Model Order Reduction

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Include your code in an appendix at the end of the report. Submit a single pdf file.

Standard plagiarism (and late submission) policies apply.

Solve points 1 to 6. Points 7 and 8 are optional and do not have to be completed.

A class of linear mechanical models can be described by the second-order system

$$M\ddot{q} + C_d\dot{q} + Kq = B_d u,$$

where $q(t) \in \mathbb{R}^\kappa$, $u(t) \in \mathbb{R}$, $M \in \mathbb{R}^{\kappa\times\kappa}$, $C_d \in \mathbb{R}^{\kappa\times\kappa}$, $K \in \mathbb{R}^{\kappa\times\kappa}$ and $B_d \in \mathbb{R}^{\kappa\times 1}$, with M positive definite. This system can be written as

$$\dot{x} = Ax + Bu \qquad y = Cx, \tag{1}$$

with

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C_d \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ -M^{-1}B_d \end{bmatrix},$$

and C taken according to the specific application.

Figure 1: Los Angeles Hospital building

Consider the model of a building with 8 floors, each having three degrees of freedom, *i.e.* displacements in the $x-$ and $y-$directions, and rotation in the $xy-$plane. The resulting system (1) has a state of dimension $n = 48$. The output of the system is the 25-th component of the state which corresponds to the displacement in the $x-$direction of the first floor. The resulting matrices (A, B, C) of system (1) are given in the attached file.

1. Verify that system (1) satisfies the assumptions to apply model order reduction by moment matching (if MATLAB tells you that this is not the case, MATLAB is wrong... explain).
2. Select a matrix S and a vector L. As you complete your work, you will experience that the selection of S is fundamental to obtain a good reduced order model. You should update the selection of S and L several times. In the report explain how the selection of the matrices S and L has been performed, *i.e.* explain why and how you end up with your final selection.

3. Compute the matrix Π. Do not use the function *sylvester* in MATLAB. Write your own routine exploiting more basic MATLAB commands (for example *kron*). Compare the solution given by your routine with the solution given by the function *sylvester* (it does not need to be better).

4. For the selected S and L, take random initial conditions $x(0)$ and $\omega(0)$ and determine the output $y(t)$ when the input to (1) is given by $u(t) = L\omega(t)$. Plot the output $y(t)$ and the steady-state output $y_{ss}(t) = C\Pi\omega(t)$. Explain the results of the plot.

5. Select a set of eigenvalues Σ and determine a reduced order model (F, G, H) such that F has the eigenvalues Σ. Plot the output $y(t)$ and the output of the reduced order model $y_r(t)$ (always for some random initial conditions). Explain the results of the plot.

6. Compare the Bode plots of system (1) and of your reduced order model. Adjust S and Σ to *improve* the reduced order model. Explain your reasoning.

7. Using the matrix S determined in the previous step, use the output sequence $\{y(t)\}$ and the sequence $\{\omega(t)\}$ to compute an approximation of $C\Pi$ using the data-driven algorithm. Show the success of your algorithm by means of figures and/or simulations.

8. Read the paper [2]. Exploiting the results from point 6, estimate a transient sequence $y_{tr}(t)$ and use it to estimate an optimal $\widetilde{F}$ and $\widetilde{H}$ (you may use the function *ssest* of MATLAB). Comment on all the numerical problems that you face (if any). Compare the Bode plots of system (1) and of the new reduced order model and make also comparisons with the previous results.

References

[1] G. Scarciotti, A. Astolfi. *Nonlinear Model Reduction by Moment Matching*. Foundations and Trends in Systems and Control: Vol. 4: No. 3-4, pp 224-409.

[2] G. Scarciotti, Z.-P. Jiang, A. Astolfi. *Constrained optimal reduced-order models from input/output data*, 55th IEEE Conference on Decision and Control, Pages: 7453-7458, 2016