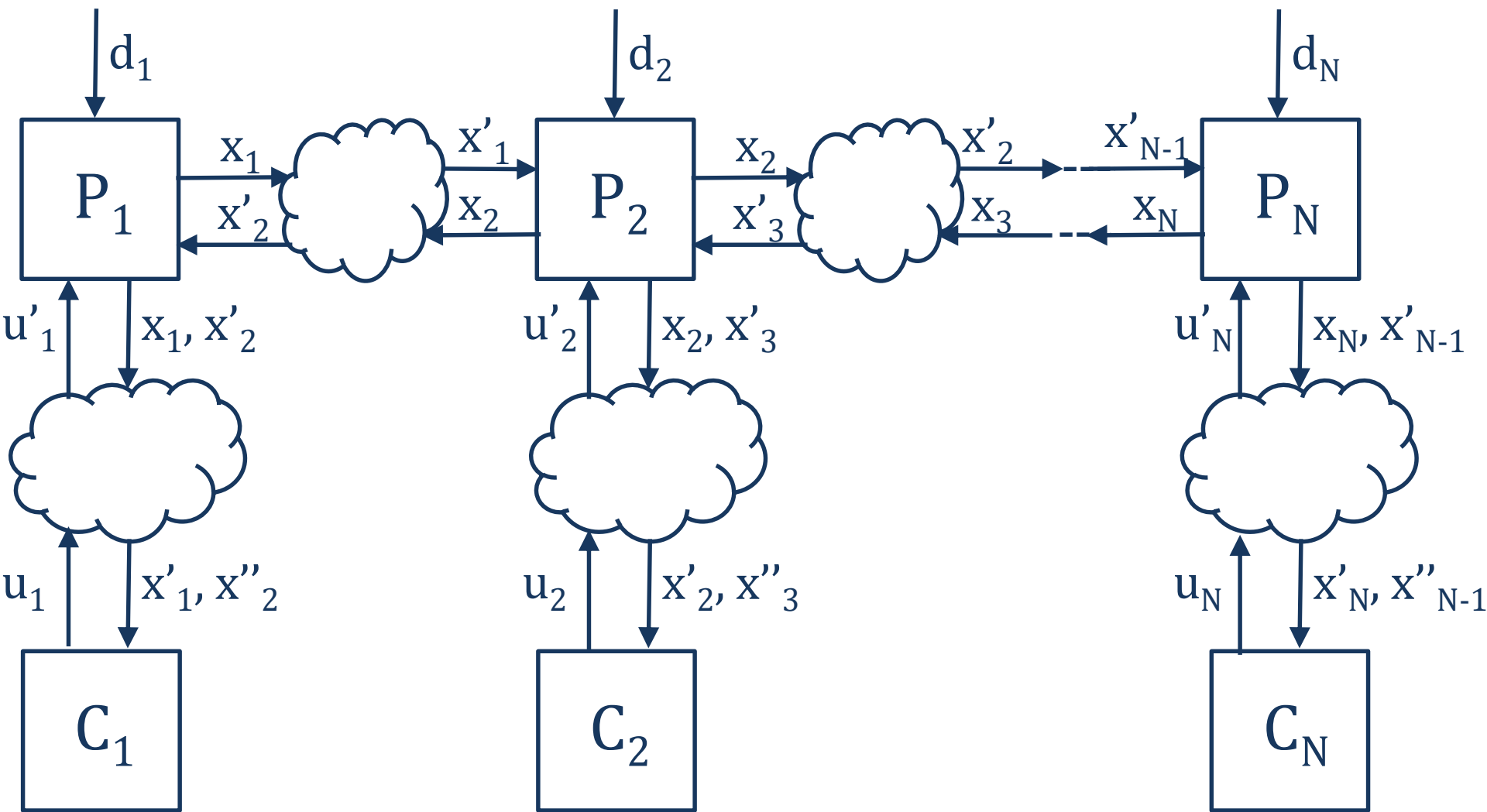


Basilica di Santa Maria di Collemaggio, L'Aquila (Italy), 1287

Lecture L15: Conclusions

Speaker: **Giordano Pola**

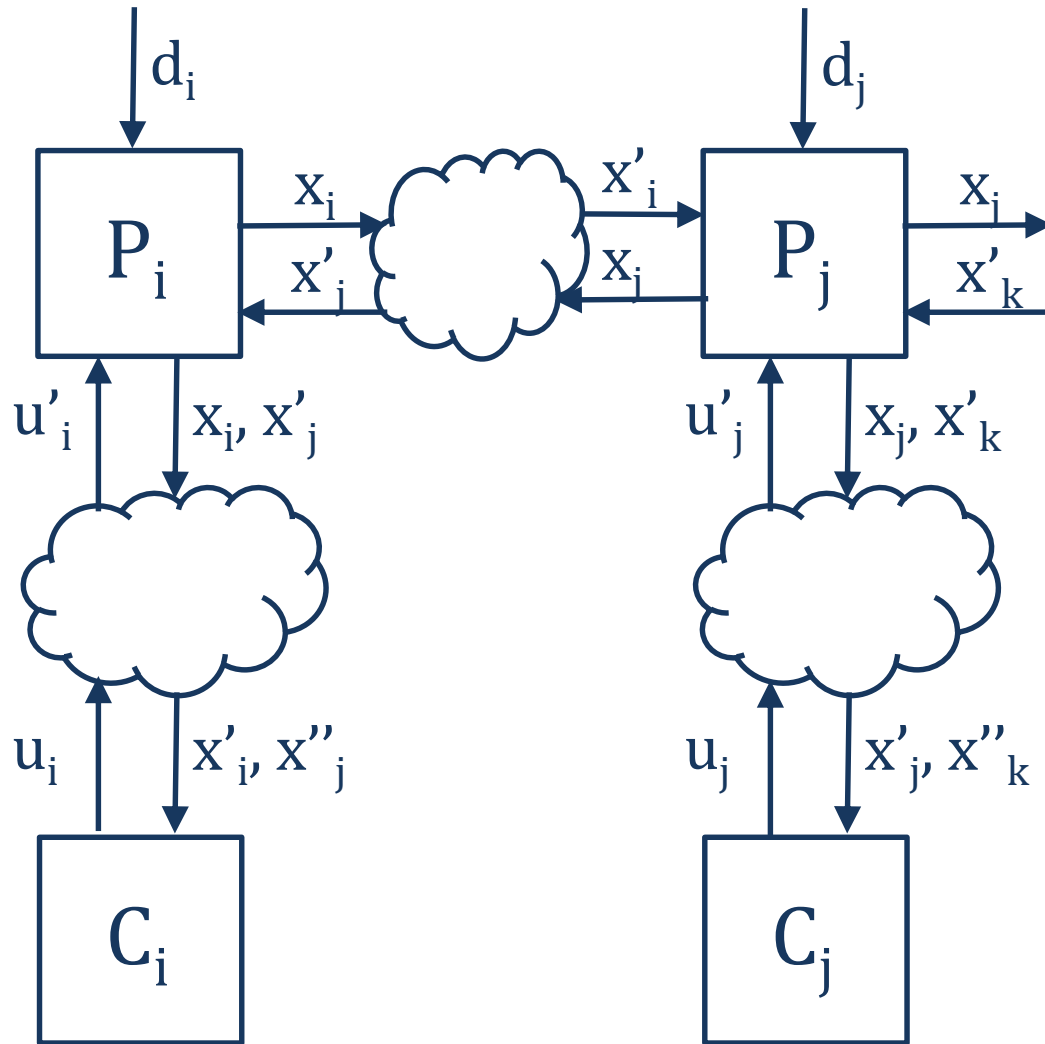
We considered a fairly general model of CPSoS



Critical aspects of CPSoS ...

- Heterogeneity: plants, controllers and specifications described in different mathematical frameworks
- Non-ideal communication infrastructure: control action delivered with delay on the basis of delayed and corrupted measure of the states of the plants, lack of information (packet drops), etc.
- Complexity: large number of systems composed of several, possibly distributed sub-systems
- Logic specifications

The approach we took ...



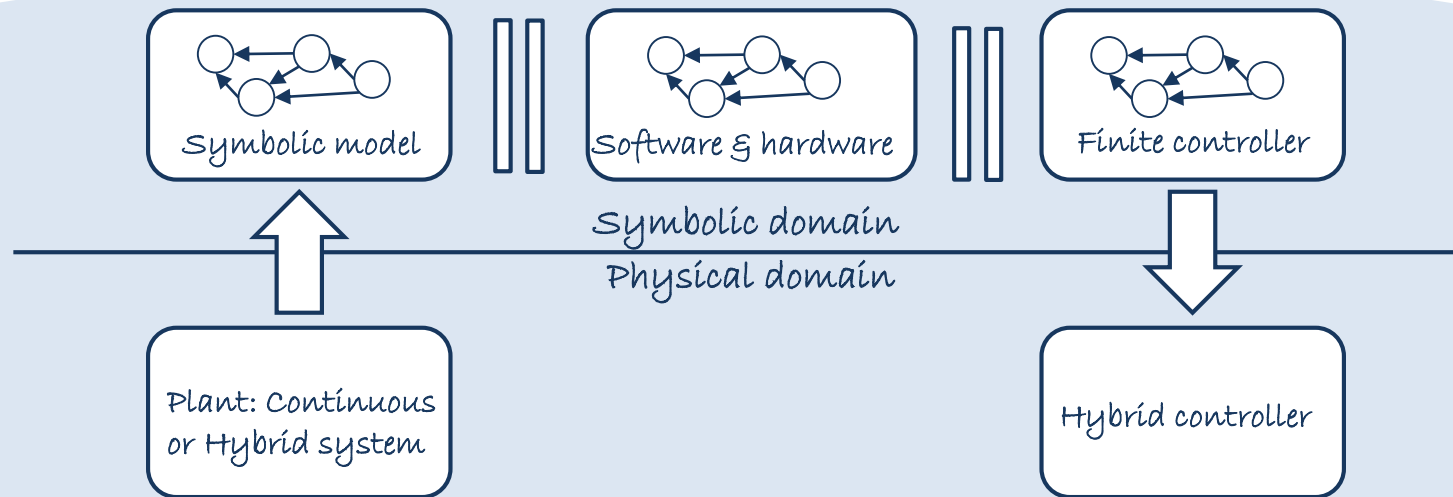
... a complementary approach :

- Single plants with no disturbances and delays
- Control design with logic specifications
- Efficient algorithms for control design
- Single plants with disturbances
- Single plants with delays
- Single, possibly unstable, plants
- Single plant, controller and communication infrastructure
- Decentralized control of networks of control systems with logic specifications

Solution based on Formal Methods

A three phases process :

- #1. Construct the finite/symbolic model T approximating the plant system P
- #2. Design a finite/symbolic controller C that solves the specification S for T
- #3. Refine the controller C to the controller C' to be applied to P



Advantages :

- Integration of software and hardware constraints in the control design of purely continuous processes
- Logic specifications can be addressed

Advantages of the approach

- Correct-by-design approach handling nonidealities of CPSoS
- Systematic approach to enforce logic specifications
- The «completeness property»
 - A control strategy enforcing a (logic) specification on one system exists if and only if it exists on a bisimilar one
 - If the system is incrementally stable then an approximated bisimilar symbolic model can be effectively constructed
 - Example 1 in Lecture L6a shows that unstable linear systems do not admit approximated bisimilar symbolic models
- Incremental forward completeness
 - Mild assumption: at present there is no example of a FC nonlinear system that is not δ -FC !
 - Since symbolic models obtained are in simulation (and not bisimulation) with the original model, here the completeness property is lost in general

Disadvantages of the approach: Computational complexity!

Approaches to tame complexity:

Methodological approaches:

s.c.c. = space computational complexity t.c.c. = time computational complexity

- On-the-fly algorithms
Example in L7b with s.c.c. gain 57491 and t.c.c. gain 419 !
- Decentralized control architectures
Example in L13 with t.c.c. gain 1046 !
- Multi-resolution approaches (see e.g. [Girard et al., TAC16])
- ...

Software tools:

- PESSOA (use of BDD)
Example in L11 where construction of symbolic model of 34020088 transitions + control (without on-the-fly techniques) in 536s !
- SCOTS (performs even better than PESSOA !)
- ...

Dedicated computing architectures:

- Parallel computing (not explored yet !)
- ...

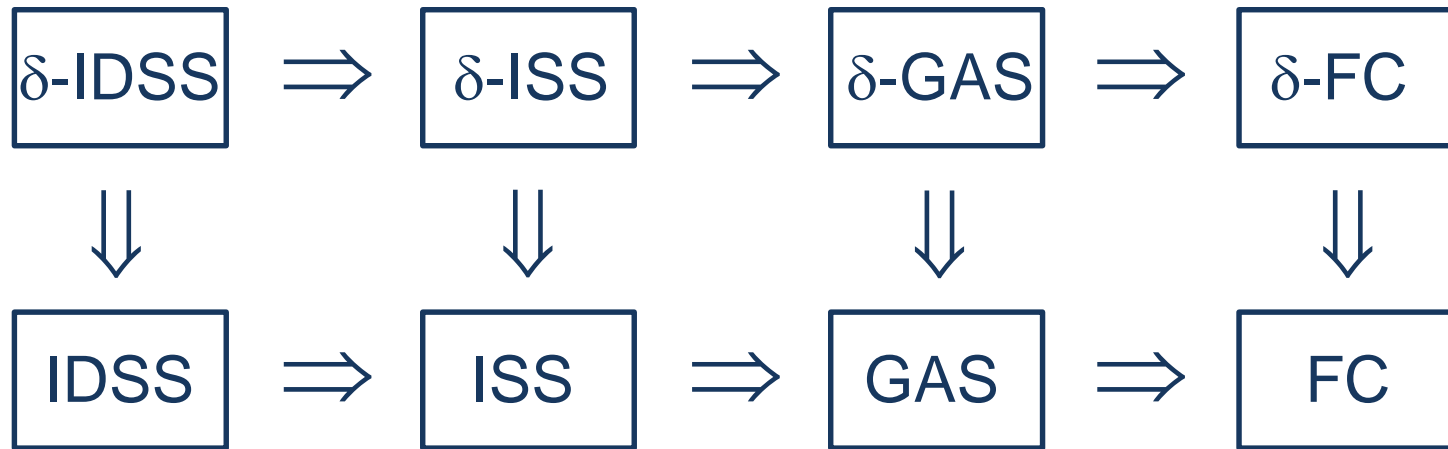
Control of CPSoS with an inter-disciplinary approach ...

Methods from:

- **Control theory**
nonlinear and time delay-systems, incremental stability, ...
- **Discrete-event systems**
regular languages, operators on automata, ...
- **Theoretical computer science**
bisimulation theory and its variants, ...
- **Mathematics**
approximation of functional spaces, ...
- **Telecommunication**
modeling of nonideal communication infrastructures, ...

Some advanced methods used from control theory

Hierarchy of stability notions for nonlinear (and time-delay) systems



Some additional topics on Formal Methods @ UNIVAQ

■ **Control of PWA systems with logic specifications**

by **G Pola and M D Di Benedetto**

- Pola, Di Benedetto, Symbolic Models and Control of Discrete-Time Piecewise Affine Systems: An Approximate Simulation Approach, TAC-2014
- Pola, Di Benedetto, Sequences of Discrete Abstractions for Piecewise Affine Systems, ADHS-2012

■ **Approximate diagnosability for metric and nonlinear control systems**

by **G Pola, M D Di Benedetto and Elena De Santis (UNIVAQ)**

- Pola, De Santis, Di Benedetto, Approximate diagnosability of metric transition systems, SEFM-2017, accepted
- De Santis, Pola, Di Benedetto, On Approximate Diagnosability of Nonlinear Control Systems, CDC-2017, submitted

■ **Stochastic equivalences and control for linear and descriptor systems**

by **G Pola, M D Di Benedetto, Costanzo Manes (UNIVAQ) and Arjan J. van der Schaft (Univ. of Groningen, NL)**

- Pola, Manes, van der Schaft, Di Benedetto, Equivalence notions for discrete-time stochastic linear control systems, TAC-2017, provisionally accepted as Paper
- Pola, On Achievable Behavior of Stochastic Descriptor Systems, CDC-2017, submitted
- Pola, Manes, Di Benedetto, On external behavior equivalence of continuous-time stochastic linear control systems, CDC-2016
- Pola, Manes, van der Schaft, Di Benedetto, Model reduction of continuous-time stochastic linear control systems via bisimulation equivalence, CDC-2016
- Pola, Manes, van der Schaft, Di Benedetto, Equivalence Notions for Discrete-Time Stochastic Control Systems, CDC-2015

■ ...

Future directions

Methodological issues

- Stochastic CPSoS
- Co-design of control algorithms with communication infrastructures
- Software tools
- ...

Application domains

- Autonomous vehicle control with nonideal communication infrastructures
- Other applications: Micro-grid, Systems biology, ...
- ...

International collaborations on formal methods

- Paulo Tabuada, UCLA (USA)
- Antoine Girard, CNRS (France)
- Karl Henrik Johansson, KTH (Sweden)
- Majid Zamani, University of Munchen (Germany)
- Manuel Mazo, TUDelft (The Netherlands)
- Dimos Dimarogonas, KTH (Sweden)



References on formal methods for the control of CPSoS @ UNIVAQ

- [Pola et al., TAC17] Pola, G., Pepe, P., Di Benedetto, M.D., Decentralized Approximate Supervisory Control of Networks of Nonlinear Control Systems, submitted, 2017
- [Pola et al., TAC16] Pola, G., Pepe, P., Di Benedetto, M.D., Symbolic Models for Networks of Control Systems, *IEEE Transactions on Automatic Control*, 61(11):3663-3668, November 2016
- [Pola et al., IJRN15] Pola, G., Pepe, P., Di Benedetto, M.D., Symbolic Models for Time-Varying Time-Delay Systems via Alternating Approximate Bisimulation, *International Journal of Robust and Nonlinear Control*, 25:2328–2347, September 2015
- [Pola et al., TAC14] Pola, G., Di Benedetto, M.D., Symbolic Models and Control of Discrete-Time Piecewise Affine Systems: An Approximate Simulation Approach, *IEEE Transactions on Automatic Control*, 59(1):175-180, January 2014
- [Borri et al., IJCC12] Borri, A., Pola, G., Di Benedetto, M.D., Symbolic models for nonlinear control systems affected by disturbances, *International Journal of Control*, 85(10):1422-1432, September 2012
- [Zamani et al., TAC12] Zamani, M., Pola, G., Mazo, M., Tabuada, P., Symbolic models for nonlinear control systems without stability assumptions, *IEEE Transactions on Automatic Control*, 57(7):1804-1809, July 2012
- [Pola et al., TAC12] Pola, G., Borri, A., Di Benedetto, M.D., Integrated design of symbolic controllers for nonlinear systems, *IEEE Transactions on Automatic Control*, 57(2):534-539, February 2012
- [Pola et al., SCL10] Pola, G., Pepe, P., Di Benedetto, M.D., Tabuada, P., Symbolic models for nonlinear time-delay systems using approximate bisimulation, *Systems & Control Letters* 59(6): 365-373, June 2010
- [Girard et al., TAC10] Girard, A., Pola, G., Tabuada, P., Approximately bisimilar symbolic models for incrementally stable switched systems, *IEEE Transactions on Automatic Control*, 55(1):116-126, January 2010
- [Pola et al., SIAM09] Pola, G., Tabuada, P., Symbolic models for nonlinear control systems: Alternating approximate bisimulations, *SIAM Journal on Control and Optimization*, 48(2):719-733, 2009
- [Pola et al., AUT08] Pola, G., Girard A., Tabuada, P., Approximately bisimilar symbolic models for nonlinear control systems, *Automatica*, 44(10):2508-2516, October 2008
- [Di Benedetto et al., EPTCS13] Di Benedetto, M.D., Pola, G., Networked Embedded Control Systems: from Modelling to Implementation, *Electronic Proceedings in Theoretical Computer Science (EPTCS)* 124, pp. 9-13, Bortolussi L., Bujorianu M.L., Pola G. (Eds.): HAS 2013, doi:10.4204/EPTCS
- [Girard et al., HSCC08] Girard, A., Pola, G., Tabuada, P., Approximately bisimilar symbolic models for Incrementally Stable Switched Systems, *Hybrid Systems: Computation and Control 2008*, M. Egerstedt and B. Mishra, Eds., *Lecture Notes on Computer Information Sciences*, Springer Verlag, vol. no. 4981, pp. 201-214
- [Borri et al., ERCIM14] Borri, A., Di Benedetto, M.D., Pola, G., Towards a Unified Theory for the Control of CPS: A Symbolic Approach, *ERCIM News No. 97*, April 2014, Special theme: Cyber-Physical Systems, Guest editors M. D. Di Benedetto, F. L. Lagarrigue, E. Schoitsch
- [Pola et al., CDC16] Pola, G., Borri, A., Di Benedetto, M.D., On Symbolic Control Design of Discrete-Time Nonlinear Systems with State Quantized Measurements, *55th IEEE Conference on Decision and Control*, Las Vegas, USA, December 2016, pp. 6571-6576
- [Pola et al., ACC14] Pola, G., Pepe, P., Di Benedetto, M.D., Symbolic Models for Networks of Discrete-Time Nonlinear Control Systems, *American Control Conference*, Portland, Oregon, USA, June 2014, pp. 1787-1792
- [Borri et al., NECSYS13] Borri, A., Dimarogonas, D.V., Johansson, K.H., Di Benedetto, M.D., Pola, G., Decentralized symbolic control of interconnected systems with application to vehicle platooning, *4th IFAC Workshop on Distributed Estimation and Control in Networked Systems*, Koblenz, Germany, September 2013, pp. 285-292
- [Borri et al., CDC12] Borri, A., Pola, G., Di Benedetto, M.D., Integrated Symbolic Design of Unstable Nonlinear Networked Control Systems, *51st IEEE Conference on Decision and Control*, Maui, Hawaii, USA, December 2012, pp. 1374-1379
- [Borri et al., HSCC12] Borri, A., Pola, G., Di Benedetto, M.D., A symbolic approach to the design of nonlinear networked control systems, *Hybrid Systems: Computation and Control 2012*, Beijing, China, April 2012, pp. 255-264, I. Mitchell and T. Dang, Eds.
- [Pola et al., ADHS12] Pola, G., Di Benedetto, M.D., Sequences of Discrete Abstractions for Piecewise Affine Systems, *4th IFAC Conference on Analysis and Design of Hybrid Systems*, Eindhoven, The Netherlands, June 2012, pp. 147-152
- [Borri et al., CDC11] Borri, A., Pola, G., Di Benedetto, M.D., Alternating Approximately Bisimilar Symbolic Models for Nonlinear Control Systems affected by Disturbances, *50th IEEE Conference on Decision and Control and European Control Conference*, Orlando, Florida, USA, December 2011, pp. 552-557
- [Pola et al., IFACWC11] Pola, G., Di Benedetto, M.D., De Santis, E., A Compositional Approach to Bisimulation of Arenas of Finite State Machines, *18th IFAC World Congress*, Milan, Italy, August-September 2011, pp. 7006-7011
- [Pola et al., CDC10] Pola, G., Pepe, P., Di Benedetto, M.D., Alternating Approximately Bisimilar Symbolic Models for Nonlinear Control Systems with Unknown Time-Varying Delays, *49th IEEE Conference on Decision and Control*, Atlanta, Georgia, USA, December 2010, pp. 7649-7654
- [Borri et al., CDC10] Borri, A., Pola, G., Di Benedetto, M.D., An integrated approach to the symbolic control design of nonlinear systems with infinite states specifications, *49th IEEE Conference on Decision and Control*, Atlanta, Georgia, USA, December 2010, pp. 1528-1533
- [Zamani et al., ACC10] Zamani, M., Pola, G., Tabuada, P., Symbolic models for unstable nonlinear control systems, *American Control Conference 2010*, Baltimore, Maryland, USA, July 2010, pp. 1021-1026
- [Pola et al., CDC09] Pola, G., Pepe, P., Di Benedetto, M.D., Tabuada, P., A symbolic model approach to the digital control of nonlinear time-delay systems, *48th IEEE Conference on Decision and Control and 28th Chinese Control Conference*, Shanghai, China, December 2009, pp. 2216-2221
- [Pola et al., CDC08] Pola, G., Tabuada, P., Symbolic models for nonlinear control systems affected by disturbances, *47th IEEE Conference on Decision and Control*, Cancun, Mexico, December 2008, pp. 251-256
- [Pola et al., CDC07] Pola, G., Girard A., Tabuada, P., Symbolic models for nonlinear control systems using approximate bisimulations, *46th IEEE Conference on Decision and Control*, New Orleans, Louisiana, USA, December 2007, pp. 432- 437
- [Pola et al., CDC07] Pola, G., Tabuada, P. Symbolic models for linear control systems with disturbances, *46th IEEE Conference on Decision and Control*, New Orleans, Louisiana, USA, December 2007, pp. 4643-4647