

# Port Hamiltonian systems : Introduction

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# Personal introduction to Port Hamiltonian systems

*Personal introduction to Port Hamiltonian systems*

# Hamiltonian formulation of bond graph models?

At the First IFAC workshop NOLCOS, Capri, Italy [June, 1989 discussion with Arjan](#) around the version of the paper that I submitted: "Geometrical formulation of the Bond– Graph dynamics with application to mechanisms", [Journal of the Franklin Institute](#), vol. 328, No 5/ 6, pp. 723– 740, 1991.

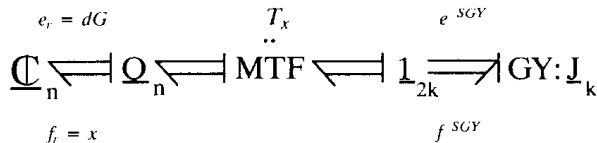


FIG. 6. A complete bond graph.

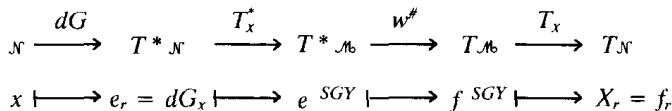
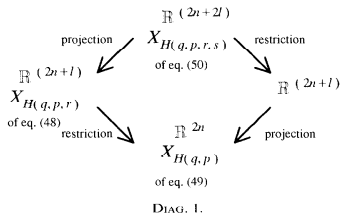


FIG. 7. Diagram of the symplectic construction of the bond graph dynamics.

# Hamiltonian formulation of network models : port Hamiltonian systems

- B. M. Maschke, A. J. van der Schaft and P. C. Breedveld, "An **intrinsic Hamiltonian formulation of network dynamics**: non– standard Poisson structures and gyrators", **Journal of the Franklin Institute**, Vol. 329, n. 5, pp. 923– 966,



1992

- B. M. Maschke, A. J. van der Schaft and P. C. Breedveld, "An **intrinsic Hamiltonian formulation of the dynamics of LC–circuits**", Trans. IEEE on Circuits and Systems, I : Fundamental Theory and Applications, Vol. 42, n° 2, pp. 73– 82, February 1995
- A.J. van der Schaft and B.M. Maschke, **Port-Hamiltonian systems on graphs**, **SIAM J. of Control and Optimization**, vol. 51, n°2, pp. 906-937, 2013



# Port Hamiltonian systems

The **fundamental conservation laws** :

- mass
- momentum
- energy
- electric field induction
- magnetic field induction

coupled by **closure relations** :

- thermodynamic properties
- reversible and irreversible laws of fluxes

Complex systems are obtained by coupling through geometric boundaries, multiphase systems, multilevel systems

**Systems of balance equations** on

- **continuous spatial domains** : infinite-dimensional Port Hamiltonian systems
- **discrete spatial domains**: port Hamiltonian systems defined on k-complexes and networks

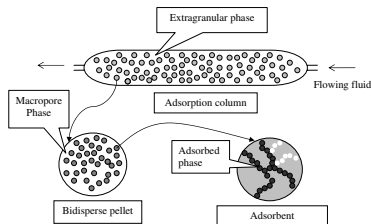
# Some examples of Port Hamiltonian models

*Some examples of Port Hamiltonian*

# Pressure swing adsorption process : scheme

Is constituted by:

- a column packed with
- bidispersed particules of adsorbants  
macroporous binder of
- small microporous crystals

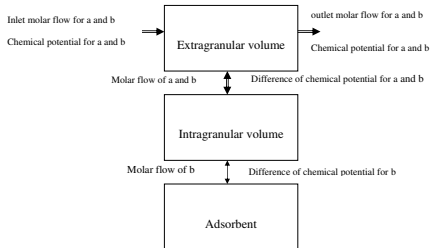


**Figure:** Schematic representation of a column a), packed with absorbent pellets b), themselves constituted by crystals c), associated with the three scales

# Pressure swing adsorption process : 3 level model

It is represented by 3-level mathematical model:

- a fluid flow dynamics at the column scale
- diffusion phenomena at the macroporous medium scale
- diffusion phenomena at the microporous medium scale



# Pressure swing adsorption process : 2 scale diffusion

Dispersed **two-phase heterogeneous mixture** with component  $\alpha$  and  $\beta$ .

The  **$\beta$ -phase** is dispersed in the  $\alpha$  one.

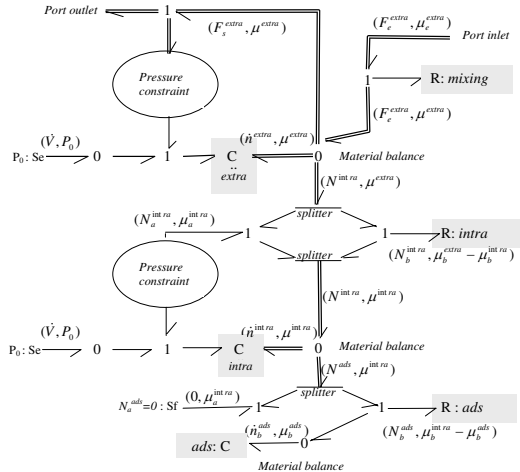
Assume there is a volume  $v_\alpha$ , characterized by a length  $L_\alpha$ , for the  $\alpha$ -phase for which:

$$\lambda_\alpha \ll L_\alpha \ll \Lambda_\alpha \text{ and } \Lambda_\beta \sim \lambda_\alpha, \quad (1)$$

where:

- $\lambda_i$  is the distance over which  $\phi_i$  varies significantly
- $\Lambda_i$  is the characteristic length of the  $i$ -phase

# Pressure swing adsorption process: mass transport model



# Application to the identification of diffusion parameters

The experimental set-up of LAGEP :

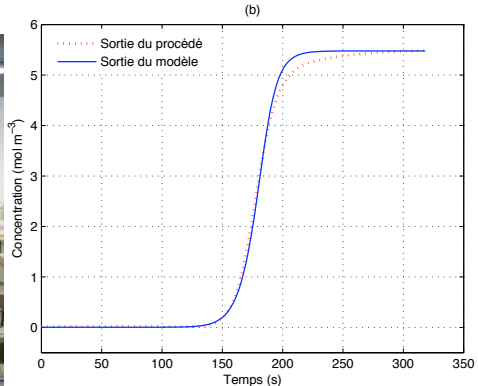


Figure: Pressure Swing Adsorption Process of LAGEP

# Model of a microchannel

*Model of a microchannel*



# A 1-D model of fluid flow: shallow water equations

One-dimensional fluid flow: the **shallow water equation**

- state variables:  $x(t) = \begin{pmatrix} p \\ q \end{pmatrix}$    
 momentum   
 section area of the water

- total energy :  $H_0(p, q) = \frac{1}{2} \int_a^b \frac{\rho g}{W} q^2 + \frac{1}{\rho} qp^2 dz$

- co-energy variables :

$$\frac{\delta H_0}{\delta x} = \begin{pmatrix} \frac{qp}{\rho} \\ \frac{p^2}{2\rho} + \frac{\rho g}{W} q \end{pmatrix} \begin{matrix} \text{volumic flow} \\ \text{hydrodynamic pressure} \end{matrix}$$

- Port Hamiltonian systems w.r.t. canonical Stokes-Dirac structure

# Shallow water equations : control design

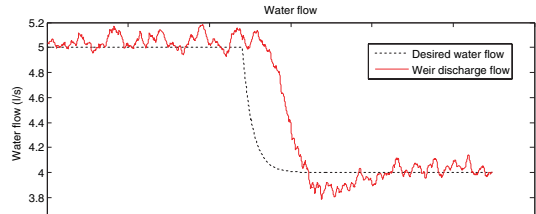
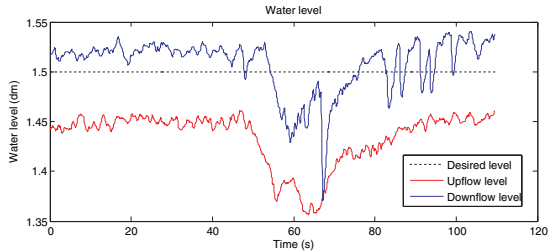
- The control acts through motion of the gates : inversion of the weir equation
- The control design is to “shaping the energy in closed-loop” by using Casimir functions :
  - the total volume
  - the total kinetic momentum

Controller :

- The feed-forward action shapes the energy of the system and stabilizes the total volume and momentum of the system
- the feedback action introduce a proportional correction for the closed loop (shaping the Dirac structure)
- an integral action is added

# Shallow water equations : experimental results

## Microchannel of ESISAR (Valence)

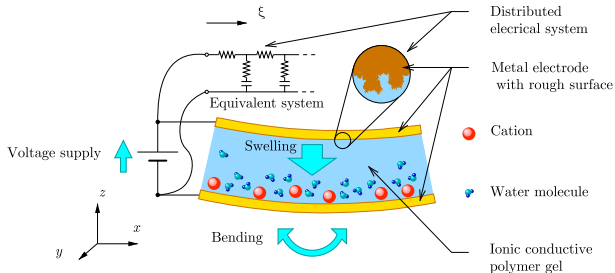


# ionic polymer metal composite (IPMC) [Nishida, Takagi]

*ionic polymer metal composite*

# Ionic polymer metal composite (IPMC)

A polyelectrolyte gel (*electro-active polymers* (EAPs)) between metal electrodes



**Fig. 2.** Physical structure of IPMC.



**Fig. 1.** IPMC (left:

# The IPMC : physical phenomena

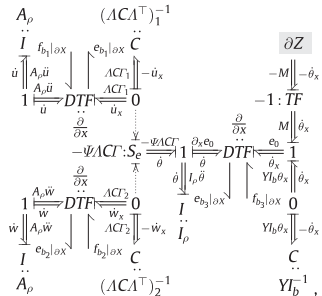
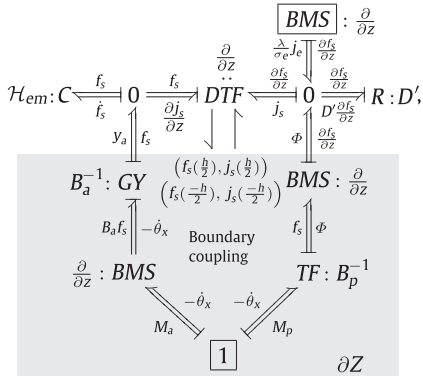
Model is the coupling of physical phenomena at **three spatial scales**

- **the electric double layer** on the interface between the polymer and the metal electrodes: spatial scale is of the order of nanometers, and the time constant is about  $0.001\text{ [s]}(\text{Na}^+)$  –  $1\text{ [s]}(\text{TEA}^+)$
- **the electro-stress diffusion coupling** with bending and relaxation dynamics that describe polyelectrolyte gels: scale of  $100\text{ [}\mu\text{m]}$ , and their time constant is about  $1\text{ [s]}(\text{Na}^+)$  –  $100\text{ [s]}(\text{TEA}^+)$
- **large mechanical deformations** : beam model: scale is  $10\text{ [cm]}$ , and their time constant is about  $0.03\text{ [s]}$ .

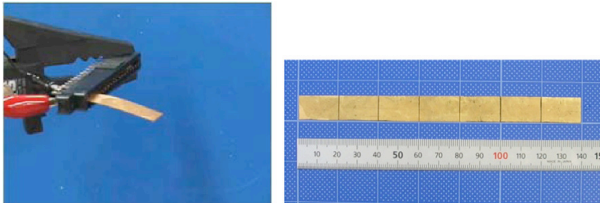
**Boundary Port Hamiltonian system:** **conservation laws coupled by interface relations on pairs of extensive-intensive variables.**

# The IPMC : bond graph

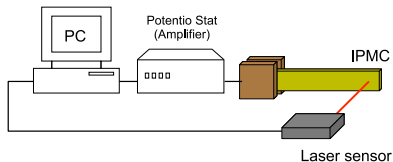
## Bond graph submodels



# Experimental results: setup



**Fig. 1.** IPMC (left: actuation with power supply, right: dimensions).



**Fig. 4.** Experimental setup.



# Experimental conditions

- The IPMC sample was made of Nafion117 (DuPont Inc.).
- The two faces in the  $xy$ -plane of the IPMC were plated with gold Au five times.
- The dimensions of the IPMC were 45 [mm] length  $L$ , 5 [mm] width  $b$ , and about 0.20 [mm] membrane thickness  $h$ .
- The deflection  $w$  at about 35 [mm] from the fixed end was measured with a laser displacement meter.
- The IPMC actuator was operated in air. In order to hydrate the IPMC sufficiently, we used a pipette for dropping deionized water during the experiment. The excess water was wiped by a disposable nonwoven fabric.
- The counterions were exchanged by soaking the sample in 0.1 [mol/l] NaOH solution or 0.2 [mol/l] TEA-Cl solution for more than 12 hours.

# Experimental results: parameters

**Table 1**

Physical parameters (Bar-Cohen, 2004; Yamaue et al., 2005).

|                             | Na <sup>+</sup>                         | TEA <sup>+</sup>                        |
|-----------------------------|---|---|
| $z$                         | 1                                       | 1                                       |
| $c$ (/m <sup>3</sup> )      | $2.487 \times 10^{27}$<br>(4.131 mol/l) | $3.228 \times 10^{27}$<br>(5.363 mol/l) |
| $q$ (C)                     | $1.602 \times 10^{-19}$                 | $1.602 \times 10^{-19}$                 |
| $\phi_p$                    | 0.7                                     | 0.7                                     |
| $a$ (nm)                    | 0.164                                   | 0.260                                   |
| $\xi_b$ (nm)                | 0.96                                    | 0.88                                    |
| $\rho$ (g/cm <sup>3</sup> ) | 1.633                                   | 1.633                                   |
| $Y$ (MPa)                   | 90                                      | 90                                      |
| $\nu$                       | 0.3                                     | 0.3                                     |

**Table 2**

Calculated parameters.

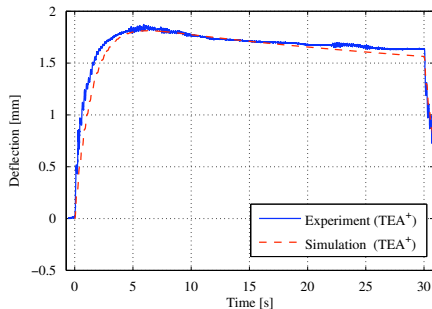
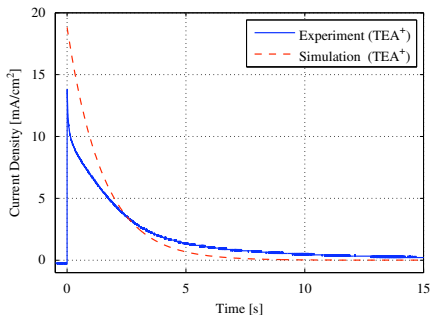
|                                 | Na <sup>+</sup>         | TEA <sup>+</sup>        |
|---------------------------------|-------------------------|-------------------------|
| $\sigma_e$ ( $\Omega^{-1}$ /cm) | 0.3334                  | 0.3274                  |
| $\lambda$                       | $9.654 \times 10^{-9}$  | $16.60 \times 10^{-9}$  |
| $\kappa$                        | $5.487 \times 10^{-18}$ | $8.530 \times 10^{-18}$ |
| $D$                             | $3.262 \times 10^{-10}$ | $1.375 \times 10^{-11}$ |

**Table 3**

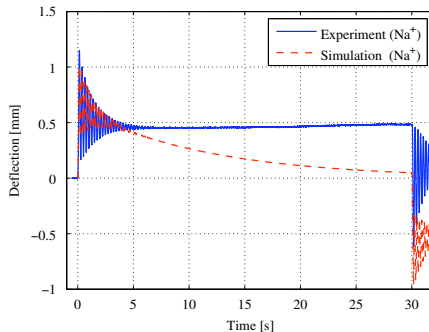
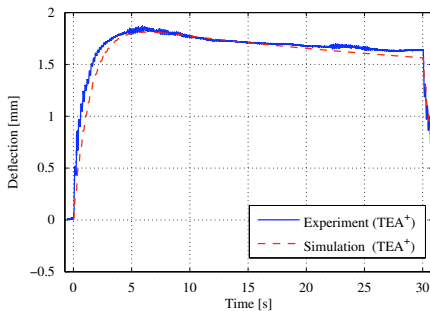
Identified parameters.

|                  | $\tau_e$ (s) | $R_m$ ( $\Omega$ cm <sup>2</sup> ) | $C_2$ (mF/cm <sup>2</sup> ) |
|------------------|--------------|------------------------------------|-----------------------------|
| Na <sup>+</sup>  | 0.0308       | 13.8                               | 2.25                        |
| TEA <sup>+</sup> | 1.73         | 71.4                               | 24.2                        |

# Experimental results: TEA+



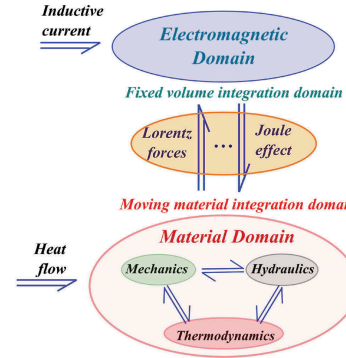
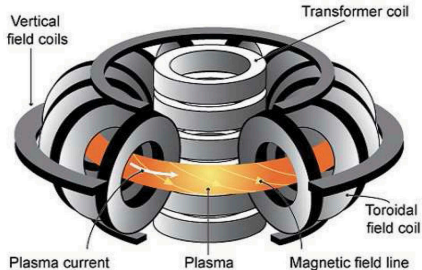
# Experimental results: TEA+ versus Na+



# Tokamak : Thermo-Magneto- Hydro- dynamical model (Lefèvre and Vu)

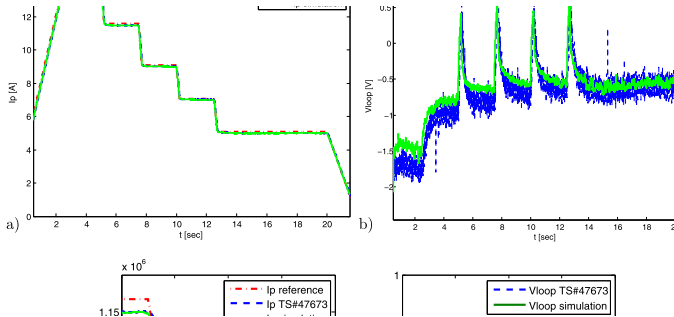
*Thermo- Magneto- Hydro- dynamical model*

# Tokamak : word Bond graph



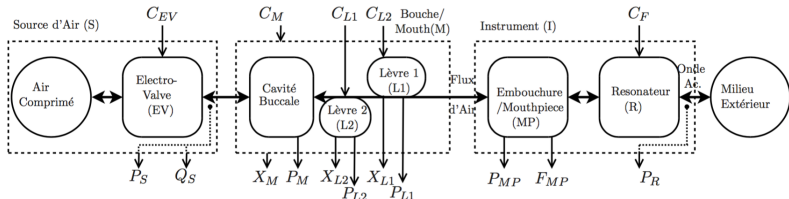
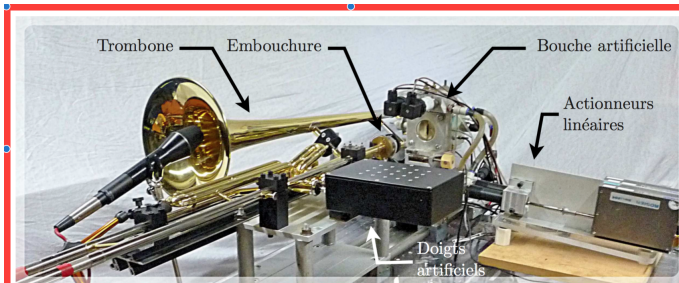


# Tokamak : experimental results





# Port Hamiltonian systems for a robotic system playing trombone [N. Lopes, IRCAM, Ph.D. defense 15 June 2016].

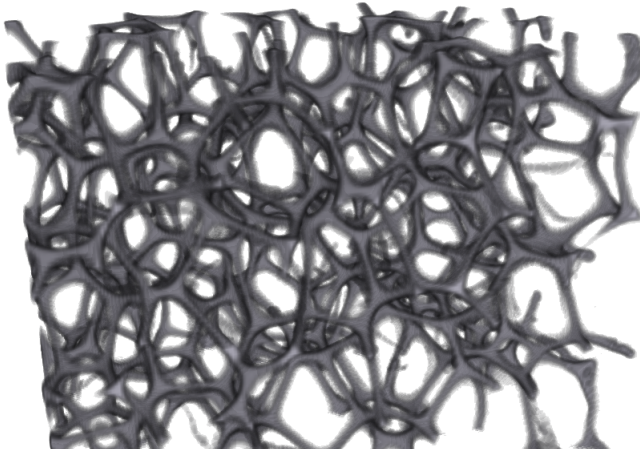


# Heat and mass transfert in catalytic foams

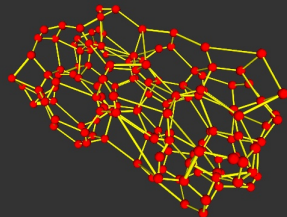
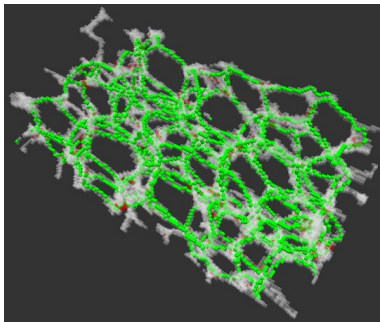
*Heat and mass transfer in catalytic foams*

# Motivational example 1 : foam structures for catalytic reactors

- 1 Continuous material with high porosity avoiding pressure loss and enhancing solid effective heat conductivity in reactors .



# Foams : from X-Ray tomograph to Graph representation



obtained by Pierre Gueth (LIRIS) and David Coeurjoly (LIRIS),  
Foam pieces experimented and provided by Marie-Line Zanota  
(LGPC) and Isabelle Pitault (LAGEP) during the ANR  
DIGITALFOAM project (2013-2015)

# Foam structures : test banks



Mass, energy and momentum balance equations

- ① Chemical Engineering pathway :
  - ① Consider the packing as an isotropic media
  - ② estimate state and parameters  
fails for highly exo-  
(endo-)thermic reactors
- ② Fluid Mechanics" pathway :
  - ① discretization of balance equations over meshing
  - ② extreme refinement of meshing at contact points  
fails to simulate complete bed and pressure drop

From  $60 \cdot 10^6$  to  $700 \cdot 10^6$  PDEs for "solid" phases and  $6 \cdot 10^6$  and  $70 \cdot 10^6$  PDEs for