Summer School SIDRA 2021
Modeling and Control of Soft Robots
Bertinoro, Italy
July 15-17, 2021

Regulation, Inversion Control, and Feedback Equivalence for Flexible Robots

Alessandro De Luca
Dipartimento di Ingegneria Informatica, Automatica e Gestionale (DIAG)

deluca@diag.uniroma1.it

SAPIENZA
UNIVERSITÀ DI ROMA

Summary

- a world of soft robots
  - flexible joints, serial elastic actuation (SEA), variable stiffness actuation (VSA), distributed link flexibility, continuum manipulators, …
- flexible joint robots
  - dynamic modeling and structural control properties
  - inverse dynamics and feedback linearization for trajectory tracking
  - regulation with partial state feedback and gravity compensation
- model-based design based on feedback equivalence
  - exact cancellation of gravity
  - damping injection on the link side
  - environment interaction via generalized impedance model
- an application of flexible joint robots: physical Human-Robot Interaction (pHRI)

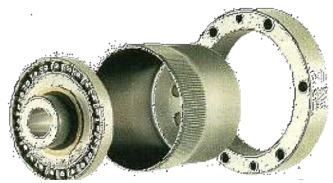
Summary

- flexible link robots
  - dynamic modeling and the role of zero dynamics
  - PD+ for regulation and input-output linearization for joint-level trajectory tracking
  - stable inversion of desired end-effector trajectories
- outlook on control of (planar) soft manipulators
  - using a piecewise continuous curvature (PCC) dynamic model

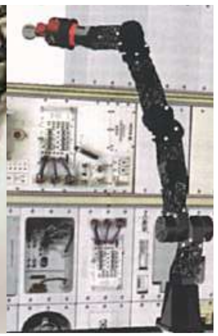
Classes of soft robots

Robots with elastic joints

- design of lightweight robots with stiff links for end-effector accuracy
- compliant elements absorb impact energy
  - elastic transmissions (HD, cable-driven, ...)
  - soft coverage of links (foam, safe bags)



- elastic joints decouple instantaneously the *larger* inertia of the driving motors from *smaller* inertia of the links (involved in contacts/collisions!)
- *relatively* soft joints need more sensing (e.g., joint torque) and better control to compensate for static deflections and dynamic vibrations

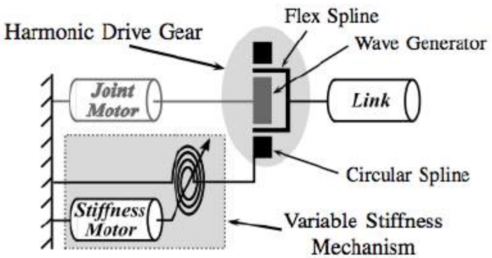


➡ torque-controlled robots (DLR LWR-III, KUKA LWR-IV & iiwa, Franka, ...)

Classes of soft robots

Robots with variable stiffness actuation (VSA)

- uncertain interaction with dynamic environments (say, *humans*) requires to adjust online the compliant behavior and/or to control contact forces
  - passive joint elasticity & active impedance control used in parallel
- nonlinear flexible joints with variable (controlled) stiffness work at best
  - can be made *stiff when moving slow* (performance), *soft when fast* (safety)
  - enlarge the set of achievable robot compliance in a task-oriented way
  - plus: mechanical **robustness**, optimal **energy use**, **explosive motion** tasks, ...



A matter of terminology ...

Different sources of elasticity, though similar robotic systems

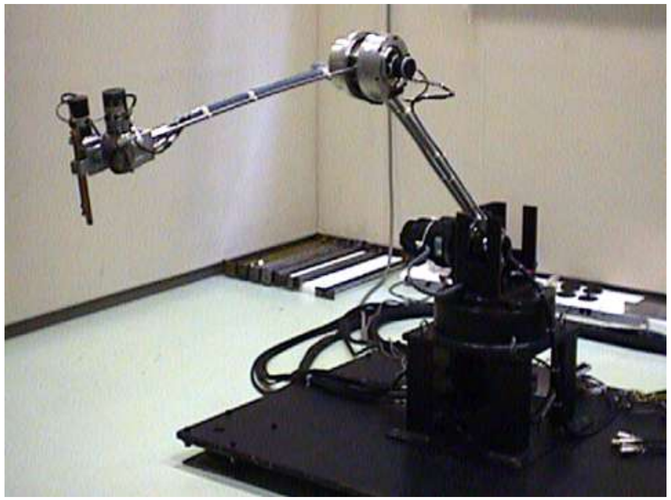
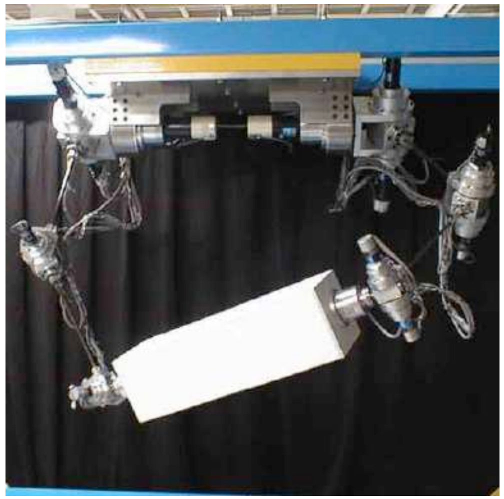
- elastic joints vs. SEA (serial elastic actuators)
  - based on the same physical phenomenon: compliance in actuation
  - compliance added on purpose in SEA, mostly a disturbance in elastic joints
  - different range of stiffness: 5-10K Nm/rad down to 0.2-1K Nm/rad in SEA
- joint deformation is often considered in the linear domain
  - modeled as a concentrated torsional spring with constant stiffness at the joint
  - nonlinear flexible joints share similar control properties
  - nonlinear stiffness characteristics & double actuation are needed in VSA
  - a (serial or antagonistic) VSA working at constant stiffness is an elastic joint
- flexible robots are usually classified as underactuated mechanical systems
  - have less commands than generalized coordinates
  - non-collocation of command inputs and controlled outputs
  - however, they are controllable in the first approximation (the *easy* case!)

Classes of soft robots

Robots with flexible links

- distributed link deformations
  - design of very long and slender arms needed in the application
  - use of lightweight materials to save weight/costs
  - due to large payloads (viz. large contact forces) and/or high motion speed
- as for joint elasticity, neglecting link flexibility will limit static (steady-state error) or dynamic (vibrations, poor tracking) performance
- control issue due to non-minimum phase nature of the end-effector output w.r.t. the torque command input … “it moves in opposite direction at start!”

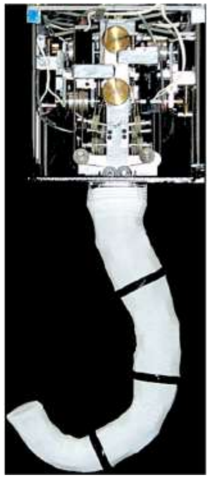


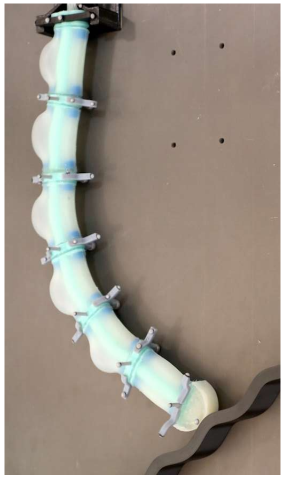
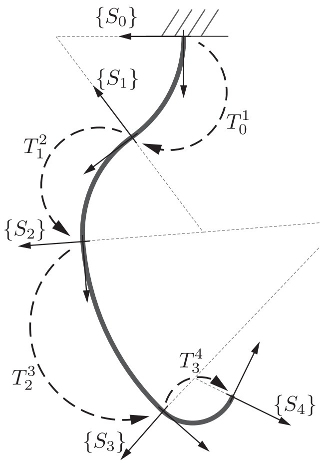



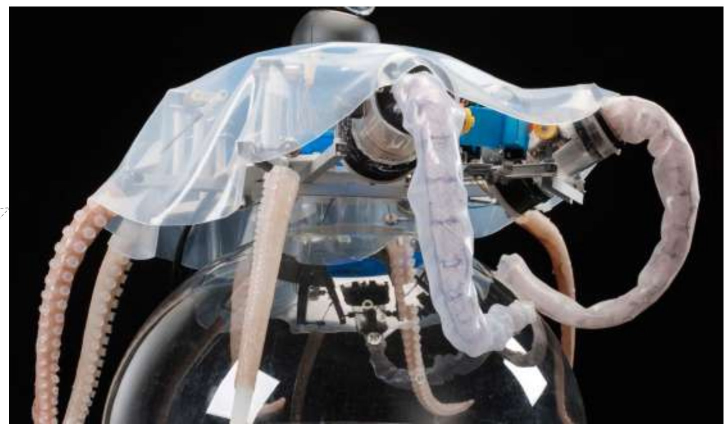
Classes of soft robots

Continuum soft manipulators

- characteristics in construction
  - long, flexible, lightweight, slender arms
  - tendon/cable-driven, multi-segmented, distributed/embedded actuation
  - energy efficient, (intentional) bio-inspired design
- useful in many special robotic applications
  - surgical, underwater, safe human interaction, cluttered environments, …
- kinematic, quasi-static, and dynamic modeling (with approximations)
- extra control issues due to task hyper-redundancy and under-actuation



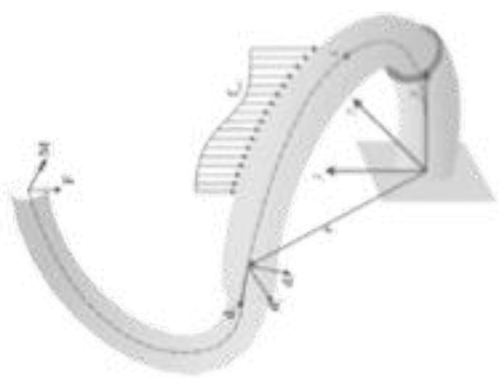





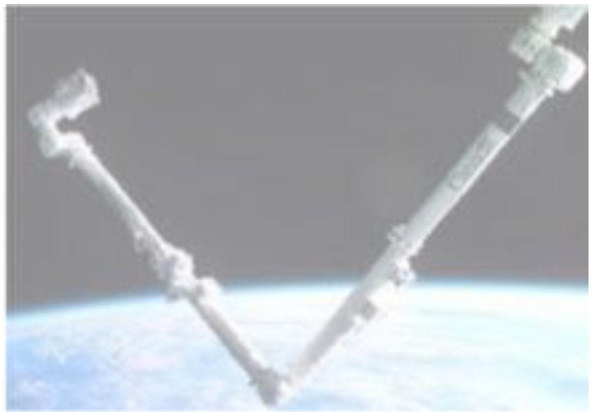
Flexible link robots vs. continuum manipulators

What are the actual (control) differences?

- continuum manipulators may assume **very complex shapes in 3D**
  - flexible link robots **not!**
- continuum manipulators may **keep a body-deformed configuration** under the action of control (apart from gravity)
  - flexible link robots **not!**
- flexible link robots are **always underactuated** mechanical systems
  - continuum manipulators also, but **possibly not!**
- **collocated vs. non-collocated** control: both may or may not have this ...



TUDOR



Canadarm2

Jan Fraas, QMU

Dynamic modeling of robots with flexible joints

Lagrangian formulation (so-called **reduced** model of [Spong, ASME JDSMC 1987])

- open chain robot with N flexible joints and N rigid links, driven by electrical actuators
- use N motor variables θ (as reflected through the gear ratios) and N link variables q
- assumptions

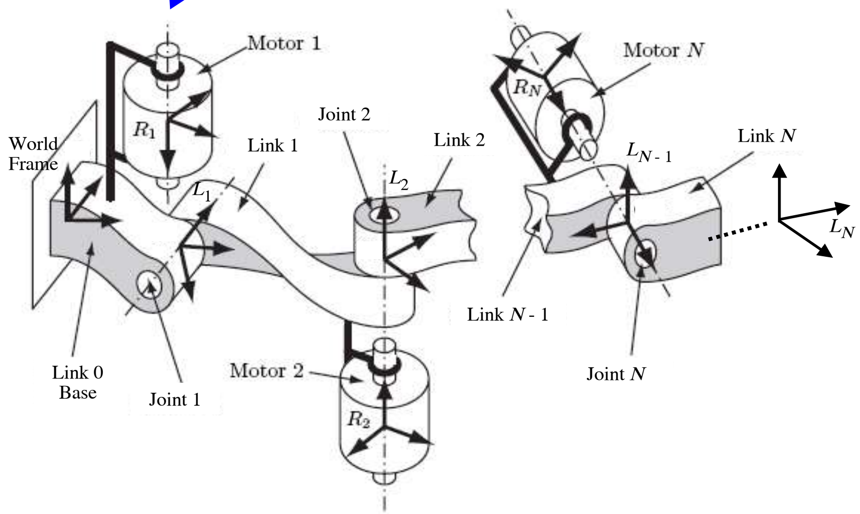
  A1) small displacements at joints (elasticity!)

  A2) axis-balanced motors

  A3) each motor is mounted on the robot in a position preceding the driven link

  A4) no inertial couplings between motors and links

center of mass of rotors on rotation axes



A4) ⇒ $2N \times 2N$ inertia matrix is block diagonal

A2) ⇒ inertia matrix and gravity vector are independent from θ

$$\begin{pmatrix} M(q) & 0 \\ 0 & B \end{pmatrix}\begin{pmatrix} \ddot{q} \\ \ddot{\theta} \end{pmatrix} + \begin{pmatrix} C(q,\dot{q})\dot{q} \\ 0 \end{pmatrix} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q-\theta) \\ K(\theta-q) \end{pmatrix} = \begin{pmatrix} 0 \\ \tau \end{pmatrix}$$

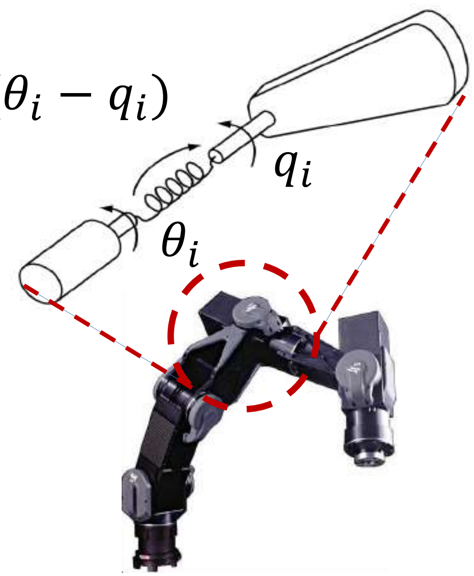
link equation

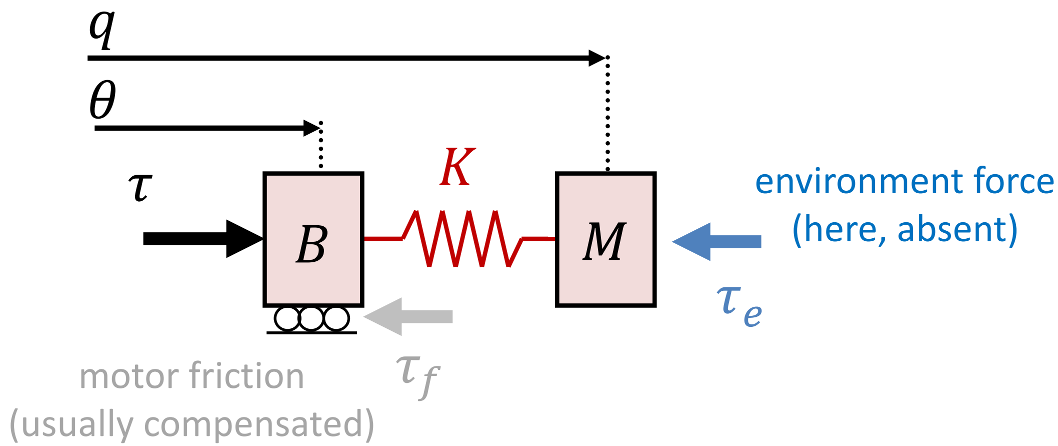
motor equation

Single elastic joint

Transfer functions of interest

$$\tau_{ei} = K_i(\theta_i - q_i)$$





we often look rather at the torque-to-velocity mappings … (eliminating one integrator)

[De Luca, Book, Springer Handbook of Robotics, 2016]

$$P_{\text{motor}}(s) = \frac{\theta(s)}{\tau(s)} = \frac{Ms^2 + K}{MBs^2 + (M+B)K}\,\frac{1}{s^2}$$

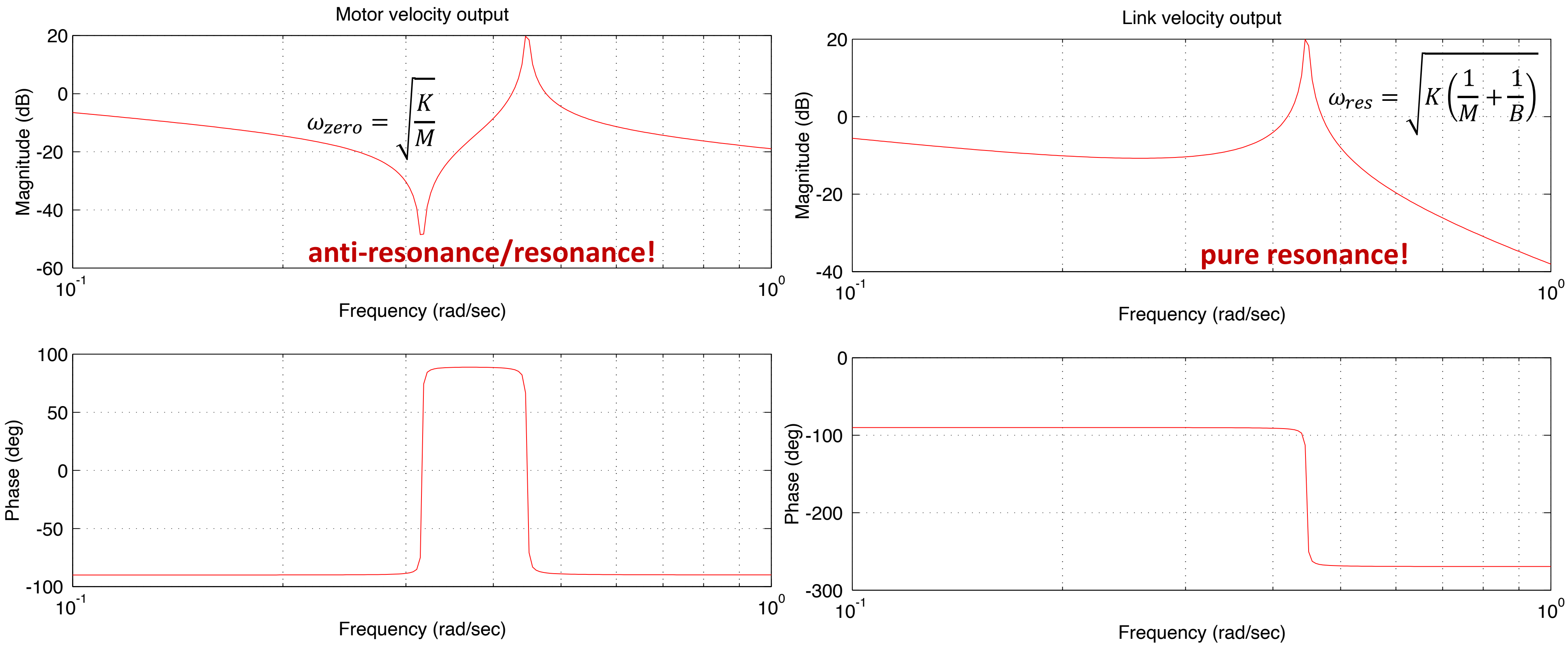
- system with stable zeros and relative degree = 2
- passive (zeros *precede* poles on imaginary axis)
- stabilization can be achieved via output θ feedback

$$P_{\text{link}}(s) = \frac{q(s)}{\tau(s)} = \frac{K}{MBs^2 + (M+B)K}\,\frac{1}{s^2}$$

- NO zeros!!
- maximum relative degree = 4

Single elastic joint

Transfer functions of interest



- typical anti-resonance/resonance on motor velocity output (minimum phase)
- pure resonance on link velocity output (weak or no zeros)

a (small) motor or link side viscous friction was added in these Bode plots

Inverse dynamics

Feedforward action for following a desired trajectory in nominal conditions

given a desired smooth link trajectory $q_d(t) \in C^4$

- compute symbolically the desired motor acceleration and, therefore, also the desired link jerk (i.e., up to the fourth time derivative of the desired motion)

$$\begin{pmatrix} M(q) & 0 \\ 0 & B \end{pmatrix}\begin{pmatrix} \ddot{q} \\ \ddot{\theta} \end{pmatrix} + \begin{pmatrix} C(q,\dot{q})\dot{q} \\ 0 \end{pmatrix} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q-\theta) \\ K(\theta-q) \end{pmatrix} = \begin{pmatrix} 0 \\ \tau \end{pmatrix}$$

$$\begin{aligned}\tau_d &= B\ddot{\theta}_d + K(\theta_d - q_d) \\ &= BK^{-1}\left[M(q_d)\, q_d^{(4)} + 2\dot{M}(q_d)\, q_d^{(3)} + \ddot{M}(q_d)\ddot{q}_d + \frac{d^2}{dt^2}\left(C(q_d,\dot{q}_d)\dot{q}_d + g(q_d)\right)\right] \\ &\quad + [M(q_d) + B]\ddot{q}_d + C(q_d,\dot{q}_d)\dot{q}_d + g(q_d)\end{aligned}$$

- the inverse dynamics can be computed efficiently in $O(N)$ using a modified Newton-Euler algorithm (with link recursions up to the 4th order) [Buondonno, De Luca IROS 2015]
- the feedforward command τ_d can be used in combination with a PD feedback control on motor position/velocity error to obtain a local but simple trajectory tracking controller

Feedback linearization

Full-state nonlinear feedback for accurate trajectory tracking tasks

- the link position q is a linearizing (flat) output (nonlinear equivalent of “no zeros”)

$$\begin{pmatrix} M(q) & 0 \\ 0 & B \end{pmatrix}\begin{pmatrix} \ddot{q} \\ \ddot{\theta} \end{pmatrix} + \begin{pmatrix} C(q,\dot{q})\dot{q} \\ 0 \end{pmatrix} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q-\theta) \\ K(\theta-q) \end{pmatrix} = \begin{pmatrix} 0 \\ \tau \end{pmatrix} \Longleftrightarrow \boxed{q^{(4)} = u}$$

- differentiating twice the link equation and using the motor acceleration yields

$$\tau = BK^{-1}M(q)u + K(\theta - q) + B\ddot{q} + BK^{-1}\left(2\dot{M}q^{(3)} + \ddot{M}\ddot{q} + \frac{d^2}{dt^2}(C\dot{q} + g(q))\right)$$

- an exactly linear and I-O decoupled system (“chains of 4 integrators”) is obtained
  - to be stabilized with standard techniques for linear dynamics (pole placement, LQ, ...)
- requires higher derivatives of q —— $q, \dot{q}, \ddot{q}, q^{(3)}$
- however, these can be computed from the model using state measurements
- requires higher derivatives of the dynamics components —— $\ddot{M}, \ddot{C}, \ddot{g}$
- A $O(N^3)$ Newton-Euler recursive numerical algorithm is available for this problem

Feedback linearization

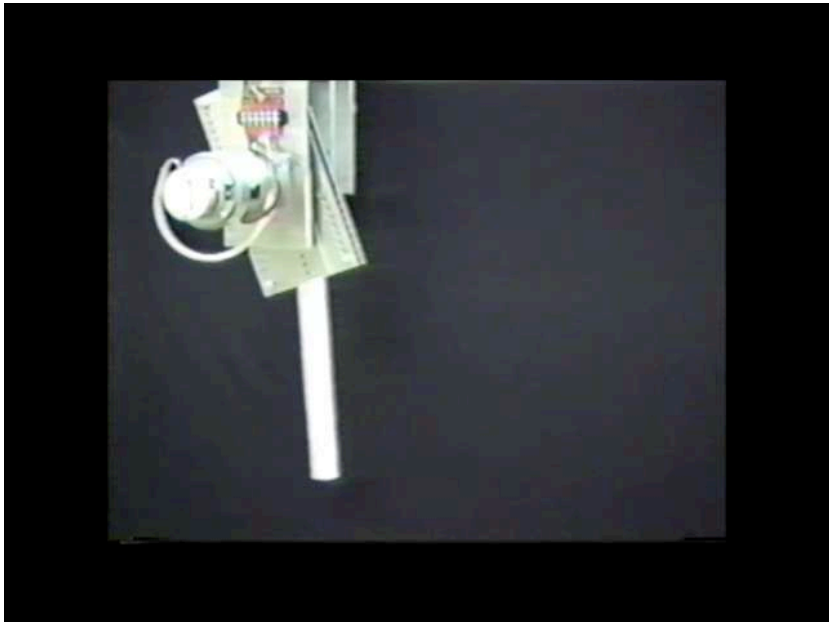
Based on the rigid model only vs. when including joint elasticity

$$\tau = M(q)(\ddot{q}_d + K_D(\dot{q}_d - \dot{q}) + K_P\,(q_d - q)) + C(q,\dot{q})\dot{q} + g(q)$$

$$\tau \;= BK^{-1}M(q)u + K(\theta - q) + B\ddot{q} + BK^{-1}\left(2\dot{M}q^{(3)} + \ddot{M}\ddot{q} + \frac{d^2}{dt^2}\left(C\dot{q} + g(q)\right)\right)$$

$$u = \left(q_d^{[4]} + K_J(\dddot{q}_d - \dddot{q}) + K_A(\ddot{q}_d - \ddot{q}) + K_D(\dot{q}_d - \dot{q}) + K_P\,(q_d - q)\right)$$

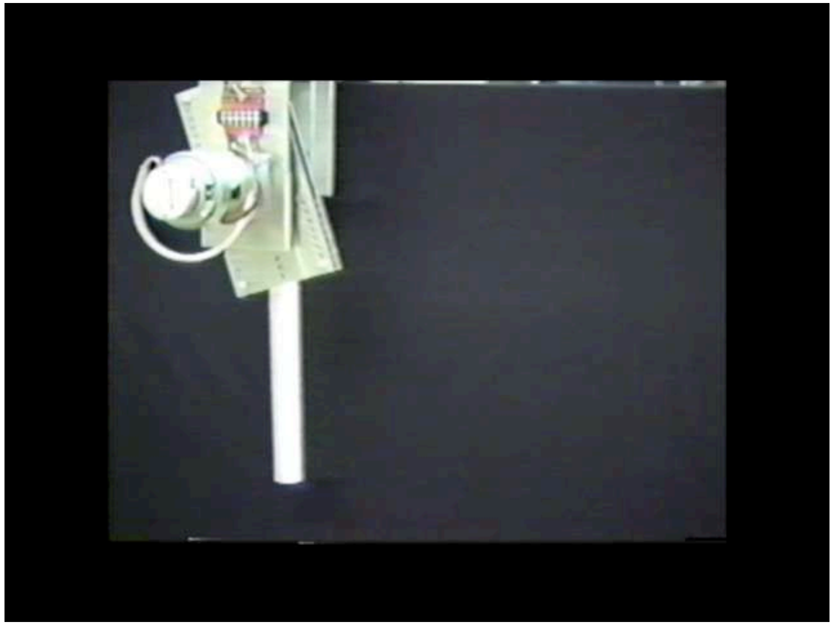
video



rigid computed torque

[Spong, ASME JDSMC 1987]

video



elastic joint feedback linearization

Feedback linearization

Benefits on an industrial KUKA KR-15/2 robot (235 kg) with joint elasticity

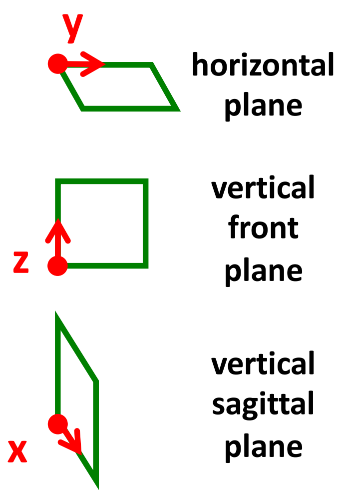
4 videos

conventional industrial robot control

feedback linearization + high-damping

trajectory tracking with model-based control

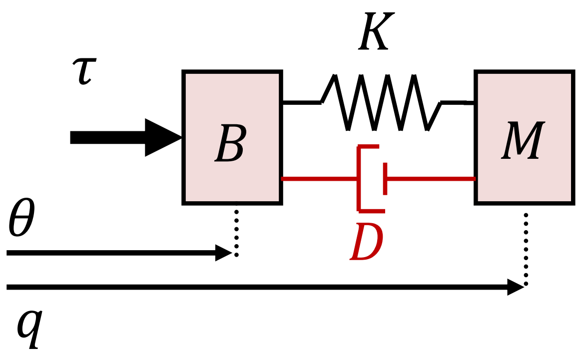
three squares in:



[Thümmel, PhD 2007]

Visco-elasticity at the joints

Introduces a structural change …



on Spong model

$$\begin{pmatrix} M(q) & 0^{*} \\ 0^{*} & B \end{pmatrix}\begin{pmatrix} \ddot{q} \\ \ddot{\theta} \end{pmatrix} + \begin{pmatrix} C(q,\dot{q})\dot{q} \\ 0 \end{pmatrix}^{*} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q-\theta) + D(\dot{q}-\dot{\theta}) \\ K(\theta-q) + D(\dot{\theta}-\dot{q}) \end{pmatrix} = \begin{pmatrix} 0 \\ \tau \end{pmatrix}$$

coupling type	control consequence for the model
stiffness	basic elastic coupling, maximum relative degree (= 4) of output q
damping	reduced relative degree (= 3), only I-O linearization by static feedback
inertia*	reduced relative degree, exact or I-O linearization needs dynamic feedback

* the so-called complete dynamic model includes off-diagonal inertial couplings between motors and links
[De Luca, Lucibello, ICRA 1998]

Regulation tasks

Using a minimal PD+ action on the motor side

for a desired constant link position q_d

- evaluate the associated desired motor position θ_d at steady state
- collocated (partial state) feedback preserves passivity, with stiff K_P gain dominating gravity
- focus on the term for **gravity compensation** (acting on link side) from motor measurements

$$\theta_d = q_d + K^{-1} g(q_d) \qquad \tau = \tau_g + K_P(\theta_d - \theta) - K_D \dot{\theta} \qquad K_D > 0$$

τ_g	gain criteria for stability	
$g(q_d)$	$\lambda_{min}\begin{bmatrix} K & -K \\ -K & K + K_P \end{bmatrix} > \alpha$	[Tomei, IEEE T-AC 1991]
$g(\theta - K^{-1} g(q_d))$	$\lambda_{min}\begin{bmatrix} K & -K \\ -K & K + K_P \end{bmatrix} > \alpha$	[De Luca, Siciliano, Zollo, ASME JDSMC 2004]
$g(\bar{q}(\theta)),\ \bar{q}(\theta):\ g(\bar{q}) = K(\theta - \bar{q})$	$K_P > 0, \quad \lambda_{min}(K) > \alpha$	[Ott *et al*, ICRA 2004]
$g(q) + BK^{-1}\ddot{g}(q)$	$K_P > 0, \quad K > 0$	[De Luca, Flacco, CDC 2010]

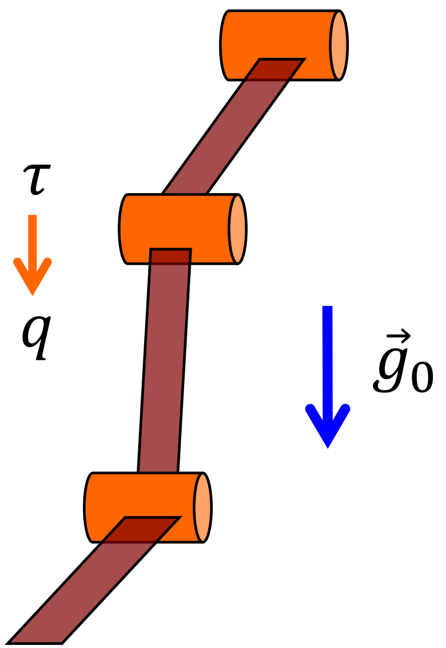
exact gravity cancellation
(with **full state** feedback)

$$\alpha = \max_q \left\| \frac{\partial g(q)}{\partial q} \right\|$$

Exact gravity cancellation

A slightly different view

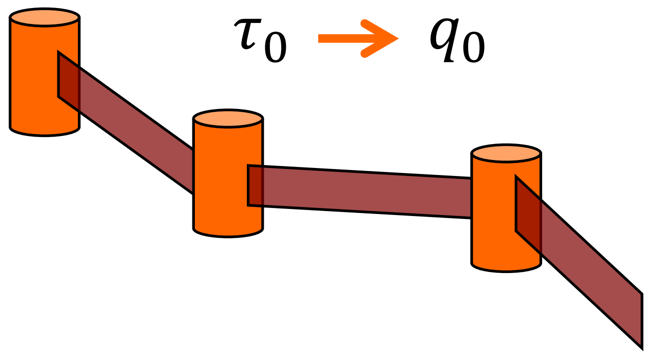
- for rigid robots this is trivial, due to collocation



$$\tau = \tau_g + \tau_0$$

$$\tau_g = g(q)$$

$$q \equiv q_o$$



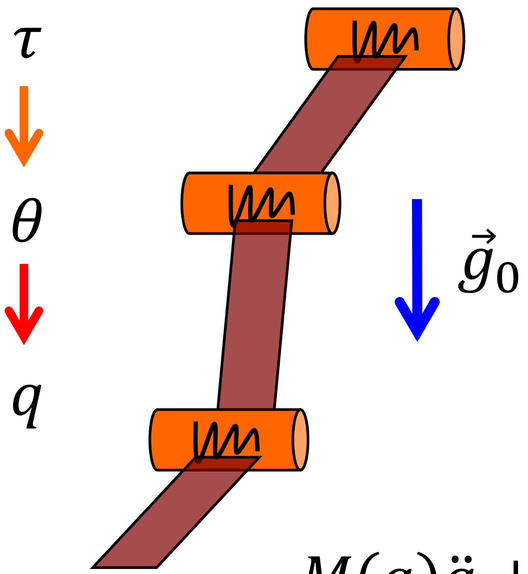
$$M(q)\ddot{q} + c(q,\dot{q}) + D\dot{q} = \tau_0$$

$$M(q)\ddot{q} + c(q,\dot{q}) + D\dot{q} + g(q) = \tau$$

Exact gravity cancellation

… based on the concept of **feedback equivalence** between nonlinear systems

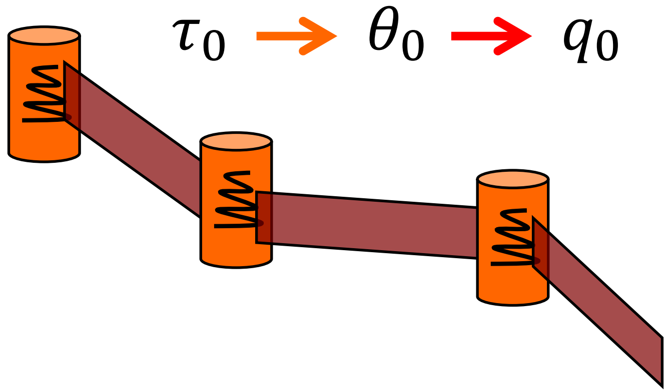
- for elastic joint robots, **non-collocation** of input torque and gravity term



??

$$\tau = \tau_g + \tau_0$$

$$q \equiv q_o$$



$$M(q)\ddot{q} + c(q,\dot{q}) + D_q\dot{q} + g(q) + K(q-\theta) = 0$$

$$B\ddot{\theta} + D_\theta\dot{\theta} + K(\theta - q) = \tau$$

$$\tau_g = g(q) + D_\theta K^{-1}\dot{g}(q) + BK^{-1}\ddot{g}(q)$$

feedback control

$$\theta_0 = \theta + K^{-1}g(q)$$

state transformation

Feedback equivalence

Exploit the system property of being feedback linearizable (without forcing it!)

$$u = \tilde{\alpha}(x) + \tilde{\beta}(x)u_0$$
$$x_0 = T_0^{-1}(T(x)) = \tilde{T}(x)$$

$$\dot{x} = f(x) + G(x)u$$

gravity-loaded system

$$\dot{x}_0 = f_0(x_0) + G_0(x_0)u_0$$

gravity-free system

feedback transformations
static **state feedback**
+ **change of coordinates**
both invertible

$$u = \alpha(x) + \beta(x)v$$
$$z = T(x)$$

$$u_0 = \alpha_0(x_0) + \beta_0(x_0)v$$
$$z = T_0(x_0)$$

$$\dot{z} = Az + Bv$$

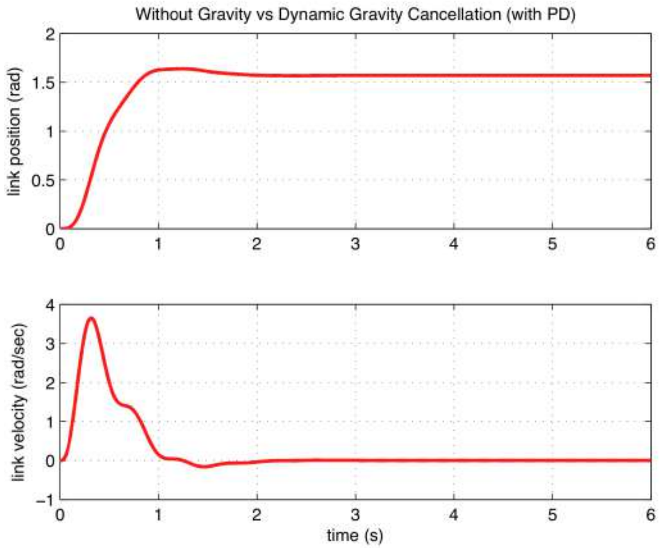
linear, controllable system

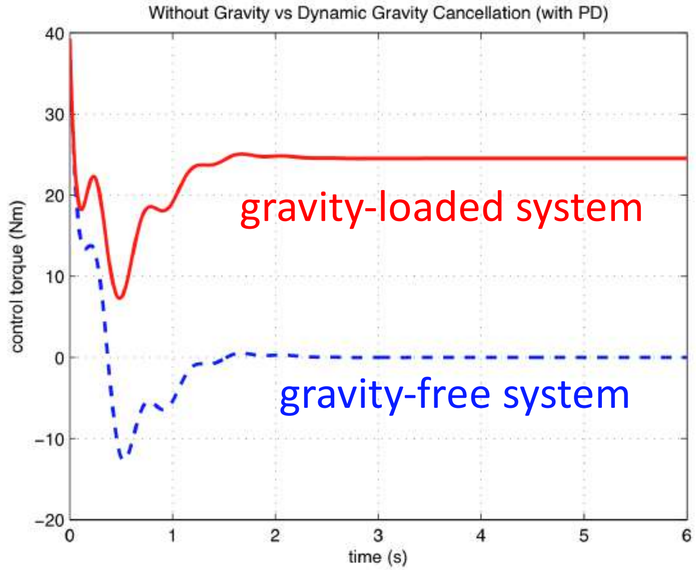
$z \approx$ **linearizing** outputs

A global PD-type regulator

Exact gravity cancellation + PD law on **modified** motor variables: A **1-DOF** arm

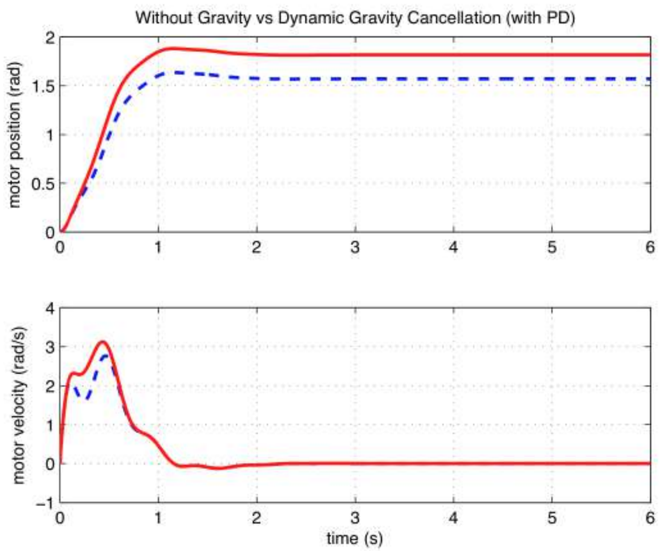
identical link behavior





total **control torque**

different motor behavior



gravity-loaded system under PD
+ gravity cancellation
vs.
gravity-free system under PD
(with **same gains**)

[De Luca, Flacco, ICRA 2011]

$K_P > 0$ $K > 0$

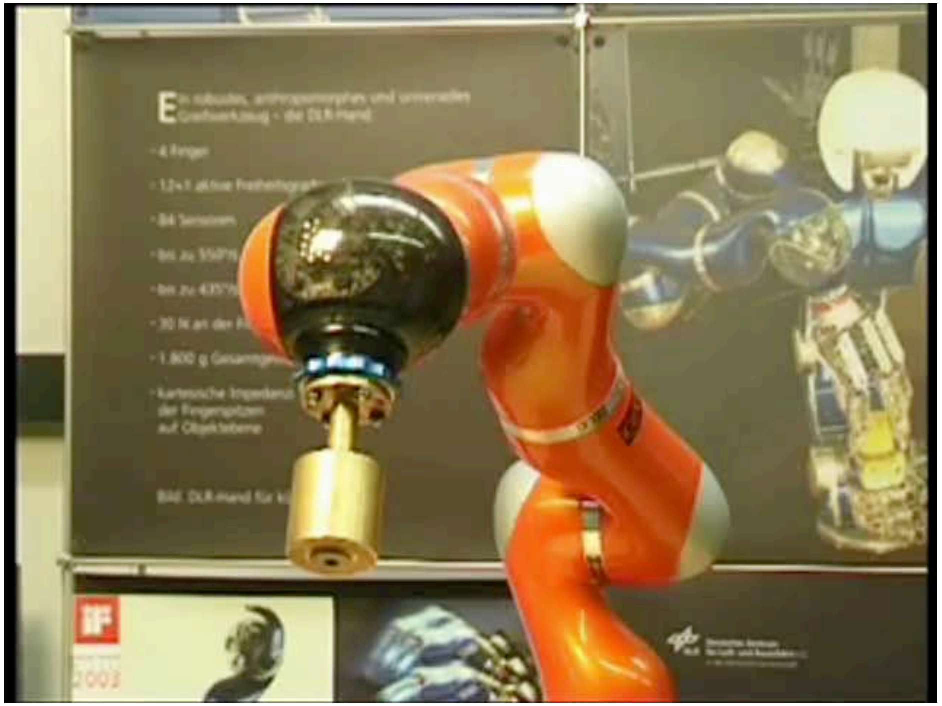
works **without** strictly positive lower bounds (**good** also for **VSA!**)

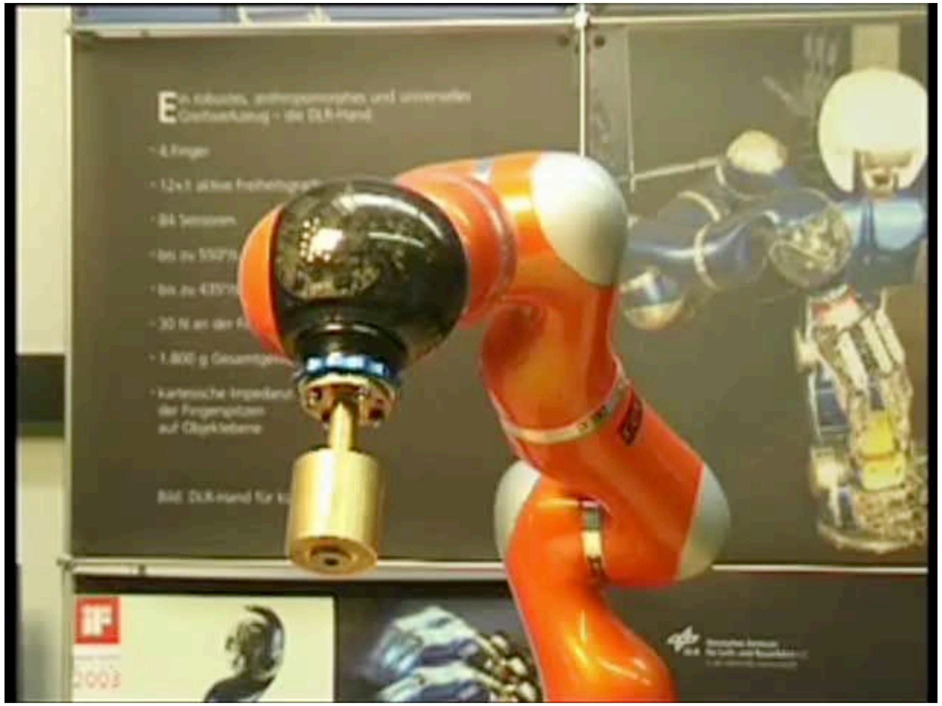
Vibration damping on lightweight robots

DLR-III or KUKA LWR-IV with relatively low joint elasticity (use of Harmonic Drives)

video

video





vibration damping **OFF**

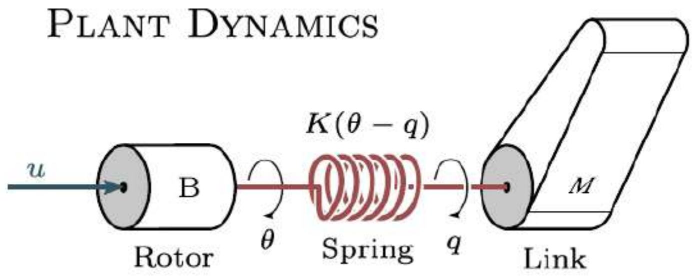
vibration damping **ON**

[Albu-Schäffer *et al*, IJRR 2007]

for relatively **large** joint elasticity (low stiffness), as encountered in VSA systems, vibration damping via joint torque feedback + motor damping is **insufficient** for high performance!

Damping injection on the link side

Method for the VSA-driven bimanual humanoid torso David



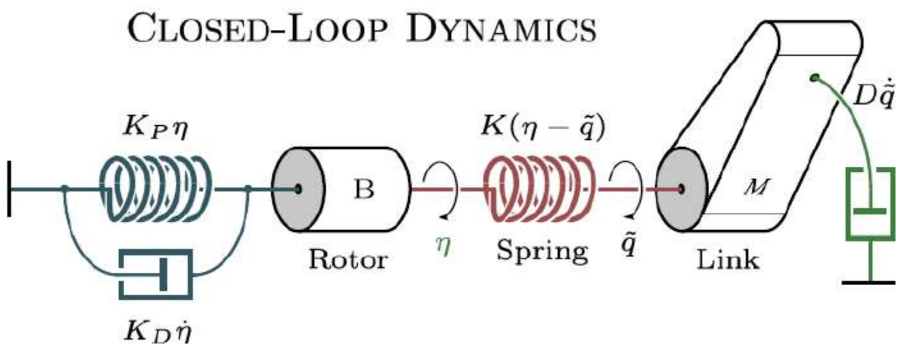
$$\begin{pmatrix} M(q) & 0 \\ 0 & B \end{pmatrix} \begin{pmatrix} \ddot{q} \\ \ddot{\theta} \end{pmatrix} + \begin{pmatrix} C(q,\dot{q})\dot{q} \\ 0 \end{pmatrix} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q-\theta) \\ K(\theta-q) \end{pmatrix} = \begin{pmatrix} 0 \\ \tau \end{pmatrix}$$

$$\theta_0 = \theta + K^{-1} D\dot{q}$$

state transformation

$$\tau = \tau_0 - D\dot{q} - BK^{-1}D\dddot{q}$$

feedback control



$$\begin{pmatrix} M(q) & 0 \\ 0 & B \end{pmatrix} \begin{pmatrix} \ddot{q} \\ \ddot{\boldsymbol{\theta}}_0 \end{pmatrix} + \begin{pmatrix} C(q,\dot{q})\dot{q} \\ 0 \end{pmatrix} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q-\boldsymbol{\theta}_0) \\ K(\boldsymbol{\theta}_0-q) \end{pmatrix} = \begin{pmatrix} -\boldsymbol{D\dot{q}} \\ \tau_0 \end{pmatrix}$$

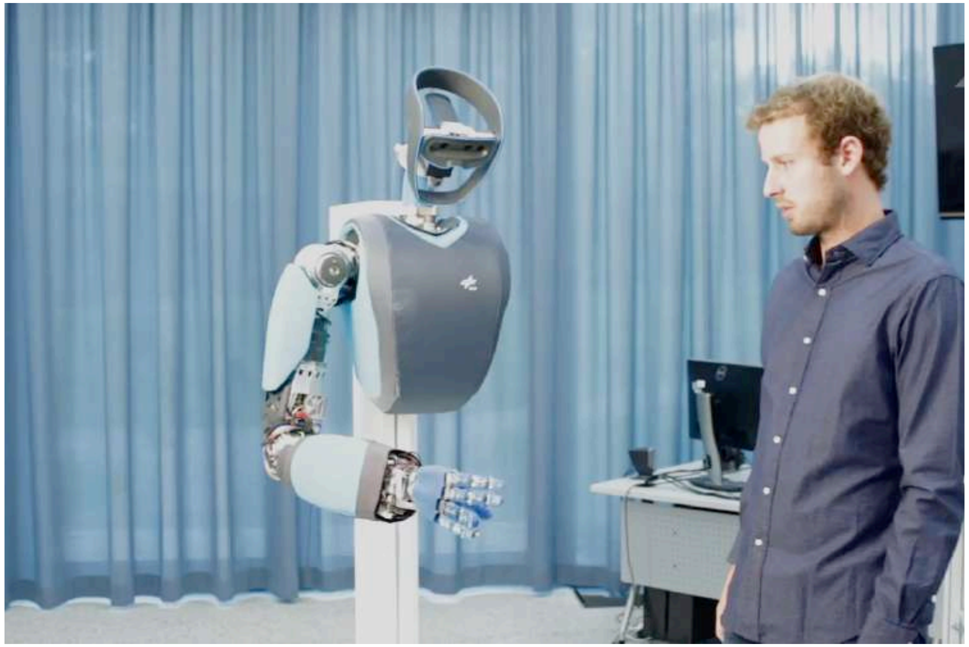
- **ESP** = Elastic Structure Preserving control by DLR [Keppler *et al,* IEEE T-RO 2018]
- same principle of feedback equivalence (including state transformation)**!**

Damping injection on the link side

Method for VSA-driven bimanual humanoid torso David at DLR

video

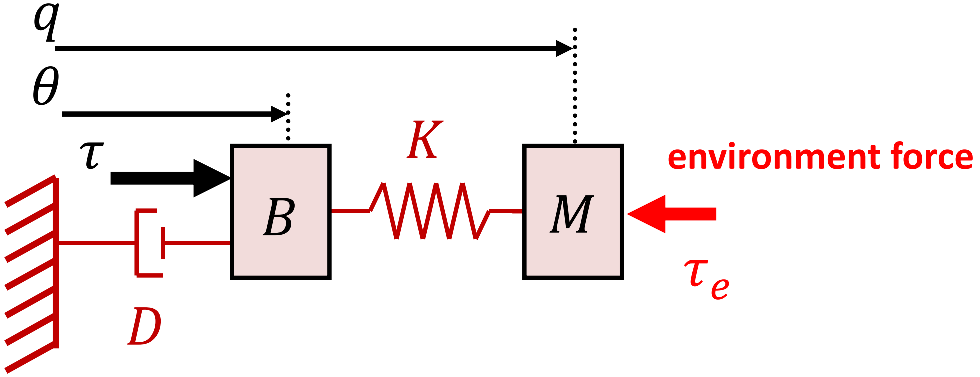
video



[Keppler *et al*, IEEE T-RO 2018]

Environment interaction via impedance control

Matching a generalized (fourth order) impedance model: A simple 1-DOF case



$$M\ddot{q} + K(q-\theta) = \tau_e$$

$$B\ddot{\theta} + D\dot{\theta} + K(\theta - q) = \tau$$

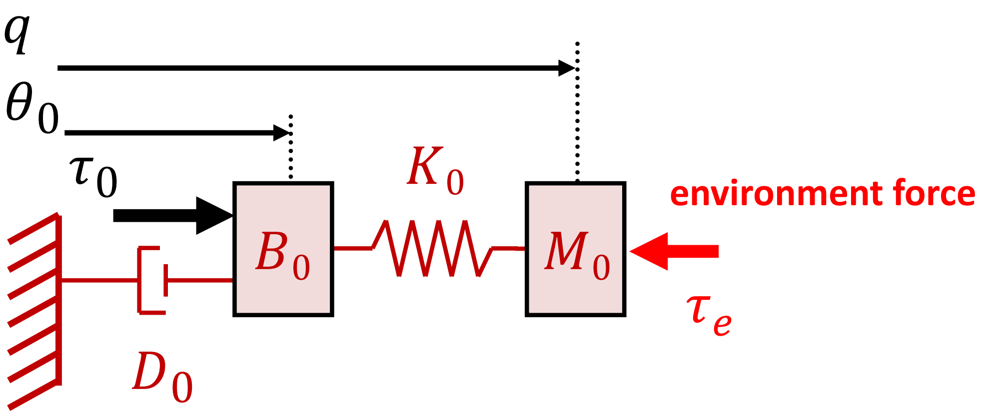
feedback control

assume that $M_0 = M$ in order to avoid derivatives of the measured force τ_e

$$\tau = K(\theta - q) + D\dot{\theta} - BK^{-1}\begin{Bmatrix} (K-K_0)M^{-1}\big(\tau_e + K(\theta - q)\big) \\ + K_0 B_0^{-1}\big(\tau_0 - D_0\dot{\theta}_0 - K(\theta - q)\big) \end{Bmatrix}$$

$$\dot{\theta}_0 = \dot{q} + KK_0^{-1}(\dot{\theta} - \dot{q})$$

state transformation



$$M_0\ddot{q} + K_0(q - \theta_0) = \tau_e$$

$$B_0\ddot{\theta}_0 + D_0\dot{\theta}_0 + K_0(\theta_0 - q) = \tau_0$$

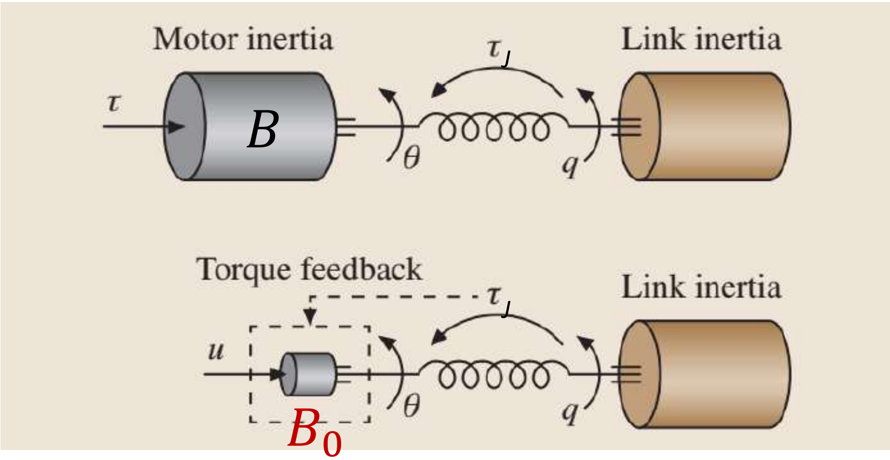
- again, by the principle of feedback equivalence (including the state transformation)

Torque feedback

An inner loop that largely reduces motor inertia (and friction)

Consider a pure proportional torque feedback (+ a derivative term for the visco-elastic case)

$$\tau = BB_0^{-1}u + \underbrace{(I - BB_0^{-1})}_{-K_T}\tau_J \;\; + \underbrace{(I - BB_0^{-1})DK^{-1}}_{-K_S}\dot{\tau}_J$$



physical interpretation:

scaling of the motor inertia and motor friction!

[Ott, Albu-Schäffer, IEEE T-RO 2008]

but also…

special case of matching by feedback equivalence**!**

original motor dynamics

$$B\ddot{\theta} + K(\theta - q) = \tau$$

visco-elastic case

$$B\ddot{\theta} + \tau_J + DK^{-1}\dot{\tau}_J = \tau$$

after the torque feedback

$$B_0\ddot{\theta} + K(\theta - q) = u$$

$$B_0\ddot{\theta} + \tau_J + DK^{-1}\dot{\tau}_J = u$$

Full-state feedback

Combining torque feedback with motor PD regulation ("torque controlled robots")

inertia scaling via torque feedback $\tau = (I + K_T)u - K_T\,\tau_J - K_S\dot{\tau}_J$

regulation via motor PD, e.g., with $u = g(\bar{q}(\theta)) + K_\theta(\theta_d - \theta) - D_\theta\dot{\theta}$

⇒ **joint level control structure** of the DLR (and KUKA) lightweight robots

dynamics feedforward and desired torque command

setpoint control

$$\tau = \tau_{J,d} - K_T\left(\tau_J - \tau_{J,d}\right) - K_S\dot{\tau}_J - K_P(\theta_d - \theta) - K_D\dot{\theta} + \tau_f + \tau_{dob}$$ (+ integral actions)

motor inertia scaling

vibration damping

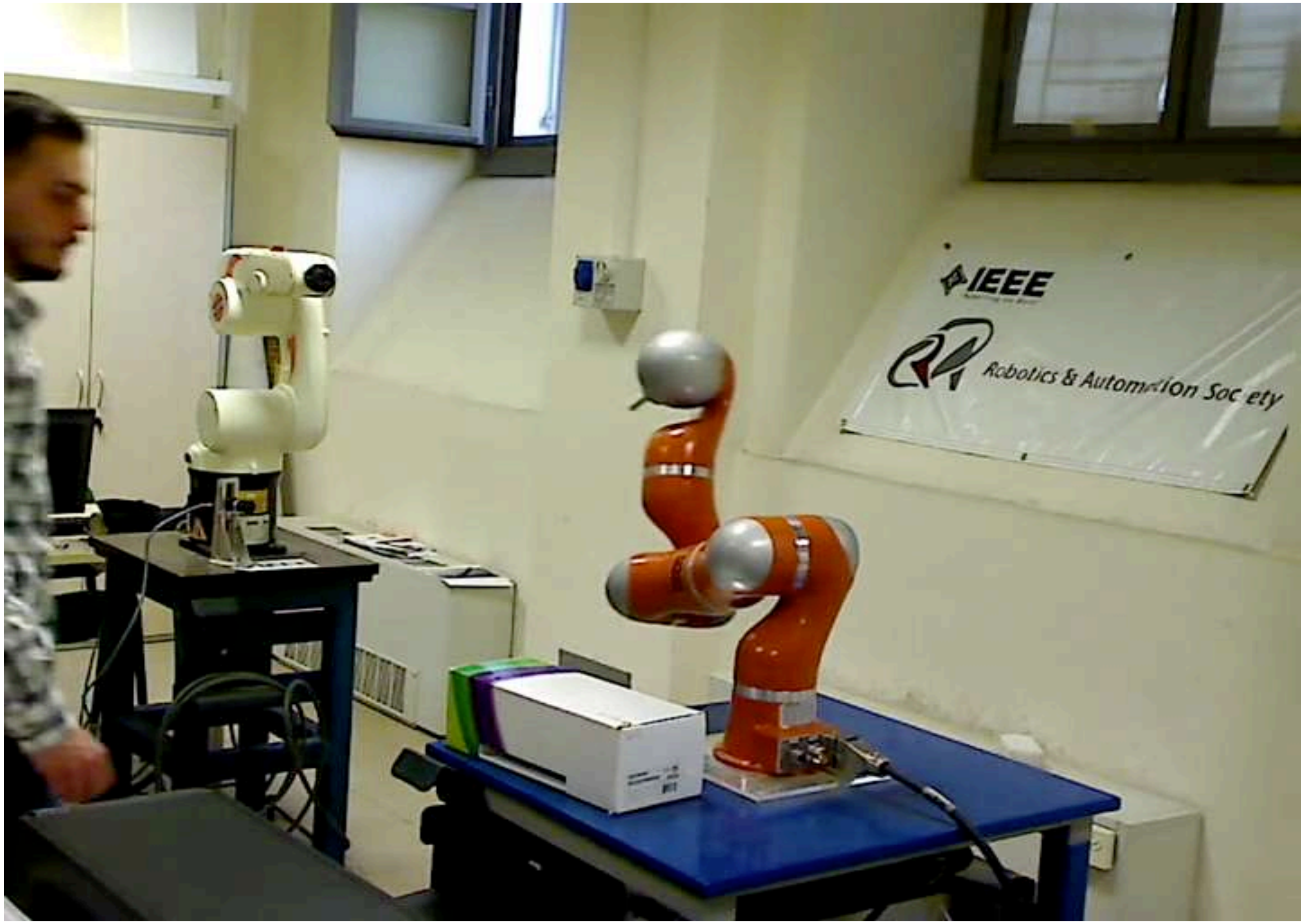
friction compensation and/or disturbance observer

torque control	**position control**	**impedance control**
$K_P = 0$	$K_P > 0$	$K_P = K_T K_\theta$
$K_D = 0$	$K_D > 0$	$K_D = K_T D_\theta$
$K_T > 0$	$K_T > 0$	$K_T = (BB_d^{-1} - I)$
$K_S > 0$	$K_S > 0$	$K_S = \left(BB_d^{-1} - I\right)DK^{-1}$
$\tau_{J,d} = \tau_d$	$\tau_{J,d} = g(q)$	$\tau_{j,d} = g(\bar{q}(\theta))$

Exploiting joint elasticity in pHRI

Detection & selective reaction in torque control mode, with momentum-based residuals

- collision detection & reaction for safety (model-based + joint torque sensing)



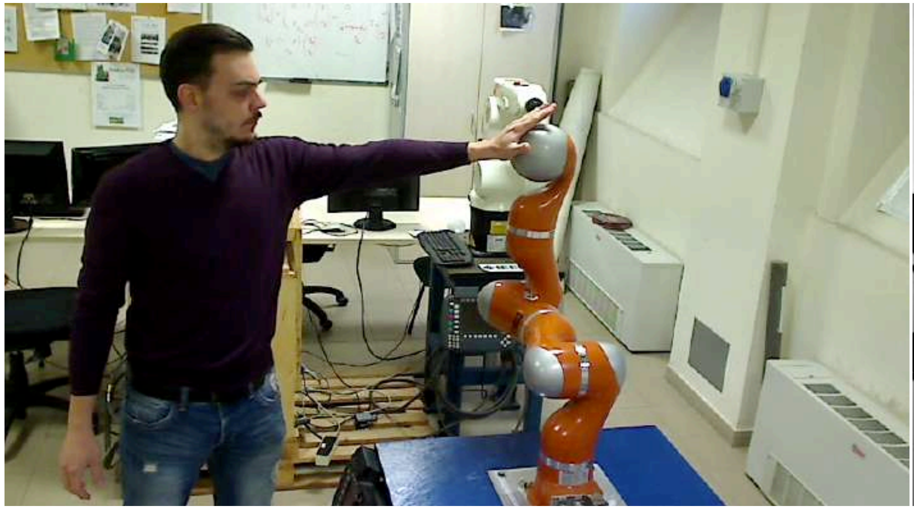
video

[De Luca *et al,* IROS 2006; Haddadin *et al,* IEEE T-RO 2017]

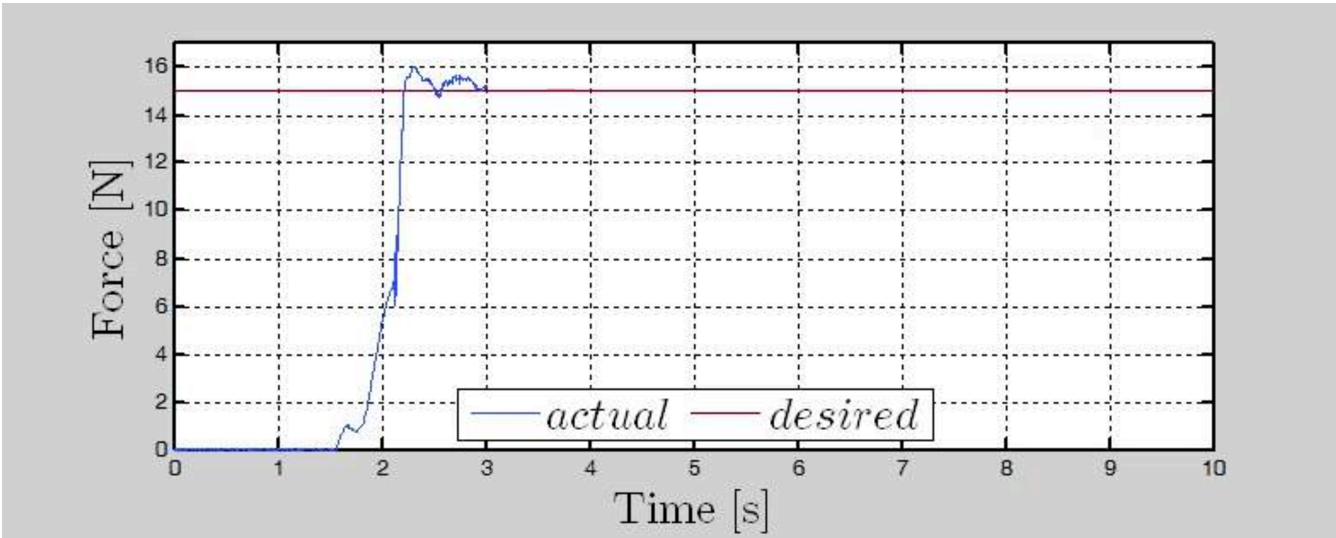
Exploiting joint elasticity in pHRI

Human-robot collaboration in torque control mode

- contact force estimation & control (virtual force sensor, anywhere/anytime)




video

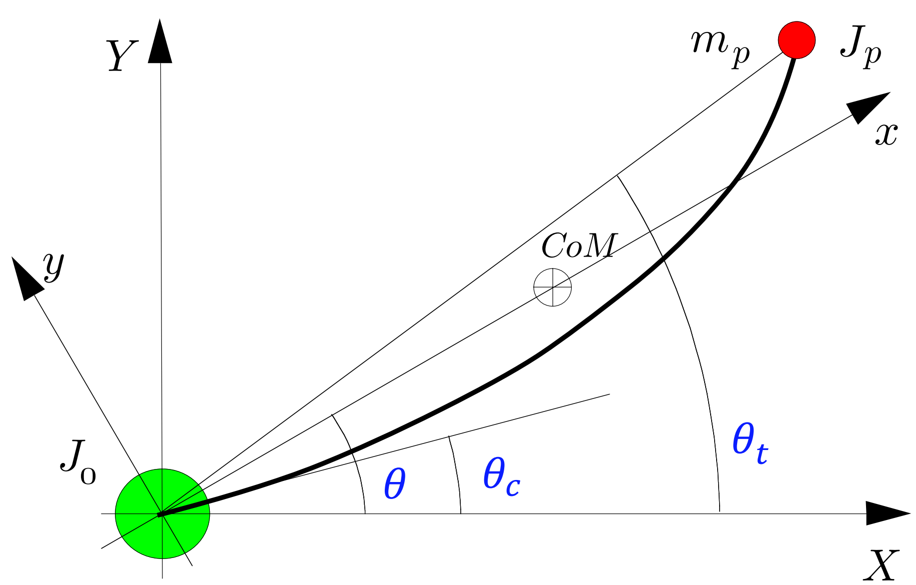


[Magrini *et al*, ICRA 2015]

Dynamic modeling of a single flexible link

Euler-Bernoulli beam [Bellezza, Lanari, Ulivi, ICRA 1990]

- beam of length l, uniform density ρ, Young modulus · cross-section inertia EI in rotation on a horizontal plane
- actuator inertia J_0 at the base and payload mass m_p and inertia J_p at the tip
- various angular variables: $\theta_c(t)$ clamped at base (measured by encoder), $\theta(t)$ pointing at CoM (very convenient!), $\theta_t(t)$ pointing at the tip (measurable and of interest)
- small deformations of pure bending $w(x,t) = \phi(x)\delta(t)$ (with space/time separation)
- Hamilton principle + calculus of variation ⇒ PDE equations, with geometric and dynamic boundary conditions



$$J\ddot{\theta}(t) = \tau(t) \qquad J = J_0 + \frac{\rho l^3}{3} + J_p + m_p l^2$$

$$EIw''''(x,t) + \rho\left(\ddot{w}(x,t) + x\ddot{\theta}(t)\right) = 0$$

$$w(0,t) = 0$$

$$EIw''(0,t) = J_0\left(\ddot{\theta}(t) + \ddot{w}'(0,t)\right) - \tau(t)$$

$$EIw''(l,t) = -J_p\left(\ddot{\theta}(t) + \ddot{w}'(l,t)\right)$$

$$EIw'''(l,t) = m_p\left(l\ddot{\theta}(t) + \ddot{w}(l,t)\right)$$

Dynamic modeling of a single flexible link

Characteristic equation and eigenfrequencies

- infinite countable roots $\beta_i, i = 1,2, \ldots$ of an eigenvalue problem

$$\left(1-\frac{m_p}{\rho^2}\beta_i^4(J_0+J_p)\right)(\cos\beta_i l \sinh\beta_i l - \sin\beta_i l \cosh\beta_i l) - \frac{2m_p}{\rho}\beta_i \sin\beta_i l \sinh\beta_i l - \frac{2J_p}{\rho}\beta_i^3 \cos\beta_i l \cosh\beta_i l$$
$$-\frac{J_0}{\rho}\beta_i^3(1+\cos\beta_i l\cosh\beta_i l) + \frac{J_0 J_p}{\rho^2}\beta_i^6(\cos\beta_i l\sinh\beta_i l + \sin\beta_i l\cosh\beta_i l) - \frac{J_0 J_p m_p}{\rho^3}\beta_i^7(1-\cos\beta_i l\cosh\beta_i l) = 0$$

- common assumed modes are special cases
  - clamped-free: $m_p = 0,\ J_p = 0,\ J_0 \to \infty \quad \Longrightarrow \quad 1+\cos\beta_i l\cosh\beta_i l = 0$
  - pinned-free: $m_p = 0,\ J_p = 0,\ J_0 = 0 \quad \Longrightarrow \quad \cos\beta_i l\sinh\beta_i l - \sin\beta_i l\cosh\beta_i l = 0$
- associated to each root β_i there is
  - an eigenfrequency (system vibrations) $\omega_i = \sqrt{EI\beta_i^4/\rho}$
  - an eigenvector (spatial mode) $\phi_i(x) = A\sin\beta_i x + B\cos\beta_i x + C\sinh\beta_i x + D\cosh\beta_i x$
  - a deformation variable $\delta_i(t)$
- finite approximation by truncation up to n_e orthonormal modes: $w(x,t) = \sum_{i=1}^{n_e}\phi_i(x)\delta_i(t)$

Dynamic model of a single flexible link

Final equations and system outputs

- linear dynamic model

$$J\ddot{\theta} = \tau$$

$$\ddot{\delta}_i + \omega_i^2 \delta_i = \phi_i'(0)\tau, \qquad i = 1, \dots, n_e$$

- including modal damping ($\zeta_i \in [0,1]$)

$$J\ddot{\theta} = \tau$$

$$\ddot{\delta}_i + 2\zeta_i\omega_i\dot{\delta}_i + \omega_i^2 \delta_i = \phi_i'(0)\tau, \qquad i = 1, \dots, n_e$$

- in matrix form

$$q = (\theta, \delta_1, \delta_2, \dots, \delta_{n_e}) \in \mathbb{R}^{n_e+1} \qquad \boxed{M\ddot{q} + D\dot{q} + Kq = B\tau}$$

$$M = \begin{pmatrix} J & 0 \\ 0 & I_{n_e} \end{pmatrix}, \quad D = \begin{pmatrix} 0 & 0 \\ 0 & 2Z\Omega \end{pmatrix}, \quad K = \begin{pmatrix} 0 & 0 \\ 0 & \Omega^2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ \Phi'(0) \end{pmatrix}$$

- system outputs

$$\theta_c = \theta + \sum_{i=1}^{n_e} \phi_i'(0)\delta_i \qquad \theta_t = \theta + \sum_{i=1}^{n_e} \frac{\phi_i(l)}{l}\delta_i$$

clamped joint level: always minimum phase — tip level: typically non-minimum phase

Single flexible link

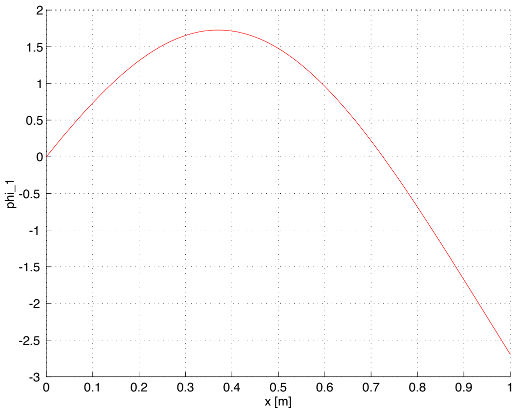
Eigenmodes

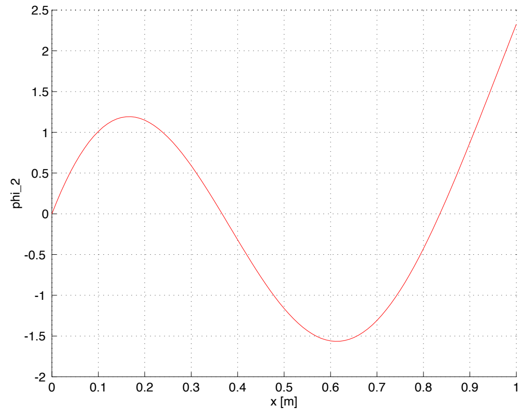
- physical data of an Euler-Bernoulli model

$$l = 1, \qquad \rho = 0.5, \qquad EI = 1, \qquad J_0 = 0.002 \qquad (m_p = J_p = 0)$$

- first four exact mode shapes (normalized) –k-th mode has k nodes w.r.t. rigid axis

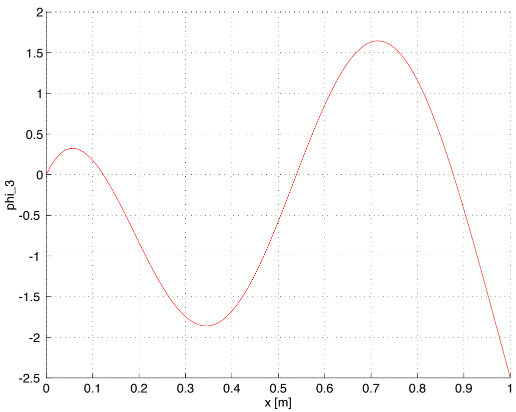
$\phi_1(x)$
at $f_1 = 3.27$ Hz

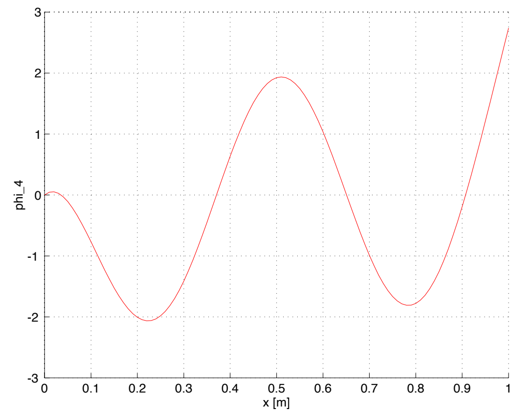




$\phi_2(x)$
at $f_2 = 8.89$ Hz

$\phi_3(x)$
at $f_3 = 16.13$ Hz





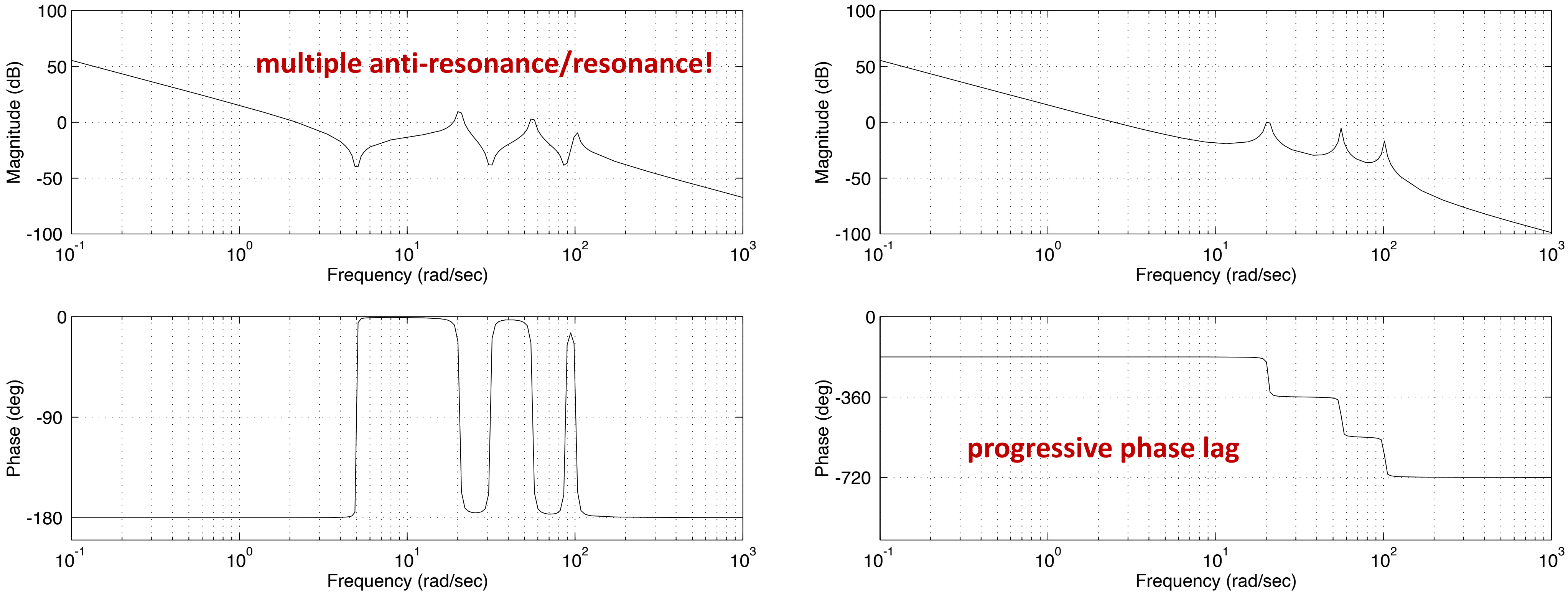
$\phi_4(x)$
at $f_4 = 28.28$ Hz

Single flexible link

Transfer functions of interest and frequency responses

$$P_c(s) = \frac{\theta_c(s)}{\tau(s)} = \frac{1}{Js^2} + \sum_{i=1}^{n_e} \frac{\phi_i'(0)^2}{s^2 + 2\zeta_i\omega_i s + \omega_i^2} \qquad P_t(s) = \frac{\theta_t(s)}{\tau(s)} = \frac{1}{Js^2} + \sum_{i=1}^{n_e} \frac{\phi_i'(0)\phi_i(l)/l}{s^2 + 2\zeta_i\omega_i s + \omega_i^2}$$

$n_e = 3$ modes



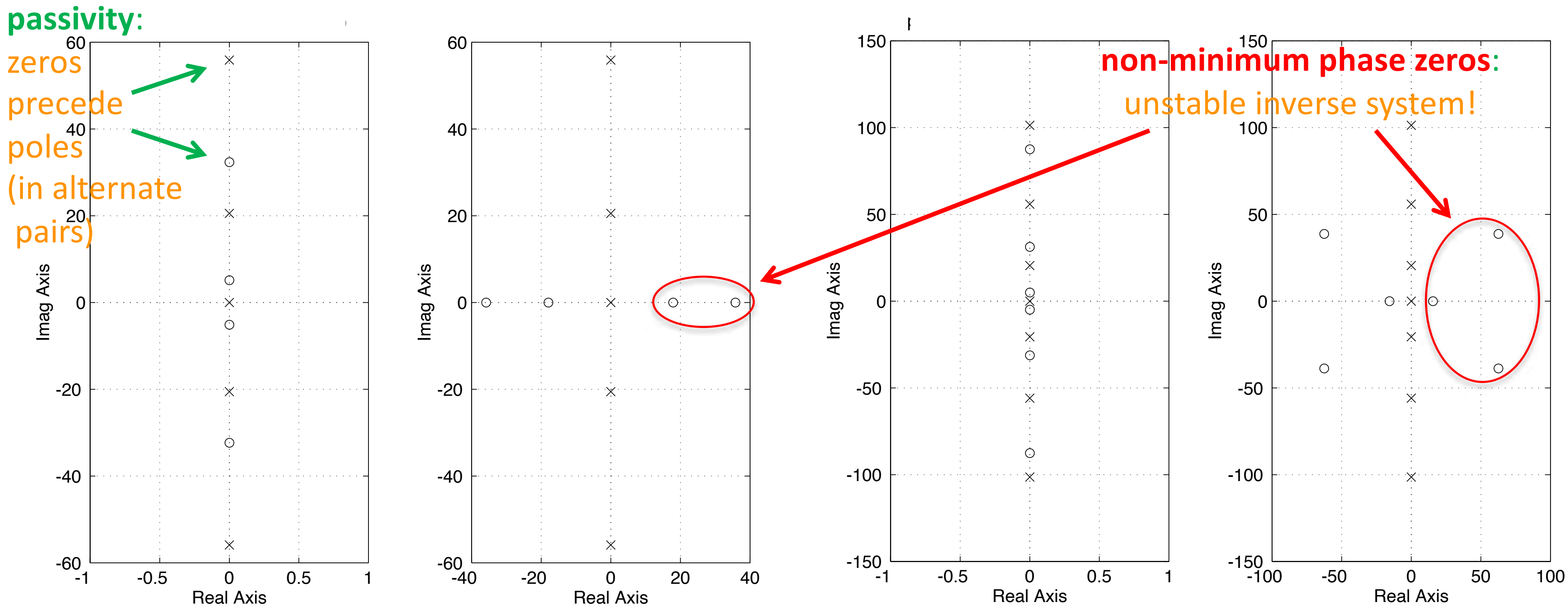
clamped joint level: always minimum phase   tip level: typically non-minimum phase

Single flexible link

Pole-zero patterns

- in the absence of modal damping

$n_e = 2$ modes $\qquad\qquad$ $n_e = 3$ modes

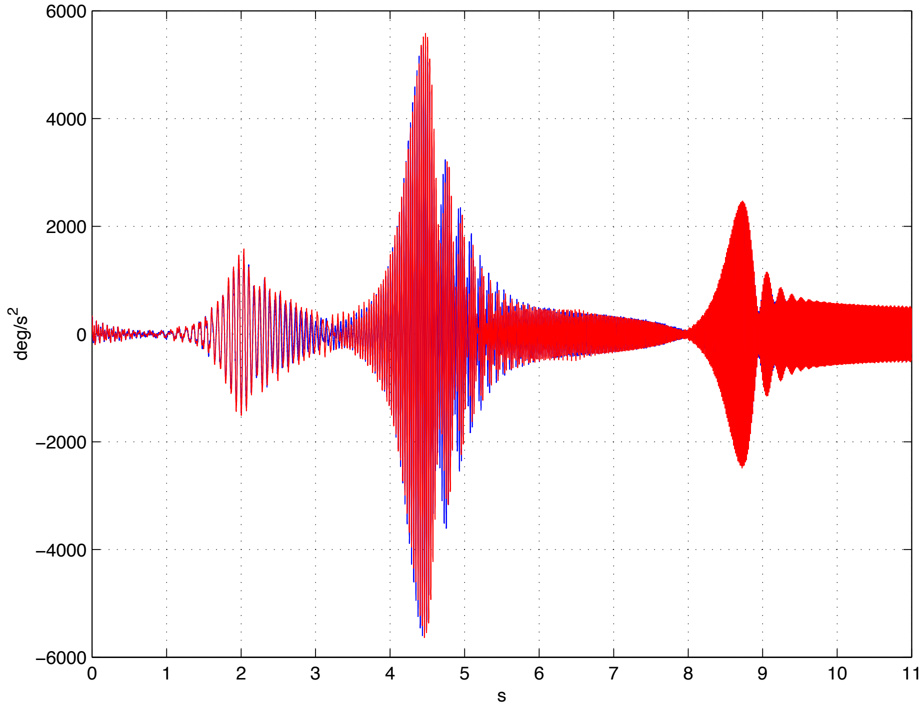


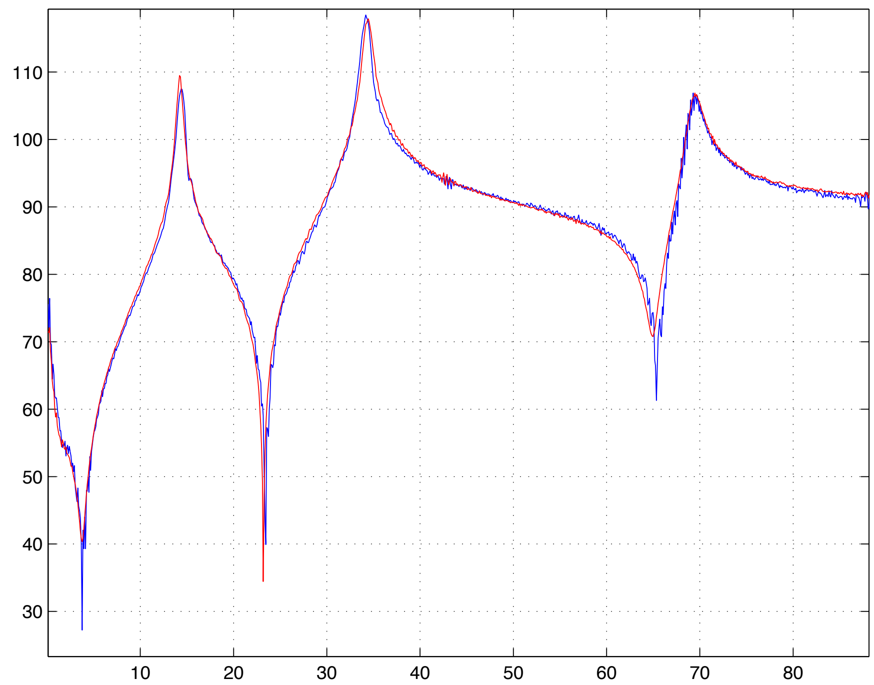
clamped joint level $\qquad$ tip level $\qquad$ clamped joint level $\qquad$ tip level

Single flexible link

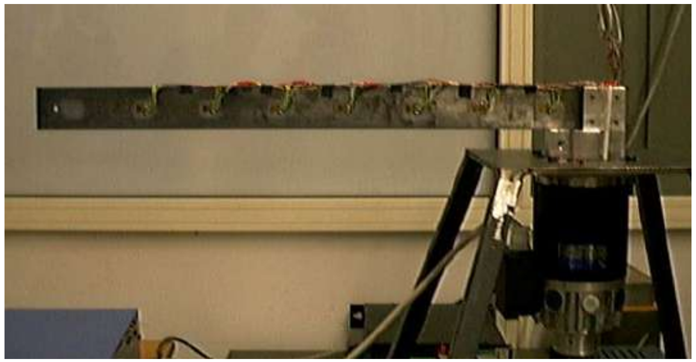
Experimental model identification

- in the frequency domain





sweep joint acceleration excitation signal: plant vs. model



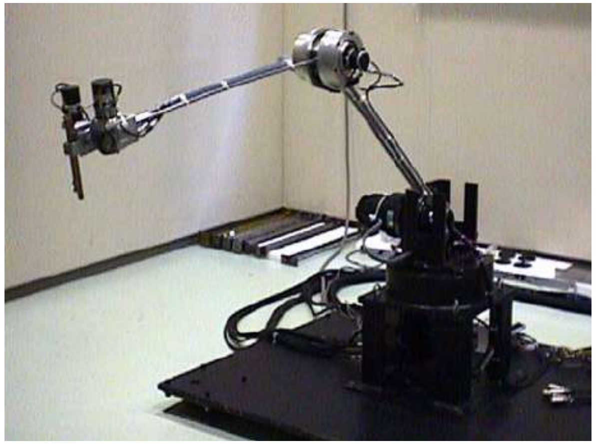
joint acceleration frequency response: plant vs. model

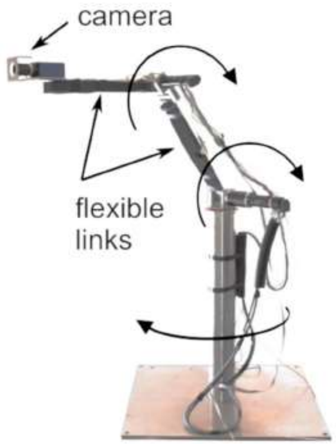
matching (≤ 1%) of resonances at $f_1 = 14.4, f_1 = 34.2, f_1 = 69.3$ Hz

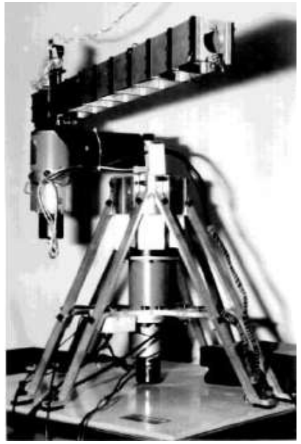
Dynamic modeling of robots with flexible links

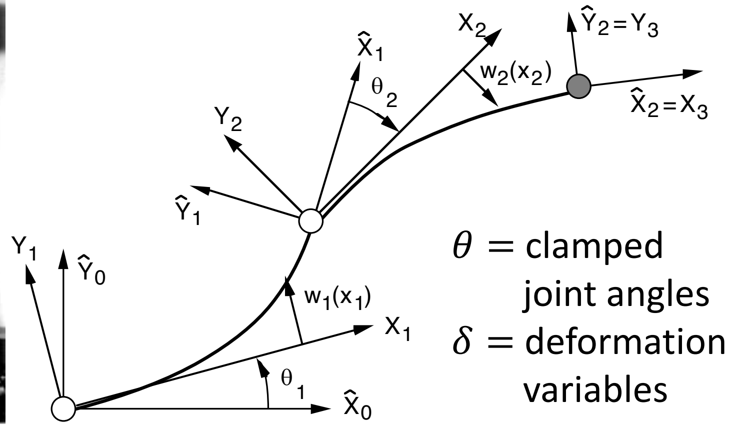
Lagrangian formulation (finite-dimensional)

- open chain robot with N flexible links, each with $n_{e,i}$ deformation variables (a total of N_e)
- single-link modeling results embedded with caution for each of the multiple flexible links
- in general, 2D bending + torsion (to limit model complexity, only planar structures here)
- typical use of simpler assumed modes to describe spatial deformation









θ = clamped joint angles
δ = deformation variables

$(N + N_e) \times (N + N_e)$
full inertia matrix

rigid equations

$$\begin{pmatrix} M_{\theta\theta}(\theta,\delta) & M_{\theta\delta}(\theta,\delta) \\ M_{\theta\delta}^T(\theta,\delta) & M_{\delta\delta}(\theta,\delta) \end{pmatrix} \begin{pmatrix} \ddot{\theta} \\ \ddot{\delta} \end{pmatrix} + \begin{pmatrix} c_\theta(\theta,\delta,\dot{\theta},\dot{\delta}) \\ c_\delta(\theta,\delta,\dot{\theta},\dot{\delta}) \end{pmatrix} + \begin{pmatrix} g_\theta(\theta,\delta) \\ g_\delta(\theta,\delta) \end{pmatrix} + \begin{pmatrix} 0 \\ K\delta + D\dot{\delta} \end{pmatrix} = \begin{pmatrix} \tau \\ 0 \end{pmatrix}$$

[De Luca, Siciliano, IEEE T-SMC 1991]

flexible equations

Dynamic modeling of robots with flexible links

Simplifications in model (possibly, for control use)

- in matrix form

$$q = (\theta, \delta) \in \mathbb{R}^{N+N_e} \qquad M(q)\ddot{q} + c(q,\dot{q}) + g(q) + \begin{pmatrix} 0 \\ D\dot{\delta} + K\delta \end{pmatrix} = \begin{pmatrix} \tau \\ 0 \end{pmatrix}$$

- common simplifications in mechanics
  - small deformations (in the linear domain) → $g_\delta(\theta)$
  - kinetic energy evaluated in the undeformed ($\delta = 0$) configuration of the arm → $M(\theta)$
  - $M_{\delta\delta}$ often constant

$$\begin{pmatrix} M_{\theta\theta}(\theta) & M_{\theta\delta}(\theta) \\ M_{\theta\delta}^T(\theta) & M_{\delta\delta} \end{pmatrix}\begin{pmatrix} \ddot{\theta} \\ \ddot{\delta} \end{pmatrix} + \begin{pmatrix} c_\theta(\theta,\dot{\theta}) \\ 0 \end{pmatrix} + \begin{pmatrix} g_\theta(\theta,\delta) \\ g_\delta(\theta) \end{pmatrix} + \begin{pmatrix} 0 \\ K\delta + D\dot{\delta} \end{pmatrix} = \begin{pmatrix} \tau \\ 0 \end{pmatrix}$$

- flexible link manipulators are underactuated systems
  - less command inputs τ than generalized coordinates q
  - we consider as many controlled outputs y as commands (‘squaring the I-O problem’)
  - problems, however, with the associated zero dynamics (in a linear or nonlinear setting)

Control problems for flexible link robots

A compact overview (moving in free space) …

- regulation to a desired equilibrium state $(q, \dot{q}) = (\theta_d, \delta_d, 0{,}0)$
  - only the desired joint/rigid variable θ_d is assigned: δ_d has to be determined
  - θ_d may come from a (numerical) kineto-static inversion of a Cartesian pose y_d
  - forward kinematics of flexible robots is a complete function $y = \text{kin}(\theta, \delta)$
  - global stabilization results with joint PD + gravity compensation
- tracking of a joint trajectory $\theta_d(t)$
  - the easy case, solved by I-O inversion (stable/minimum phase zero dynamics)
  - solution stiffens the arm at the bases of the flexible links, rejecting vibrations
- tracking of an end-effector trajectory $y_d(t)$
  - the difficult case, facing the unstable/non-minimum phase zero dynamics
  - non-causal solution designed in frequency or time domain (feedforward + local stabilizing feedback)
  - causal solution by nonlinear regulation (avoiding critical cancellations)
- rest-to-rest motion between two equilibria in assigned time T

Control solutions for flexible link robots

Main results – 1

- global asymptotic stabilization to a desired equilibrium state $(\theta_d, \delta_d, 0{,}0)$

$$\tau = K_P(\theta_d - \theta) - K_D\dot{\theta} + g_\theta(\theta_d, \delta_d)$$

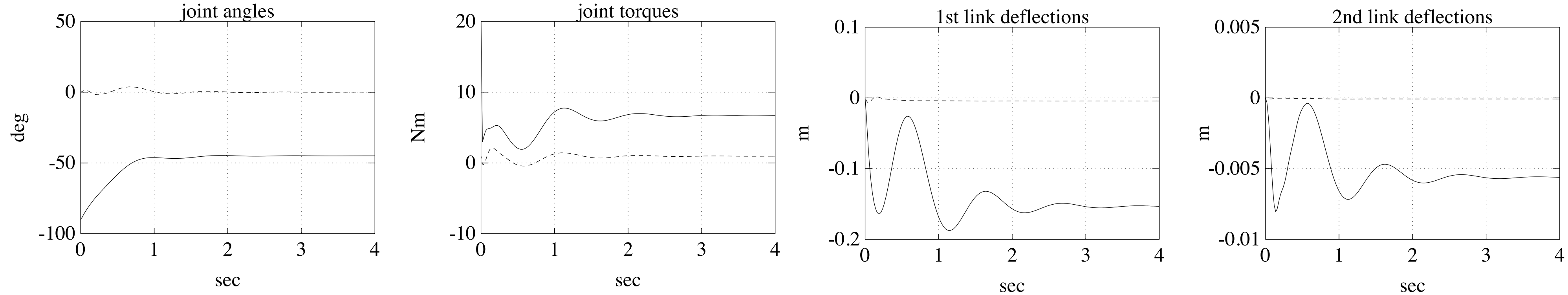
$$\delta_d = -K^{-1}g_\delta(\theta_d) \qquad \lambda_{min}\left\{\begin{pmatrix} K_P & 0 \\ 0 & K \end{pmatrix}\right\} > \alpha \qquad K_D > 0$$

possibly by iterative solution of $\text{kin}(\theta, -K^{-1}g_\delta(\theta)) = y_d$

upper bound on $\left\|\frac{\partial g(q)}{\partial q}\right\|$

[De Luca, Siciliano, IEEE T-RO 1993a]

- two-link flexible arm with two bending modes for each link under gravity



from $\theta(0) = (-90°, 0°)$
to $\theta_d = (-45°, 0°)$

$f_{11} = 1.4, f_{12} = 5.1,$
$f_{21} = 5.2, f_{22} = 32.4$ [Hz]

Control solutions for flexible link robots

Main results – 2

- tracking of a joint trajectory $\theta_d(t)$ via I-O feedback linearization

$$\tau = \left(M_{\theta\theta} - M_{\theta\delta}M_{\delta\delta}^{-1}M_{\theta\delta}^T\right)a + c_\theta + g_\theta - M_{\theta\delta}M_{\delta\delta}^{-1}\left(c_\delta + g_\delta + K\delta + D\dot{\delta}\right)$$

resulting closed-loop system

[De Luca, Siciliano, AIAA JGCD 1993b]

$$\ddot{\theta} = a$$
$$\ddot{\delta} = -M_{\delta\delta}^{-1}\left(M_{\theta\delta}^T a + c_\delta + g_\delta + K\delta + D\dot{\delta}\right)$$

trajectory error (exponential) stabilization

$$a = \ddot{\theta}_d + K_D\left(\dot{\theta}_d - \dot{\theta}\right) + K_P(\theta_d - \theta), \qquad K_P, K_D > 0$$

- the zero dynamics, when the output $\theta(t) \equiv 0$, is asymptotically stable (via Lyapunov argument)

$$\ddot{\delta} = -M_{\delta\delta}^{-1}\left(c_\delta + g_\delta + K\delta + D\dot{\delta}\right)$$

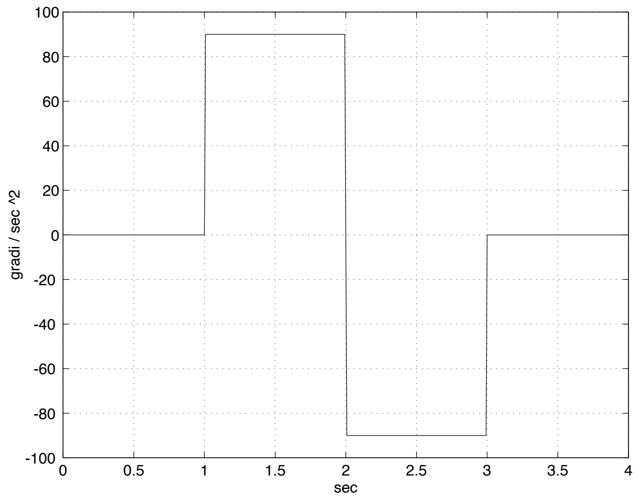
- the clamped dynamics, when the output $\theta(t) \equiv \theta_d(t)$, is bounded

$$\ddot{\delta} = -A_2(t)\dot{\delta} + A_1(t)\delta + f_\delta(t)$$

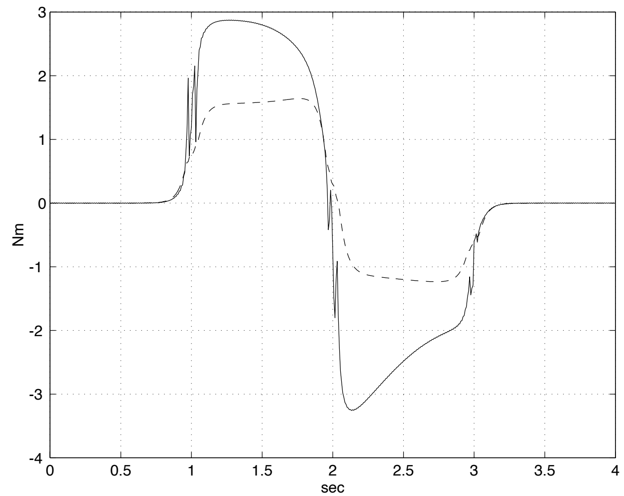
Control solutions for flexible link robots

Main results – 3

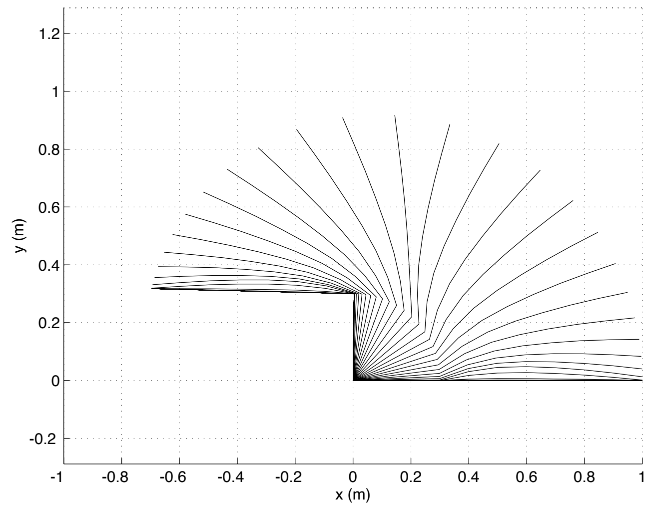
- tracking of an end-effector trajectory $y_d(t)$
  - non-causal command designed in frequency domain ⇒ desired acceleration as part of a periodic profile, bounded inversion via Fourier transform (or FFT)
    [Bayo, JRobSyst 1987]
  - … designed in time domain ⇒ forward/backward time integration of stable/unstable parts of the inverse system
    [Kwon, Book, ASME JDSMC 1994]
  - both extended from linear to nonlinear case via numerical/iterative methods



bang-bang acceleration in T = 2 s for both system outputs



control torques, with pre-charge and discharge intervals (T_τ = 2.5 s)



stroboscopic motion of the 2R FLEXARM under E-E control

Control solutions for flexible link robots at Sapienza

Main results – 4 (oldies but goldies…)

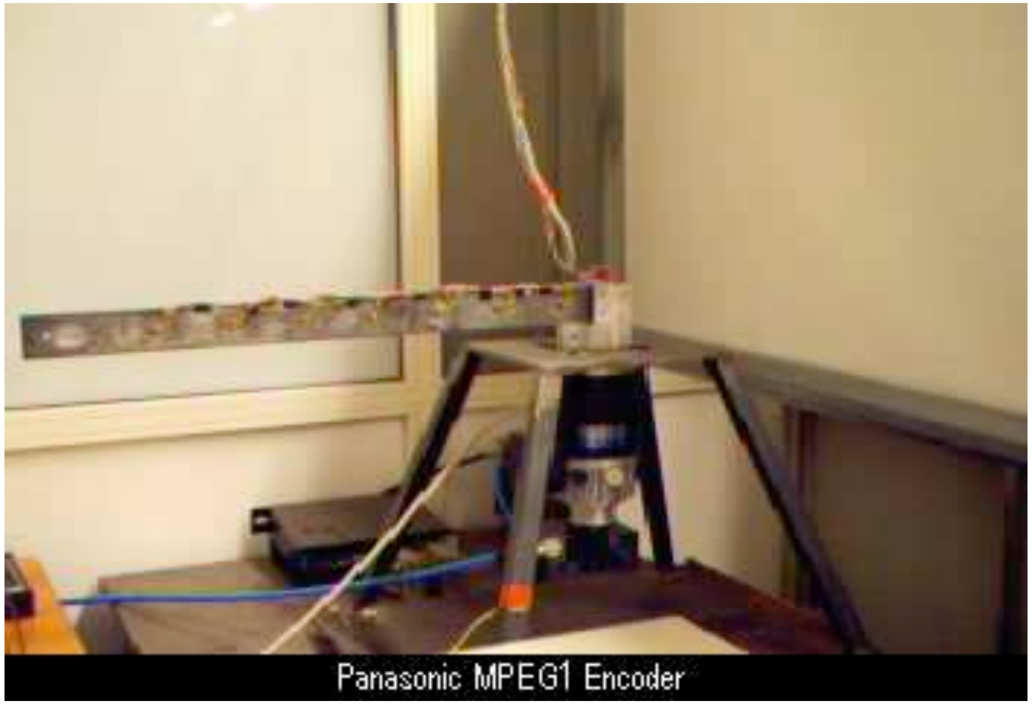
- stable nonlinear regulation of end-effector trajectory for the 2R FLEXARM
- rest-to-rest slew motion in assigned time for a one-link flexible beam

video

45° for (rigid) link 1 and 45° for tip of flexible forearm in T = 1.5 s

[De Luca *et al*, CDC 1990, ICRA 1998]

video



90° slew in T = 2 s (flat output design)

[De Luca, Di Giovanni, AIM 2001; De Luca, Caiano, Del Vescovo, ISER 2002]

Control solutions for flexible link robots

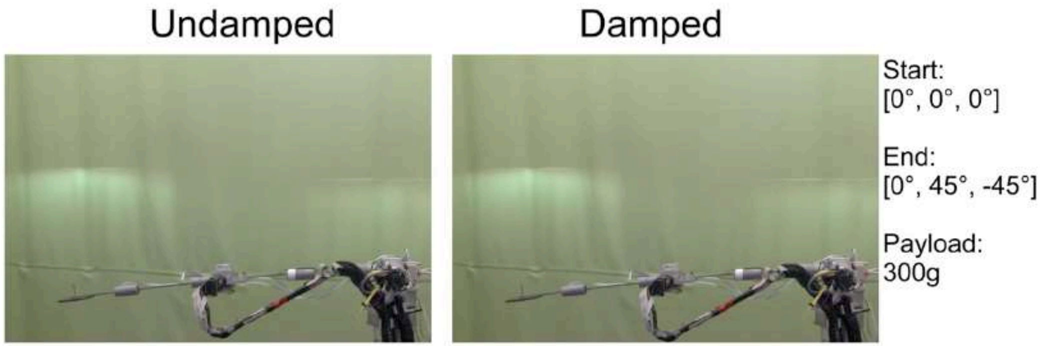
More results, including physical interaction

- 3R arm with flexible links TUDOR (TU Dortmund Omni-elastic Robot)
- vibration damping by strain gauge feedback during motion (or after impact)

video

technische universität dortmund

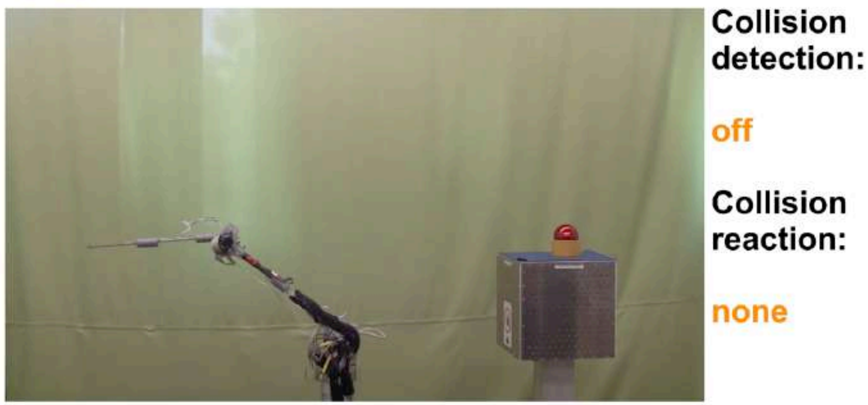
[Malzahn *et al*, IEEE ROBIO 2011]



[Malzahn, Bertram, IFAC World Congr 2014]

video

technische universität dortmund



- collision detection and reaction based on generalized momentum observer

same residual method as in elastic joint robots!!

Outlook on control of soft manipulators

Continuum planar arms with PCC

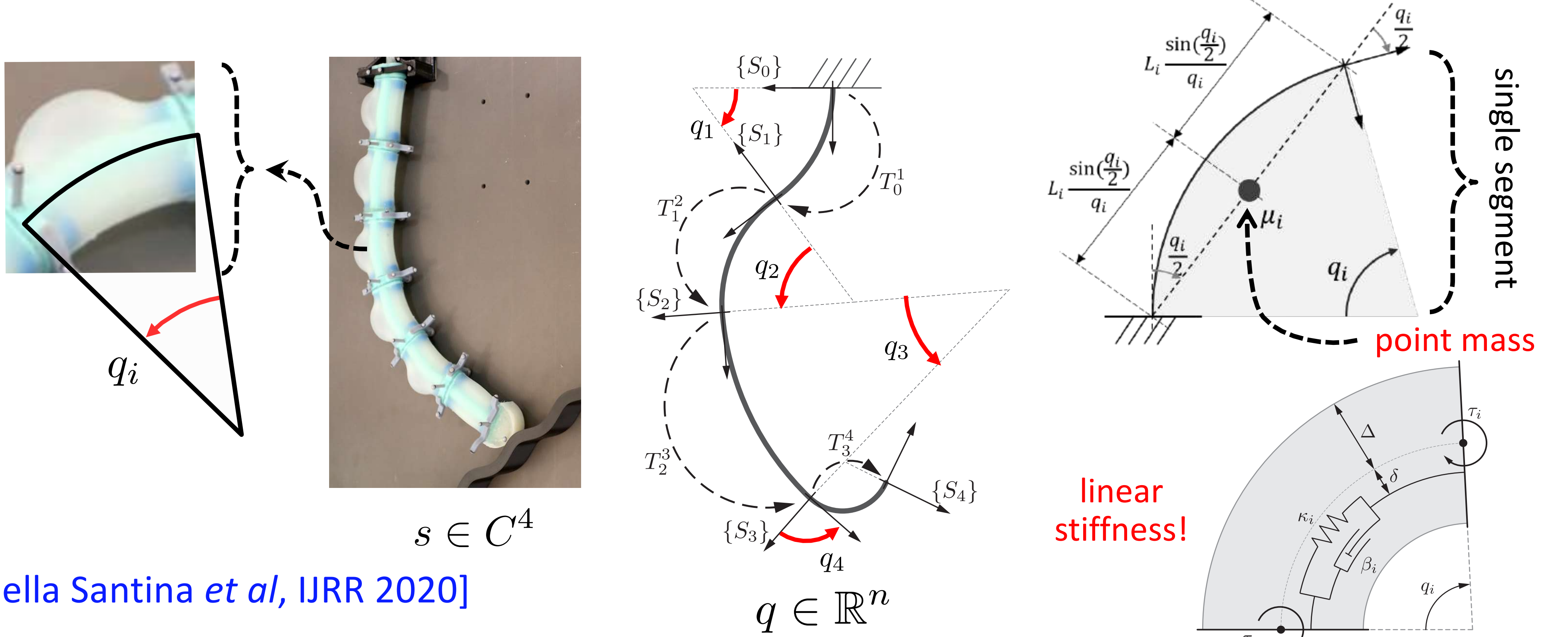
- dynamic modeling assumptions

  A1) [kinematics] approximated as a series of n segments, each with a curvature q_i

  A2) [inertia] each segment can be described by an equivalent point mass

  A3) [impedance] continuous distribution of infinitesimal springs and dampers

- fully actuated on each segment ⇔ underactuated with $m < n$ input commands



[Della Santina *et al*, IJRR 2020]

Dynamic model of planar soft manipulator

Full actuation vs. underactuation in PCC model

- actuated on each of the n segments

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + Kq + D\dot{q} = \tau$$

with the usual properties ($M > 0$, $\dot{M} - 2C$ skew-symmetric, g bounded in norm, …)

⇒ regulation, curvature trajectory tracking, Cartesian stiffness control, preserving (in nominal conditions) stiffness and damping of the soft system

[Della Santina *et al*, IJRR 2020]

- underactuated with only $m < n$ input commands
  - let $q = (q_a, q_u)$, possibly after relabeling of segments, being $q_a \in \mathbb{R}^m$ the curvature of active segments and $q_u \in \mathbb{R}^{n-m}$ that of the unactuated segments
  - dropping dependencies, with active commands $\tau \in \mathbb{R}^m$ and suitable partitions

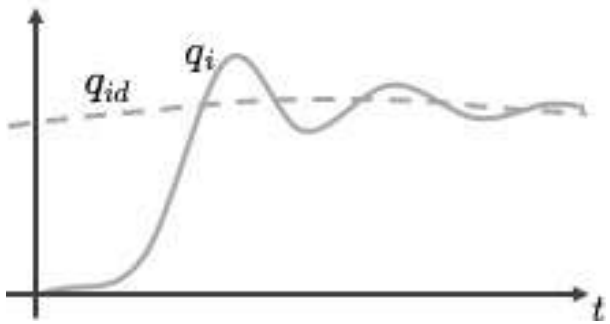
$$\begin{pmatrix} M_{aa} & M_{au} \\ M_{au}^T & M_{pu} \end{pmatrix}\begin{pmatrix} \ddot{q}_a \\ \ddot{q}_u \end{pmatrix} + \begin{pmatrix} C_{aa} & C_{au} \\ C_{ua} & C_{uu} \end{pmatrix}\begin{pmatrix} \dot{q}_a \\ \dot{q}_u \end{pmatrix} + \begin{pmatrix} g_a \\ g_u \end{pmatrix} + \begin{pmatrix} K_a & 0 \\ 0 & K_u \end{pmatrix}\begin{pmatrix} q_a \\ q_u \end{pmatrix} + \begin{pmatrix} D_a & 0 \\ 0 & D_u \end{pmatrix}\begin{pmatrix} \dot{q}_a \\ \dot{q}_u \end{pmatrix} = \begin{pmatrix} \tau \\ 0 \end{pmatrix}$$

⇒ a few preliminary results … [joint work with Pietro Pustina, 2021]

Regulation and trajectory tracking

Full actuation: moving from joint configuration space to local curvature space

- regulation to a (quasi-static) q_d



feedforward (soft robot stiffness & damping)

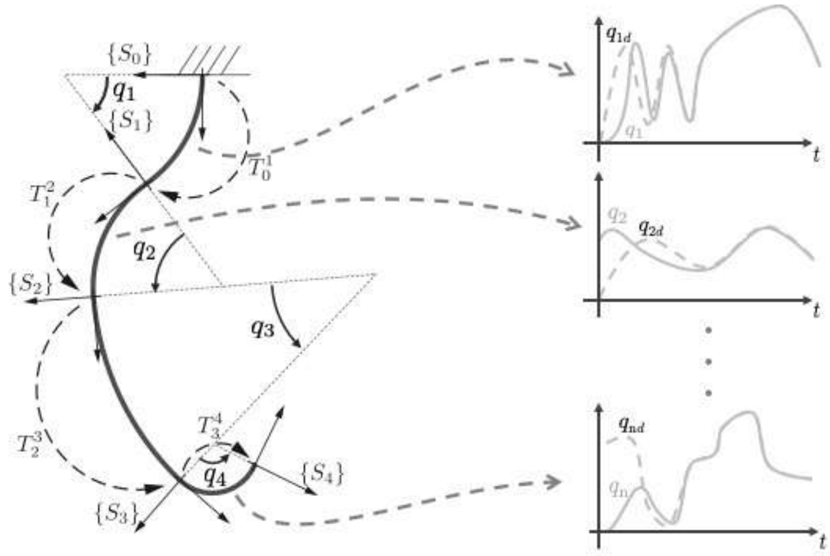
gravity feedback cancellation

$$\tau = Kq_d + D\dot{q}_d + g(q)$$
$$+ K_P(q_d - q) + K_D(\dot{q}_d - \dot{q})$$

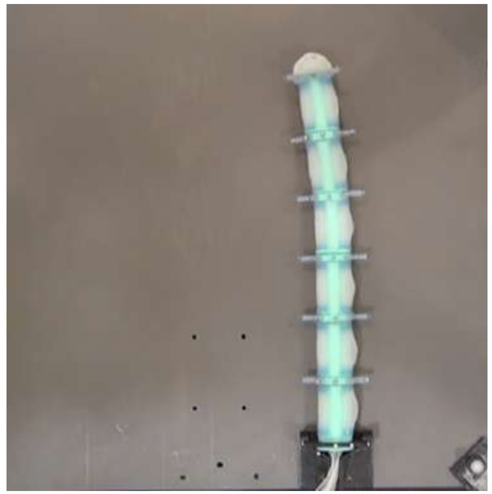
robustifying PD action

[Della Santina *et al,* IJRR 2020]

- tracking of $q_d(t)$, with $\dot{q}_d \neq 0$, $\ddot{q}_d \neq 0$



video



passivity-based tracking controller

$$\tau = Kq_d + D\dot{q}_d + g(q) + C(q,\dot{q})\dot{q}_d + M(q)\ddot{q}_d$$
$$+ K_P(q_d - q) + K_D(\dot{q}_d - \dot{q})$$

Zero dynamics and regulation

Underactuated planar PCC model, without and with gravity

- zero dynamics when the output is $y = q_a \in \mathbb{R}^m$
  - in the absence of gravity ($g(q) \equiv 0$), the unique state $(q_u, \dot{q}_u) = (0,0)$ is globally asymptotically stable for the zero dynamics of the soft robot
  - in the presence of gravity (e.g., in a vertical plane), the trajectories of the zero dynamics remain bounded and converge to $(q_u, \dot{q}_u) = \left(q_{u.eq}, 0\right)$, being $q_{u,eq}$ a solution of

$$K_u q_u + g_u(0, q_u) = 0$$

  - proofs by Lyapunov/La Salle analysis

- regulation to $q_d = (q_{a,d}, 0) \in \mathbb{R}^n$, $q_{a,d} \in \mathbb{R}^m$, in the absence of gravity

$$\tau = K_P\left(q_{a,d} - q_a\right) - K_D \dot{q}_a + K_a q_{a,d} \qquad K_P, K_D > 0$$

- regulation to $q_d = (q_{a,d}, q_{p,d}) \in \mathbb{R}^n$, $q_{a,d} \in \mathbb{R}^m$, in the presence of gravity

$$\begin{cases} \tau = K_P\left(q_{a,d} - q_a\right) - K_D \dot{q}_a + g_a(q_d) + K_a q_{a,d} \\ \tau^g = K_P\left(q_{a,d} - q_a\right) - K_D \dot{q}_a + g_a(q_{a,d}, q_u) + K_a q_{a,d} \end{cases}$$

$K_P > 0$, sufficiently large

$q_{u,d}$ unique solution to

$$K_u q_u + g_u(q_{a,d}, q_u) = 0$$

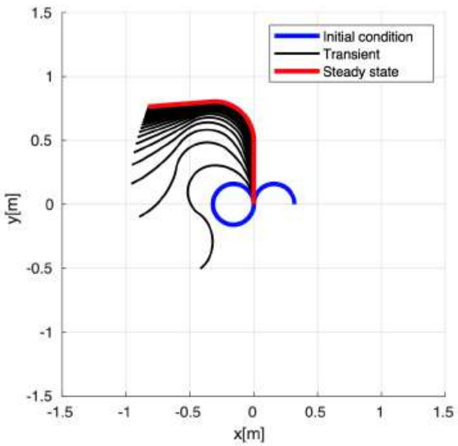
Simulation results

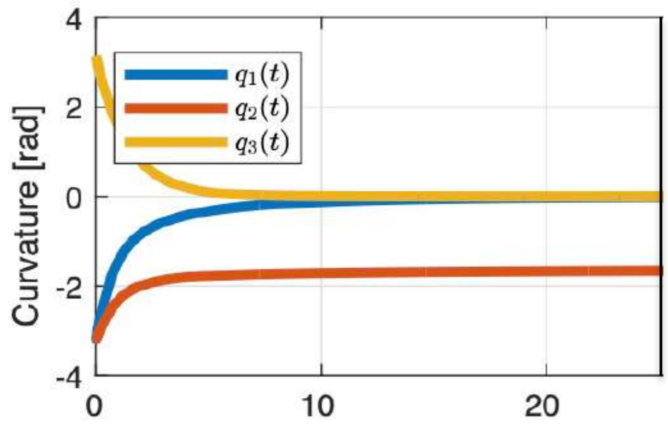
Underactuation with $n = 3$ segments, $m = 2$ actuated: $\boldsymbol{q_a} = (q_1, q_3)$, $\boldsymbol{q_u} = q_2$

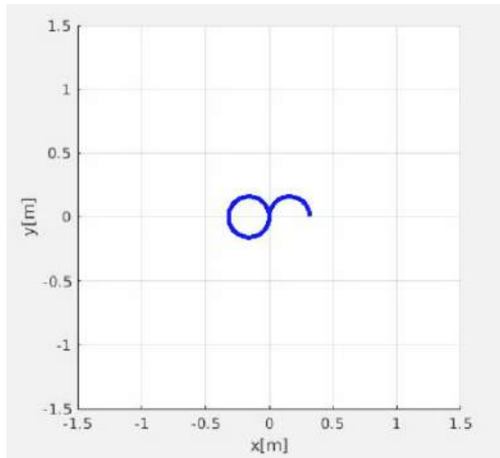
- regulation to $q_{a,d} = (0,0)$ from $q(0) = (-\pi, -\pi,\ \pi)$ using τ^g, in the presence of gravity

video

$\vec{g}_0$



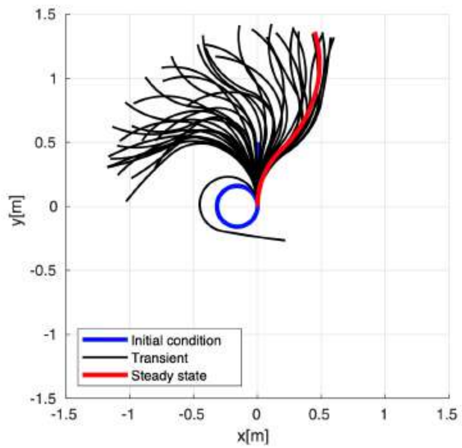


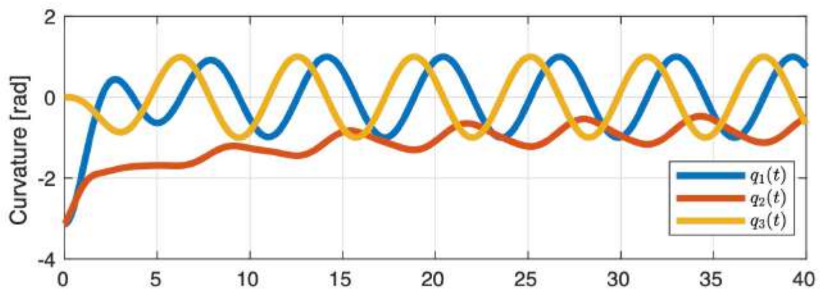


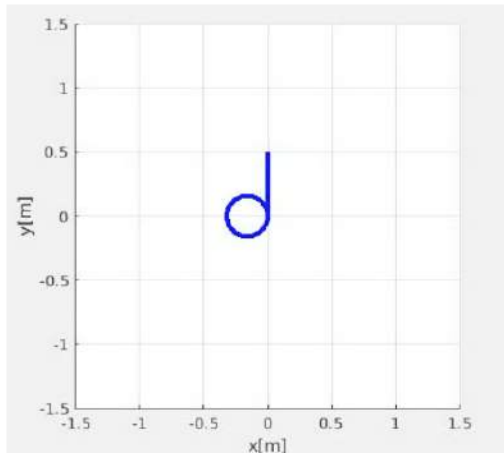
- tracking of $q_{a,d}(t) = (\sin t\,, \cos t)$ starting from $q(0) = (-\pi, -\pi,\ 0)$, using a partial feedback linearization control τ^{PFL}, in the presence of gravity

video

$\vec{g}_0$







Take home messages

Control of soft robots in 2021+

- a "soft explosion" is revamping the mature field of flexible robot control
  - consideration of dynamics in the control design/performance of soft robots
  - combine (learned) feedforward and feedback to achieve robustness
  - iterative learning (on repetitive tasks) is available for flexible manipulators
  - optimal control (min time, min energy, max force, …) still open for fun
- revisiting model-based control design
  - do not fight against the natural dynamics of the system
    - it is unwise to stiffen what was designed/intended to be soft on purpose
  - still, don't give up too much of desirable performance!
- ideas assessed for flexible joints and links may migrate to other classes of soft-bodied robots (and applications)
  - keep in mind intrinsic task constraints and control limitations (e.g., instabilities in system inversion of tip trajectories for flexible link robots)
  - locomotion, shared manipulation, physical interaction in complex tasks, …

References

Cited in the slides – 1

pdf and videos: see also
www.diag.uniroma1.it/deluca/Publications.php

- Albu-Schäffer, Ott, Hizinger, 2007. A unified passivity-based control framework for position torque and impedance control of flexible joint robots. *Int. J. of Robotics Research*, 26(1), 23-39
- Bayo, 1987. A finite element approach to control the end-point motion of a single-link flexible robot. *J. of Robotic Systems*, 4(1), 63-75
- Bellezza, Lanari, Ulivi, 1990. Exact modeling of the flexible slewing link. *IEEE Int. Conf. on Robotics and Automation*, 734-739
- Buondonno, De Luca, 2015. A recursive Newton-Euler algorithm for robots with elastic joints and its application to control. *IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, 5526-5532
- De Luca, Albu-Schäffer, Haddadin, Hirzinger, 2006. Collision detection and safe reaction with the DLR-III lightweight manipulator arm. *IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, 1623-1630
- De Luca, Book, 2016. Robots with flexible elements. *Springer Handbook of Robotics*, chap 11, 243-282
- De Luca, Caiano, Del Vescovo, 2003. Experiments on rest-to-rest motion of a flexible arm. *Experimental Robotics VIII*, Springer Tracts in Advanced Robotics, vol 5, 338-349
- De Luca, Di Giovanni, 2001. Rest-to-rest motion of a one-link flexible arm. *IEEE/ASME Int. Conf. on Advanced Intelligent Mechatronics*, 923-928
- De Luca, Flacco, 2010. Dynamic gravity cancellation in robots with flexible transmissions. *49th IEEE Conf. on Decision and Control*, 288-295
- De Luca, Flacco, 2011. A PD-type regulator with exact gravity cancellation for robots with flexible joints. *IEEE Int. Conf. on Robotics and Automation*, 317-323

References

Cited in the slides – 2

- De Luca, Lanari, Lucibello, Panzieri, Ulivi, 1990. Control experiments on a two-link robot with a flexible forearm. *29th IEEE Conf. on Decision and Control*, 520-527
- De Luca, Lucibello, 1998. A general algorithm for dynamic feedback linearization of robots with elastic joints. *IEEE Int. Conf. on Robotics and Automation,* 504-510
- De Luca, Panzieri, Ulivi, 1998. Stable inversion control for flexible link manipulators. *IEEE Int. Conf. on Robotics and Automation*, 799-805
- De Luca, Siciliano, 1991. Closed-form dynamic model of planar multi-link lightweight robots. *IEEE Trans. on Systems, Man, and Cybernetics*, 21(4), 826-839
- De Luca, Siciliano, 1993a. Regulation of flexible arms under gravity. *IEEE Trans. on Robotics and Automation*, 9(4), 463-467
- De Luca, Siciliano, 1993b. Inversion-based nonlinear control of robot arms with flexible links, *AIAA J. of Guidance, Control, and Dynamics*, 16(6), 1169-1176
- De Luca, Siciliano, Zollo, 2005. PD control with on-line gravity compensation for robots with elastic joints: Theory and experiments. *Automatica*, 41(10), 1809-1819
- Della Santina, Katzschmann, Bicchi, Rus, 2020. Model-based dynamic feedback control of a planar soft robot: Trajectory tracking and interaction with the environment. *Int. J. of Robotics Research*, 39(4), 490-513
- Haddadin, De Luca, Albu-Schäffer, 2017. Robot collisions: A survey on detection, isolation, and identification. *IEEE Trans. on Robotics*, 33(6), 1292-1312

References

Cited in the slides – 3

- Keppler, Lakatos, Ott, Albu-Schäffer, 2018. Elastic Structure Preserving (ESP): Control for compliantly actuated robots. *IEEE Trans. on Robotics*, 34(2), 317-335
- Kwon, Book, 1994. A time-domain inverse dynamic tracking control of a single-link flexible manipulator. *ASME J. Dynamic Systems, Measurement. and Control*, 116(2), 193-200
- Magrini, Flacco, De Luca, 2015. Control of generalized contact motion and force in physical human-robot interaction," *IEEE Int. Conf. on Robotics and Automation*, 2298-2304
- Malzahn, Phung, Hoffmann, Bertram, 2011. Vibration control of a multi-flexible-link robot arm under gravity. *IEEE Int. Conf. on Robotics and Biomimetics*, 1249-1254
- Malzahn, Bertram, 2014. Collision detection and reaction for a multi-elastic-link robot arm. *19th IFAC World Congr.*, 320-325
- Ott, Albu-Schäffer, Kugi, Stramigioli, Hirzinger, 2004. A passivity based Cartesian impedance controller for flexible joint robots - Part I. *IEEE Int. Conf. on Robotics and Automation*, 2659-2665
- Pustina, 2021. *Master Thesis*, Sapienza University of Rome
- Spong, 1987. Modeling and control of elastic joint robots. *ASME J. Dynamic Systems, Measurement, and Control*, 109(4), 310-319
- Tomei, 1991. A simple PD controller for robots with elastic joints. *IEEE Trans. Automatic Control,* 36(10), 1208-1213
- Thümmel, 2007. Entwurf, Auslegung und Evaluierung einer Regelungsstruktur für Roboterarme mit elastischen Gelenken zur Erreichung hoher Performance bei Positionier- und Bahnaufgaben am Beispiel eines KUKA KR15/2. *PhD Thesis*, Technical University of Munich (*in German*)