LECTURE 6
Power and force limitation

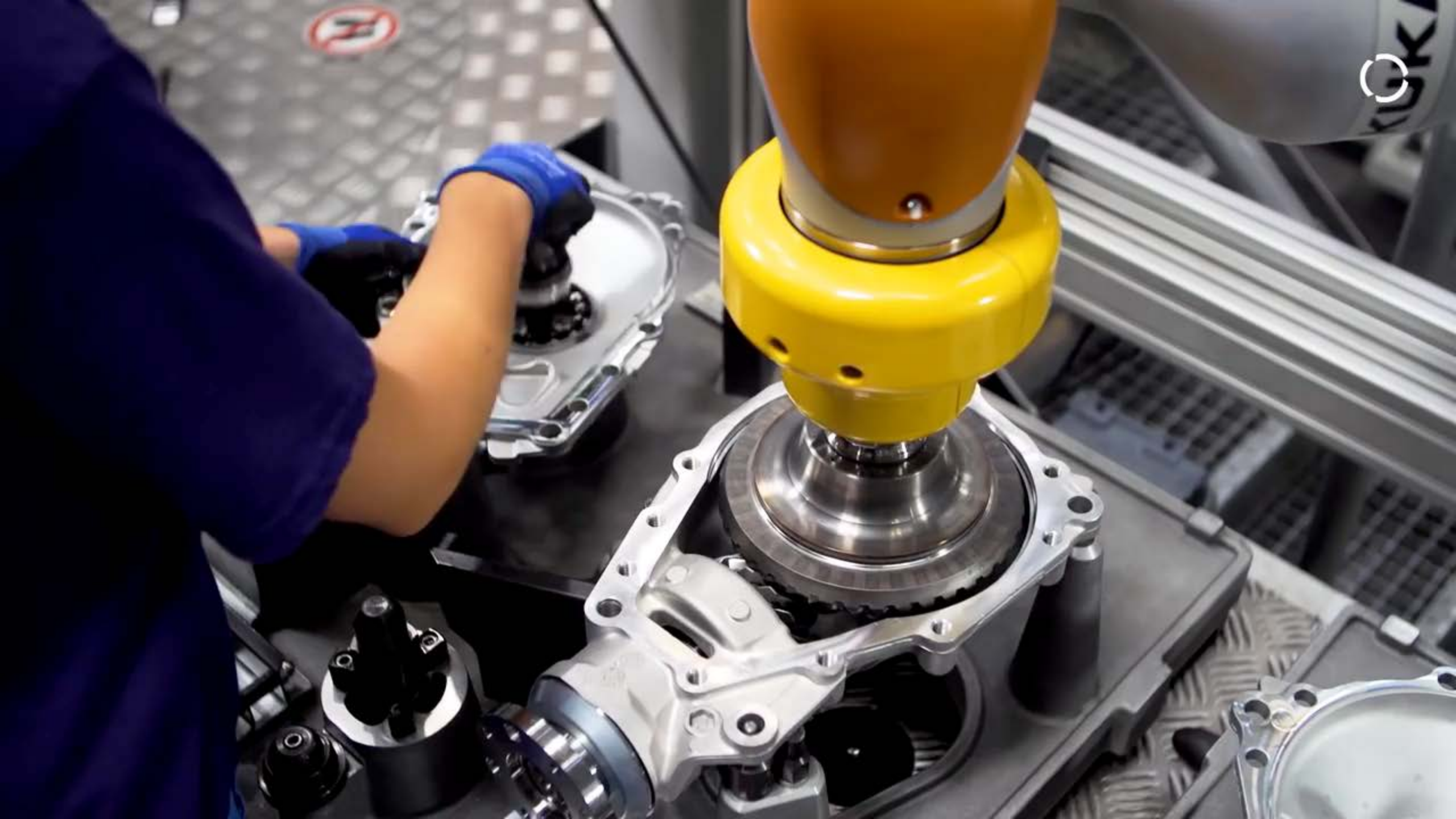
Andrea Zanchettin

Agenda

- Introduction and motivations

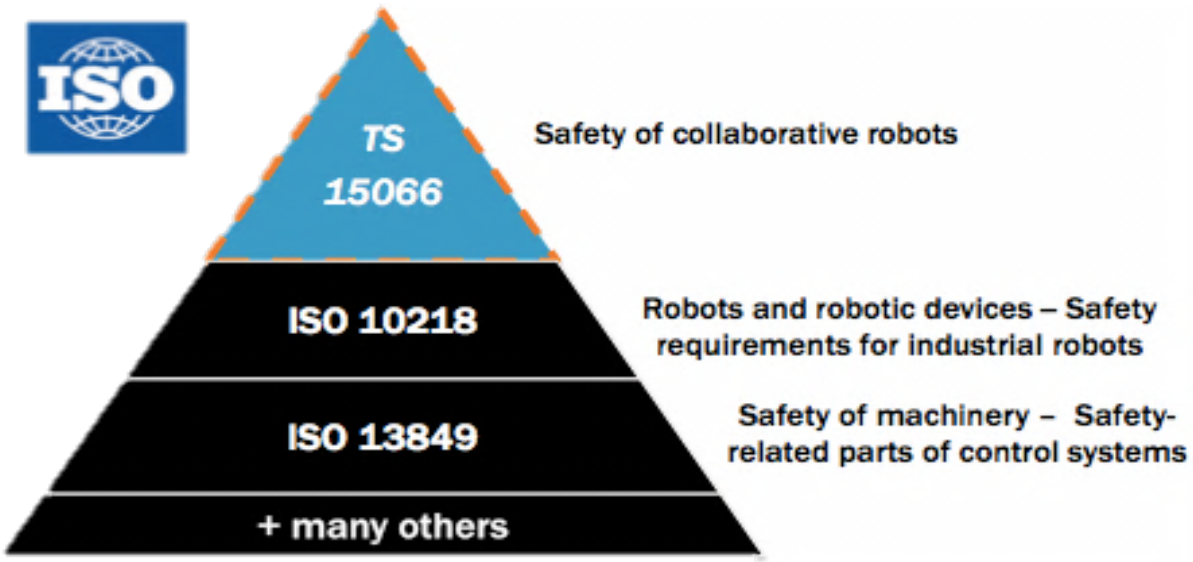
- Modelling contacts
  - Nomenclature
  - Transient contacts and quasi-static contacts
  - ISO TS 15066 thresholds

- Motion generation and control
  - Model verification
  - Speed supervision



Introduction and motivations

Introduction and motivations



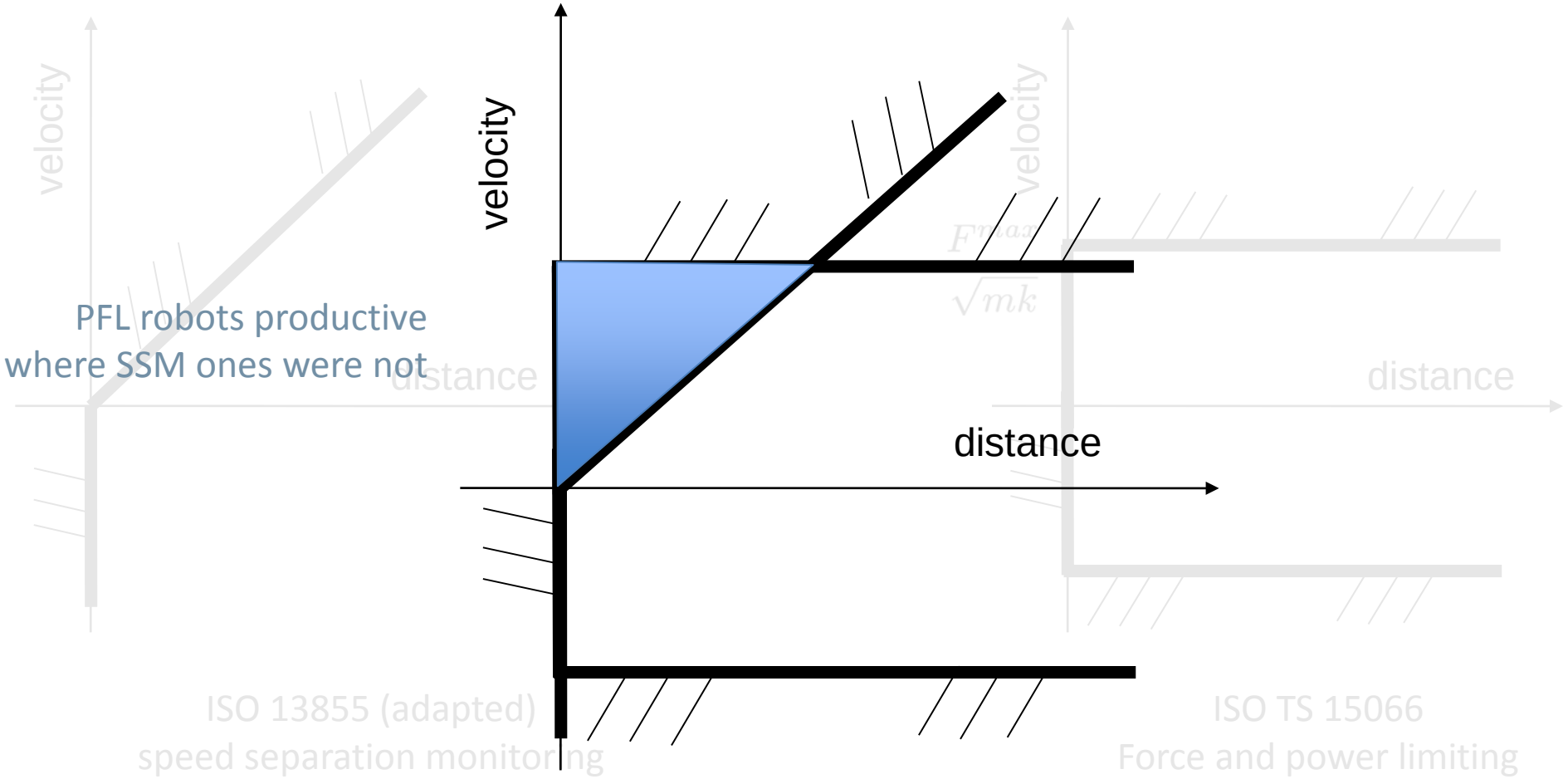
Energy transfer $\Delta T = \frac{1}{2} m v_r^2$

Energy deposit $\Delta T = \frac{1}{2} k x^2 = \frac{1}{2} \frac{F^2}{k}$

Boundary $m v_r^2 = \frac{F^2}{k} \Rightarrow v_r \leq \frac{F^{max}}{\sqrt{mk}} \approx \frac{p^{max} A}{\sqrt{mk}}$

Without sensing $|v_r| \leq \frac{p^{max} A}{\sqrt{mk}}$

Introduction and motivations



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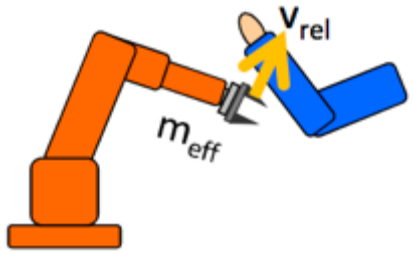
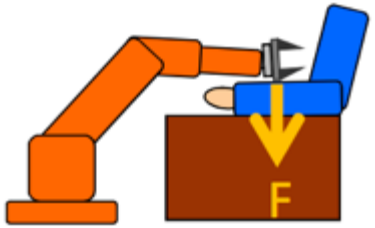
Modelling contacts - nomenclature

	Transient Contact	Quasi-Static Contact
Description	• Contact event is “short” (< 50 ms) • Human body part can usually recoil	• Contact duration is “extended” • Human body part cannot recoil, is trapped
Limit Criteria	• Peak forces, pressures, stresses • Energy transfer, power density	• Peak forces, pressures, stresses
Accessible in Design or Control	• Effective mass (robot pose, payload) • Speed (relative) • Contact area, duration	• Force (joint torques, pose) • Contact area, duration





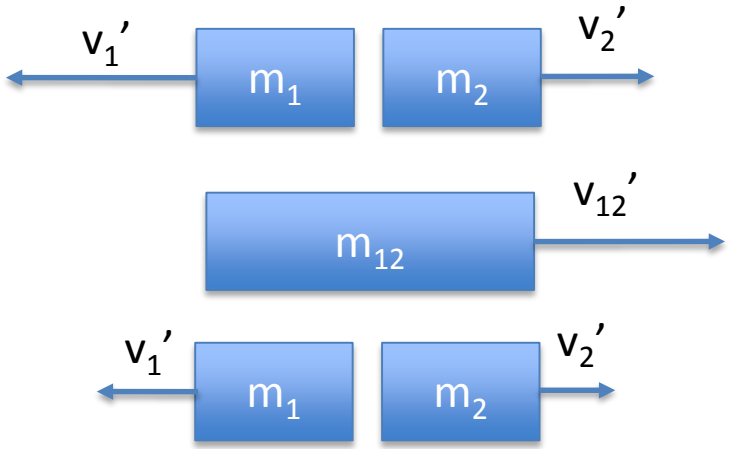
Courtesy of ABB Corporate Research (Ladenburg, Germany)

Modelling contacts - transient and quasi-static contacts

A contact between two bodies can be described looking at the discontinuity:

- Pre-contact situation:

- Post-contact situation:
  - Elastic (conservation of momentum)
  - Totally inelastic (maximum dissipation)
  - Partially inelastic (partial dissipation)



Modelling contacts - transient and quasi-static contacts

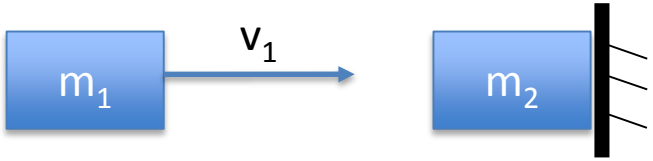
A more formal representation of the energy dissipated during a generic impact can be given as follows:

$$\Delta T = \frac{1}{2}\frac{1-e^2}{\left(m_1^{-1}+m_2^{-1}\right)^{-1}}\left(v_2-v_1\right)^2 = \frac{1}{2}\frac{1-e^2}{\left(m_1^{-1}+m_2^{-1}\right)^{-1}}v_r^2$$

where

- e = 1 corresponds to an elastic impact (no dissipation)
- e = 0 corresponds to a totally inelastic impact

Moreover, for **quasi-static** contacts we have $m_2 = \infty$.



Rossi, R., Parigi Polverini, M. Zanchettin, A.M., Rocco, P. IROS (2015)

Modelling contacts - transient and quasi-static contacts

We still need to understand what is the maximum energy transfer each human body part can tolerate.

If we assume the body part subject to contact to be modelled as a spring we have:

$$\Delta T = \frac{1}{2} k \Delta x^2 = \frac{1}{2} \frac{F^2}{k} = \frac{1}{2} \frac{A^2 p^2}{k}$$

Therefore, in the worst case (e = 0) we have

$$\frac{A^2 p^2}{k} = \frac{v_r^2}{\left(m_1^{-1} + m_2^{-1}\right)^{-1}} \Rightarrow \frac{v_r}{A} \sqrt{\frac{k}{\left(m_1^{-1} + m_2^{-1}\right)^{-1}}} \leq p^{max}$$

Modelling contacts - transient and quasi-static contacts

Back to the formula: $$\frac{v_r}{A}\sqrt{\frac{k}{\left(m_1^{-1}+m_2^{-1}\right)^{-1}}} \leq p^{max}$$

- the higher area A the better: this is way new generation robots have smooth surfaces and no pinch points;
- the lighter the better: this is way cobots are typically light-weight with reduced payload.

Modelling contacts - transient and quasi-static contacts

For the clamping case, in turn, we have: $$\frac{v_r}{A}\sqrt{\frac{k}{m_1}} \leq p^{max}$$

- the higher area A the better: this is way new generation robots have smooth surfaces and no pinch points;
- the lighter the better: this is way cobots are typically light-weight with reduced payload.

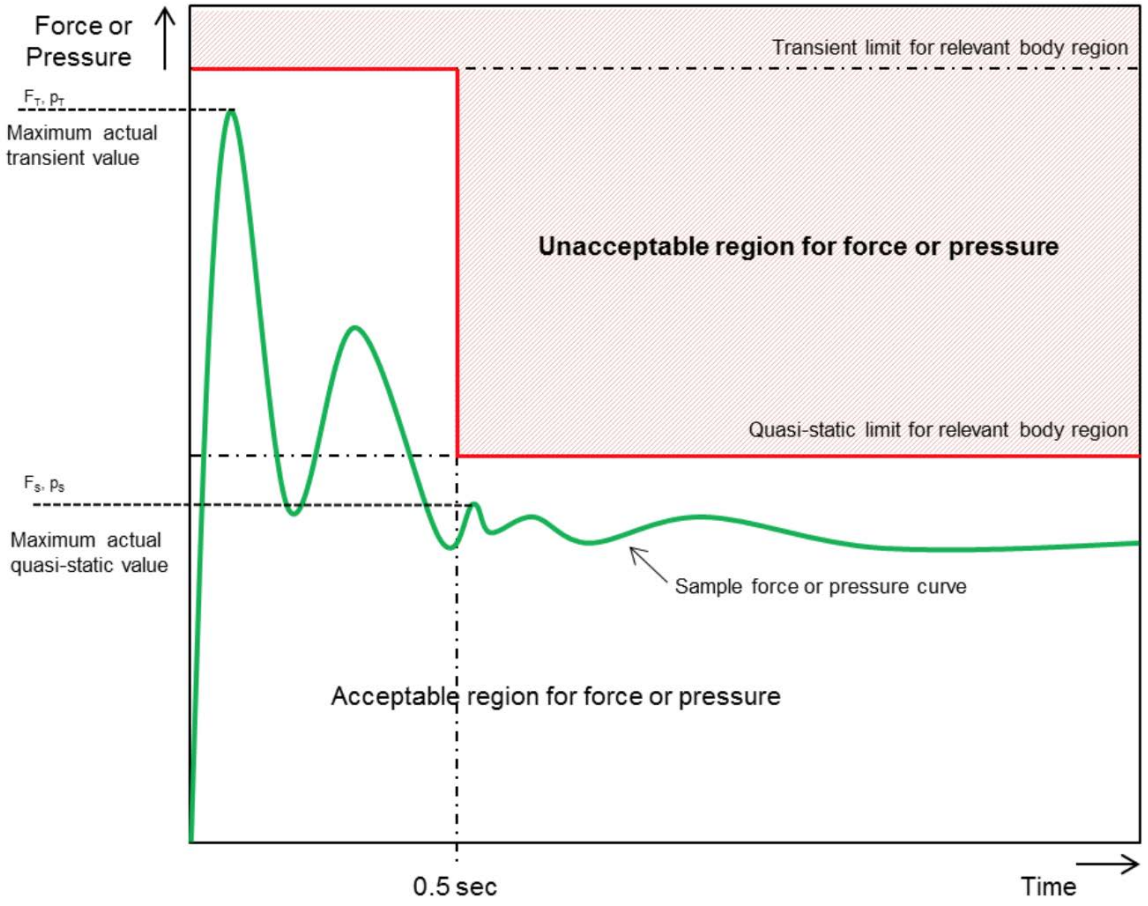
Modelling contacts - transient and quasi-static contacts

Summing up, we have:

- Transient contact: $$\frac{v_r}{A}\sqrt{\frac{k}{\left(m_1^{-1}+m_2^{-1}\right)^{-1}}} \leq p^{max} \qquad v_r\sqrt{\frac{k}{\left(m_1^{-1}+m_2^{-1}\right)^{-1}}} \leq F^{max}$$

- Quasi-static contact: $$\frac{v_r}{A}\sqrt{\frac{k}{m_1}} \leq p^{max} \qquad v_r\sqrt{\frac{k}{m_1}} \leq F^{max}$$

Modelling contacts - transient and quasi-static contacts



Based on some empirical verification, ISO TS 15066 actually suggests to double the maximum quasi-static contact force and pressure thresholds in order to obtain the transient ones.

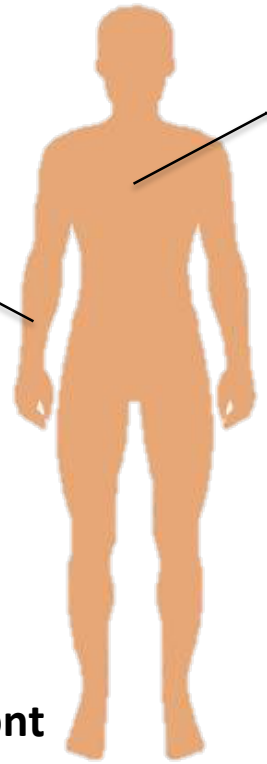
In fact, the peak force (transient contact) is approximately twice the steady-state one (quasi-static).

Modelling contacts - ISO TS 15066 thresholds

Few examples:

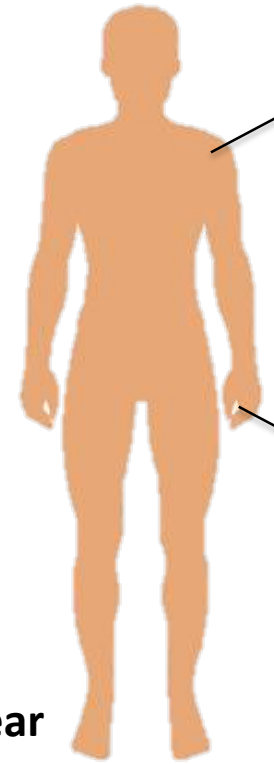
Arm nerve:
p^{max} = 180 N/cm^2
F^{max} = 160 N
k = 40 N/mm
m_2 = 2 kg

Sternum:
p^{max} = 120 N/cm^2
F^{max} = 140 N
k = 25 N/mm
m_2 = 40 kg



Front

Deltoid:
p^{max} = 190 N/cm^2
F^{max} = 150 N
k = 35 N/mm
m_2 = 40 kg



Rear

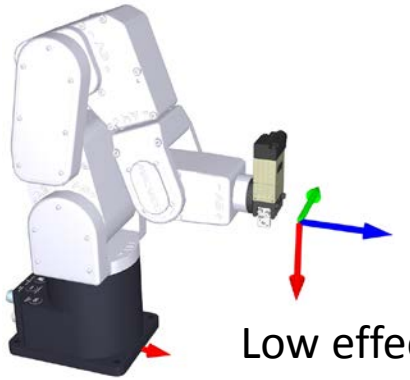
Forefinger end:
p^{max} = 270 N/cm^2
F^{max} = 140 N
k = 75 N/mm
m_2 = 0.6 kg

Modelling contacts - transient and quasi-static contacts

Finally, what is the equivalent mass of the robot?

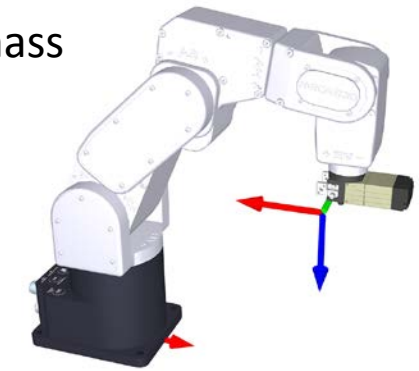
$$m_1^{-1} = \boldsymbol{n}^T \boldsymbol{J}_P(\boldsymbol{q}) \boldsymbol{B}(\boldsymbol{q})^{-1} \boldsymbol{J}_P(\boldsymbol{q})^T \boldsymbol{n}$$

Equivalent (of effective) mass depends on posture. This complicate the risk assessment as the severity of the impact depends on the position of the robot, and not only on its velocity.



Low effective mass

High effective mass



Khatib, O. IJRR (1995)

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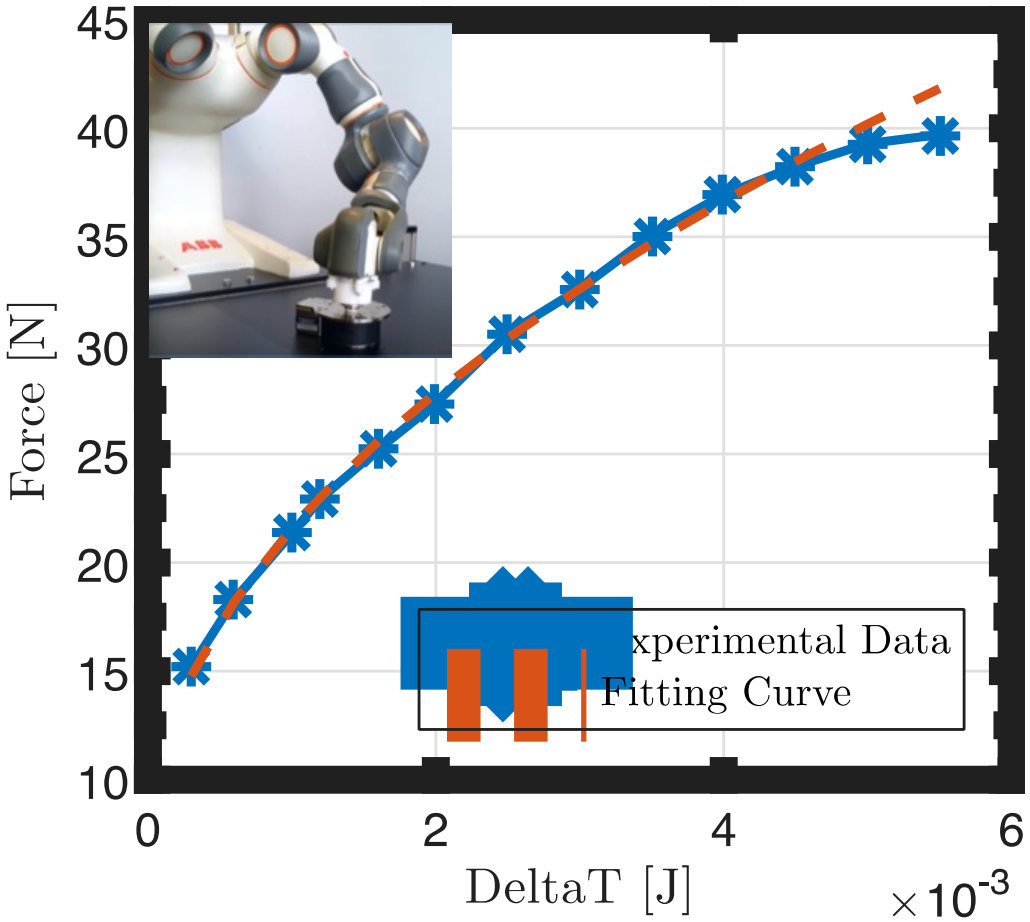
Motion generation and control - model verification

We first have to verify the model:

$$\Delta T = \frac{1}{2}\frac{F^2}{k} \Rightarrow F \propto \sqrt{2\Delta T k}$$

Assuming a quasi-static contact against a fixed object (a load cell), we have

$$\Delta T = \frac{1}{2}\dot{\boldsymbol{q}}^T \boldsymbol{B}(\boldsymbol{q})\,\dot{\boldsymbol{q}}$$



Rossi, R., Parigi Polverini, M. Zanchettin, A.M., Rocco, P. IROS (2015)

Motion generation and control - speed supervision

Assume a path is given for the position and orientation of the robot end-effector

$$\boldsymbol{x} = \boldsymbol{x}\left(s\right)$$

and an inverse kinematic mapping is available

$$\boldsymbol{q} = \boldsymbol{f}^{-1}\left(\boldsymbol{x}\left(s\right)\right) = \boldsymbol{h}\left(s\right) \qquad \dot{\boldsymbol{q}} = \frac{\partial \boldsymbol{h}}{\partial s}\dot{s} \qquad \ddot{\boldsymbol{q}} = \frac{\partial \boldsymbol{h}}{\partial s}\ddot{s} + \frac{\partial^2 \boldsymbol{h}}{\partial s^2}\dot{s}^2$$

Substituting in the dynamic equations of the robot we have

$$\boldsymbol{a}\left(s\right)\ddot{s} + \boldsymbol{b}\left(s\right)\dot{s}^2 + \boldsymbol{c}\left(s\right) = \boldsymbol{\tau}$$

Motion generation and control - speed supervision

Then accounting for the maximum motor torques, we have

$$\boldsymbol{\tau}_{min} \leq \boldsymbol{a}(s)\,\ddot{s} + \boldsymbol{b}(s)\,\dot{s}^2 + \boldsymbol{c}(s) \leq \boldsymbol{\tau}^{max}$$

$$\boldsymbol{\tau}_{min} - \boldsymbol{b}(s)\,\dot{s}^2 - \boldsymbol{c}(s) \leq \boldsymbol{a}(s)\,\ddot{s} \leq \boldsymbol{\tau}^{max} - \boldsymbol{b}(s)\,\dot{s}^2 - \boldsymbol{c}(s)$$

By collecting we obtain

$$\boldsymbol{\alpha}_{min}(s,\dot{s}) \leq \boldsymbol{a}(s)\,\ddot{s} \leq \boldsymbol{\alpha}^{max}(s,\dot{s})$$

which is a set of scalar inequalities (one per each joint).

Kang G. Shin, N. McKay, Tran AC (1985)

Motion generation and control - speed supervision

If we also include:

$$\Delta T\left(s,\dot{s}\right) \leq \Delta T^{max} = \frac{1}{2}\frac{F^{max\,2}}{k}$$

We can solve the following constrained optimization problem:

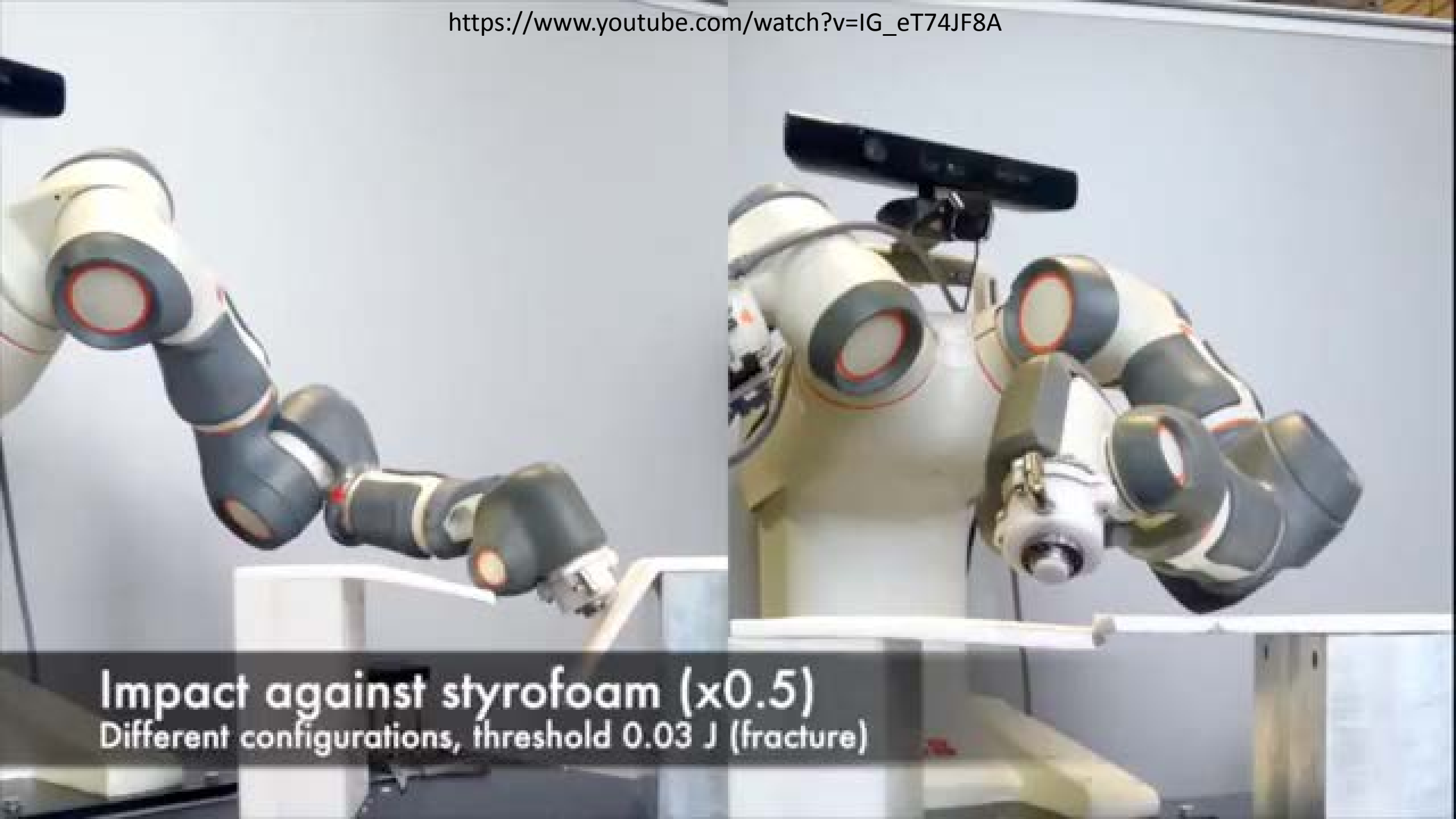
$$s\left(t\right) = \min_{\ddot{s}(\cdot)} \int_0^T dt$$

$$\boldsymbol{\alpha}_{min}\left(s,\dot{s}\right) \leq \boldsymbol{a}\left(s\right)\ddot{s} \leq \boldsymbol{\alpha}^{max}\left(s,\dot{s}\right)$$

$$\Delta T\left(s,\dot{s}\right) \leq \frac{1}{2}\frac{F^{max\,2}}{k}$$

This constraint has to be converted at acceleration level

https://www.youtube.com/watch?v=IG_eT74JF8A



Motion generation and control - speed supervision

The robot we used is redundant, hence it is also possible to minimize its kinetic energy, without compromising its velocity along the path.

Hence the selected redundancy resolution tries to reduce the effective mass by trying to minimize its gradient, i.e.

$$\min_{\boldsymbol{q}} \left(\boldsymbol{n}(s)^T \boldsymbol{J}_P(\boldsymbol{q}) \boldsymbol{B}(\boldsymbol{q})^{-1} \boldsymbol{J}_P(\boldsymbol{q})^T \boldsymbol{n}(s) \right)^{-1}$$

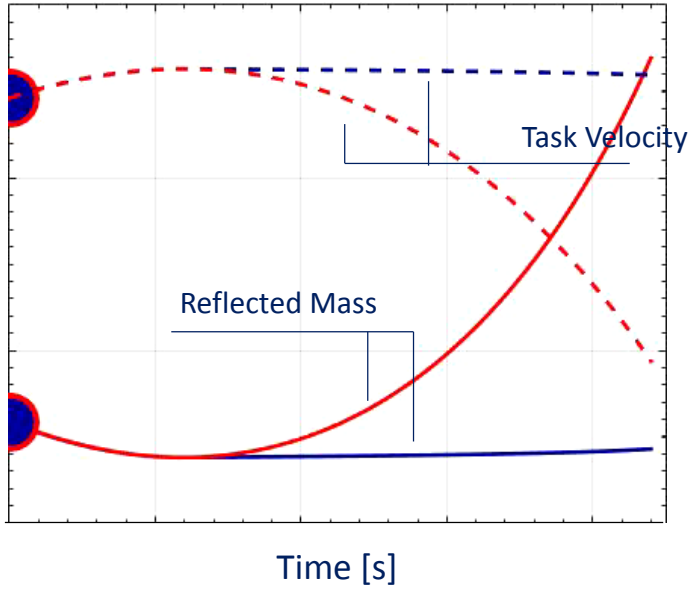
In order to solve this (lower priority) optimization problem, state of the art algorithms can be adopted.

Zanchettin, A.M., Rocco, P. Tran RO (2013)

Motion generation and control - model verification

Mass Minimization
Not Active

Mass Minimization
Active



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