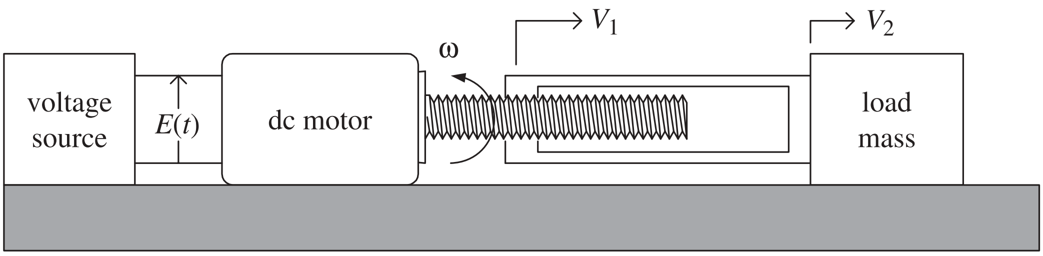
Exercises

Port-Hamiltonian modelling and passivity-based control of physical systems. Theory and applications

Exercise #1

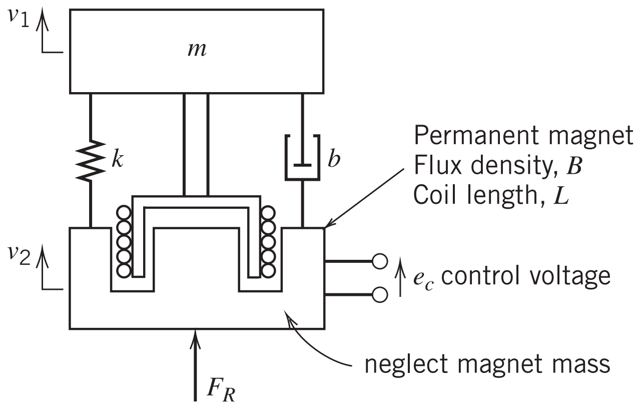
The figure shows the physical part of a load-positioning system without the sensors and the controller electronics that control the voltage source.



1. You are to make a bond graph model of the system, including the following effects:
   A. The voltage is a controlled effort source. Indicate this by including a signal or active bond with a double arrow impinging on an effort source.
   B. The dc motor has inductance and resistance in the armature coils.
   C. Include a rotary inertia for the motor armature and the screw drive.
   D. Use a resistor to model the friction moment associated with the motor and the screw drive.
   E. A constant S relates the angular velocity of the screw with the nut: $V_1 = S\,\omega$.
   F. The tube connecting the nut with the load mass is elastic with a spring constant k.
   G. The load mass has significant friction with the ground.
2. Write down the final system in port-Hamiltonian form.
3. Suppose that the control input is the torque τ generated by the dc motor. Develop an energy-shaping control law that allows to position the load mass
4. What if the control input is the voltage $E(t)$?

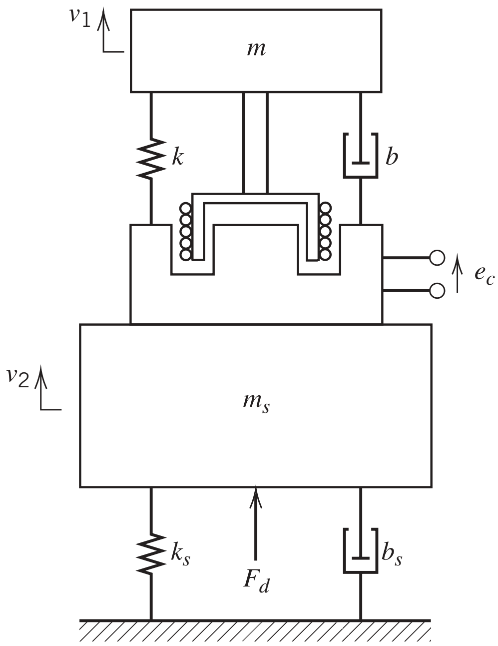
Exercise #2

An inertial actuator is a device in which, by appropriate acceleration of an internal mass, a prescribed reaction force can be generated. By attaching the actuator to a structure, the structure motion can be controlled.



The device shown is voice coil driven, where only the winding resistance is important on the electrical side. The mass m is attached to the base with spring k and damper b.

1. Here, F_R is the output force. Assume the base has prescribed motion, $v_2(t)$. Construct a bond graph model and derive state equations in port-Hamiltonian form.
2. Derive the transfer functions relating F_R to v_2 and F_R to e_c, and denote them with $G_{Fv}(s)$ and $G_{Fc}(s)$, respectively.
3. Sometimes we desire the reaction force to mimic the effect of a damper attached between the base and inertial ground, i.e. that $F_R = b_c v_2$, with b_c some controller gain. Derive the ideal control system that will yield the desired reaction force from the inherent dynamics of the device.
4. Shown here is the previous device attached to a structure consisting of a mass, m_s, spring, k_s, and damper, b_s. A force,F_d, acts upon the structure mass. Construct a bond graph model for this total system and derive state equations.



5. Derive the open-loop (i.e. with $e_c = 0$) transfer function relating output, v_2, to input, F_d. Does the result have a component similar to a damper to ground?

Exercise #3

An ideal, compressible, isentropic fluid with one dimensional domain is modelled by the following coupled PDEs that express the conservation of mass and momentum:

$$\frac{\partial \rho}{\partial t}(t,z) = -\frac{\partial(\rho v)}{\partial z}(t,z) \qquad \frac{\partial(\rho v)}{\partial t}(t,z) = -\frac{\partial}{\partial z}[\rho(t,z)v^2(t,z) + P(t,z)]$$

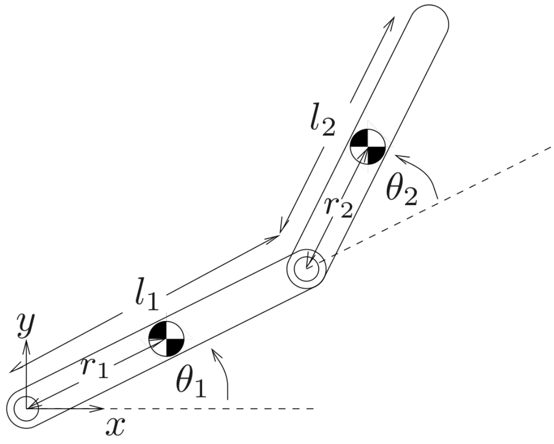
in which $\rho(t,z) > 0$ is the fluid density, $v(t,z)$ is the flow velocity, $P(t,z)$ is the flow pressure, and $z \in [a,b]$ the spatial coordinate.

1. Prove that the second relation can be equivalently rewritten as
$$\frac{\partial v}{\partial t}(t,z) = -v(t,z)\frac{\partial v}{\partial z}(t,z) - \frac{1}{\rho(t,z)}\frac{\partial P}{\partial z}(t,z)$$
2. The energy variables are in fact the fluid density ρ and its velocity v, and the total energy is
$$H(\rho,v) = \int_a^b \left[\frac{1}{2}\rho v^2 + \rho U(\rho)\right] dz$$
where $U(\rho)$ is the internal energy of the fluid. Since
$$P(t,z) = \frac{RT}{M}\rho(t,z) = \rho^2(t,z)\frac{\partial U}{\partial \rho}(\rho(t,z)),$$
where R is the universal gas constant, T is the system (constant) temperature and M is the fluid molar mass, compute the expression of the internal energy $U(\rho)$, and write the system of coupled PDEs in port-Hamiltonian form.
3. The co-energy variables are the dynamic pressure and fluid flow, that are given by
$$\frac{\delta H}{\delta \rho}(\rho,v) \qquad \frac{\delta H}{\delta v}(\rho,v)$$
Compute their expression.
4. The boundary inputs are chosen in the following way: in $z = a$, the fluid flow, while in $z = b$ the dynamic pressure. Determine the dual outputs such that the system is in impedance form.
5. For stabilisation purposes, a control law based on damping injection is implemented at both sides of the spatial domain. Write down the control law.
6. Under the condition $|v(t,z)| \le \sqrt{\frac{RT}{M}}$, prove that the original system can be mapped into
$$\frac{\partial}{\partial t}\begin{pmatrix}\bar{x}_-\\ \bar{x}_+\end{pmatrix} + \begin{pmatrix}\lambda_-(\bar{x}_-,\bar{x}_+) & 0\\ 0 & \lambda_-(\bar{x}_-,\bar{x}_+)\end{pmatrix}\frac{\partial}{\partial z}\begin{pmatrix}\bar{x}_-\\ \bar{x}_+\end{pmatrix} = 0$$
where $\bar{x}_-, \bar{x}_+, \lambda_-, \lambda_+$ have to be computed. Verify that, under the previous hypothesis, λ_- and λ_+ have opposite sign. Write down the damping injection control law in the new coordinates.

Exercise #4

The figure below shows a planar 2DOF manipulator with rotational joints where:

A. Each link can be modelled as a homogeneous rectangular bar with mass m_i and moment of inertia I_i around the z-axis.
B. Each joint is subject to a rotational viscous friction with damping coefficient b_i.
C. θ_i represents the joint variables
D. Each joint can be actuated by a torque τ_i.
E. The gravity force is along the negative y direction.



Now, answer the next questions:

1. Build the port-Hamiltonian model of the manipulator in the joint space.
2. Using the control by interconnection technique, build a controller for stabilising the configuration $(\theta_1, \theta_2) = (\pi/2, \pi/2)$. Write down the Casimir function you used for developing the controller.
3. Suppose that the robot is under-actuated, i.e. that $\tau_1 = 0$. Is it still possible to stabilise the robot at $(\theta_1, \theta_2) = (\pi/2, \pi/2)$? Which configurations can you stabilise using the control by interconnection technique?