SIDRA Summer School 2021

# Optimal Planning and Control of Articulated Soft Robots

Manolo Garabini

# a short intro

- Master Mech. Eng. – 2010
- PhD Robotics – 2014

- Assistant Prof. Automatic Control – since 2018

- H2020 Projects
  – THING (2018-present) Local PI
  – NI (2021-present) Coordinator

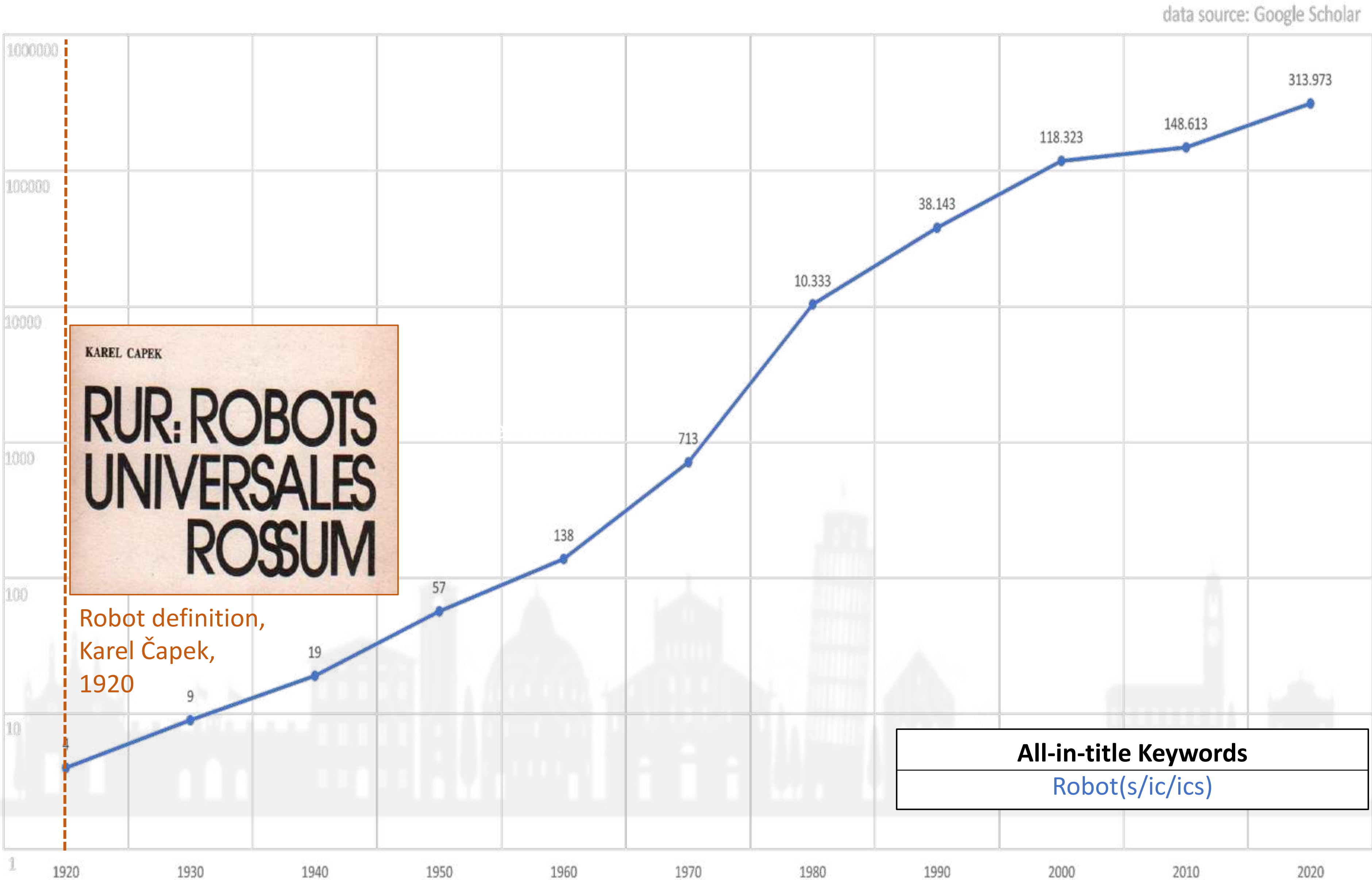
- Co-founder
  – Qbrobotics (since 2011)
  – Xstar Motion (since 2021)

# outline

- classical and new goals for robots
- soft robotics: a new solution for a new problem
- the optimal approach for planning and control of soft robots
  - Minimum Time Trajectory Planning
  - The Safe Brachistochrone
  - Max speed
  - Optimal brake
  - Robust Interaction
  - Max Efficiency in Cyclic Tasks
- what’s next?

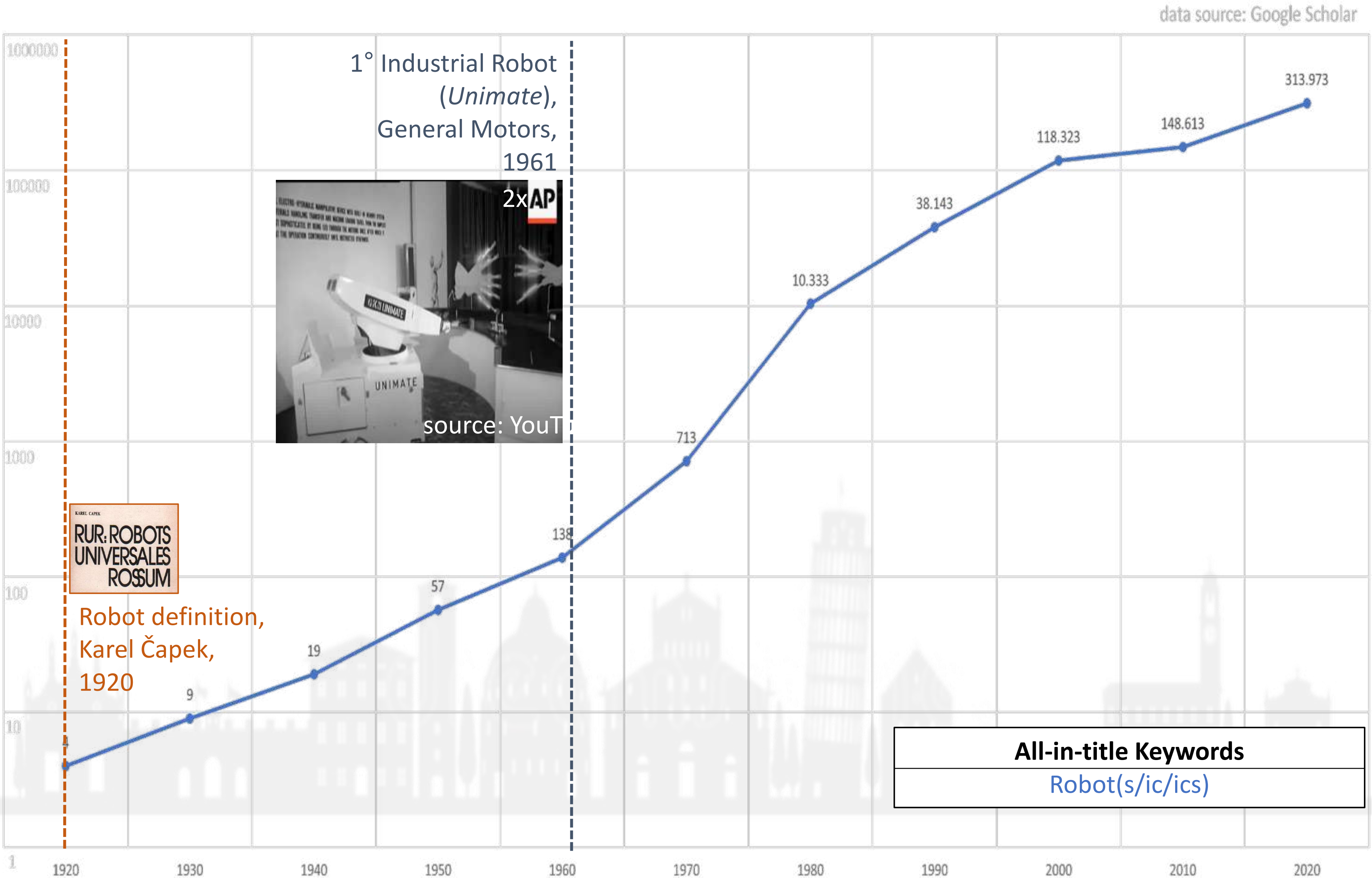
# Evolution of Robots in a Century



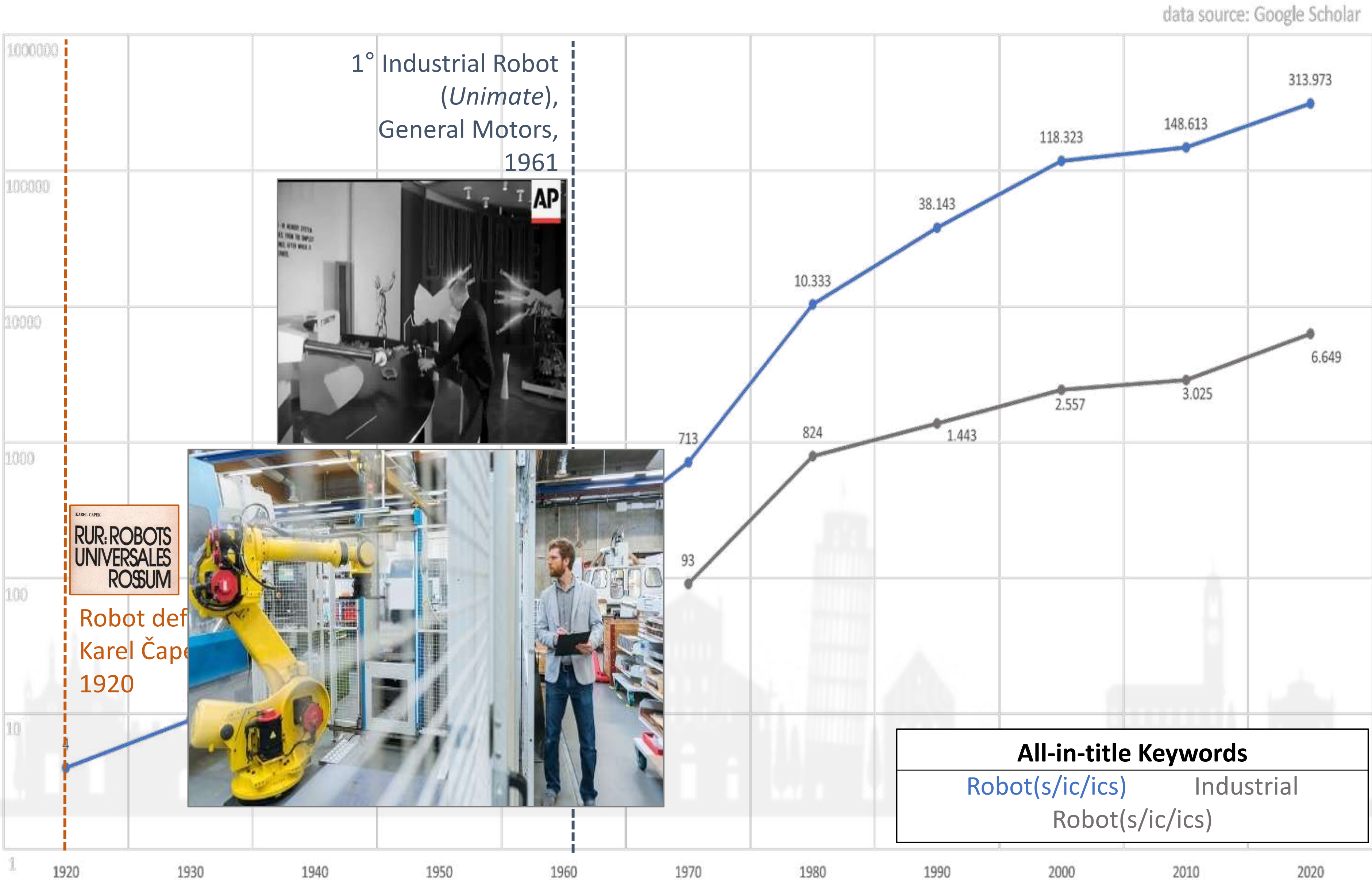


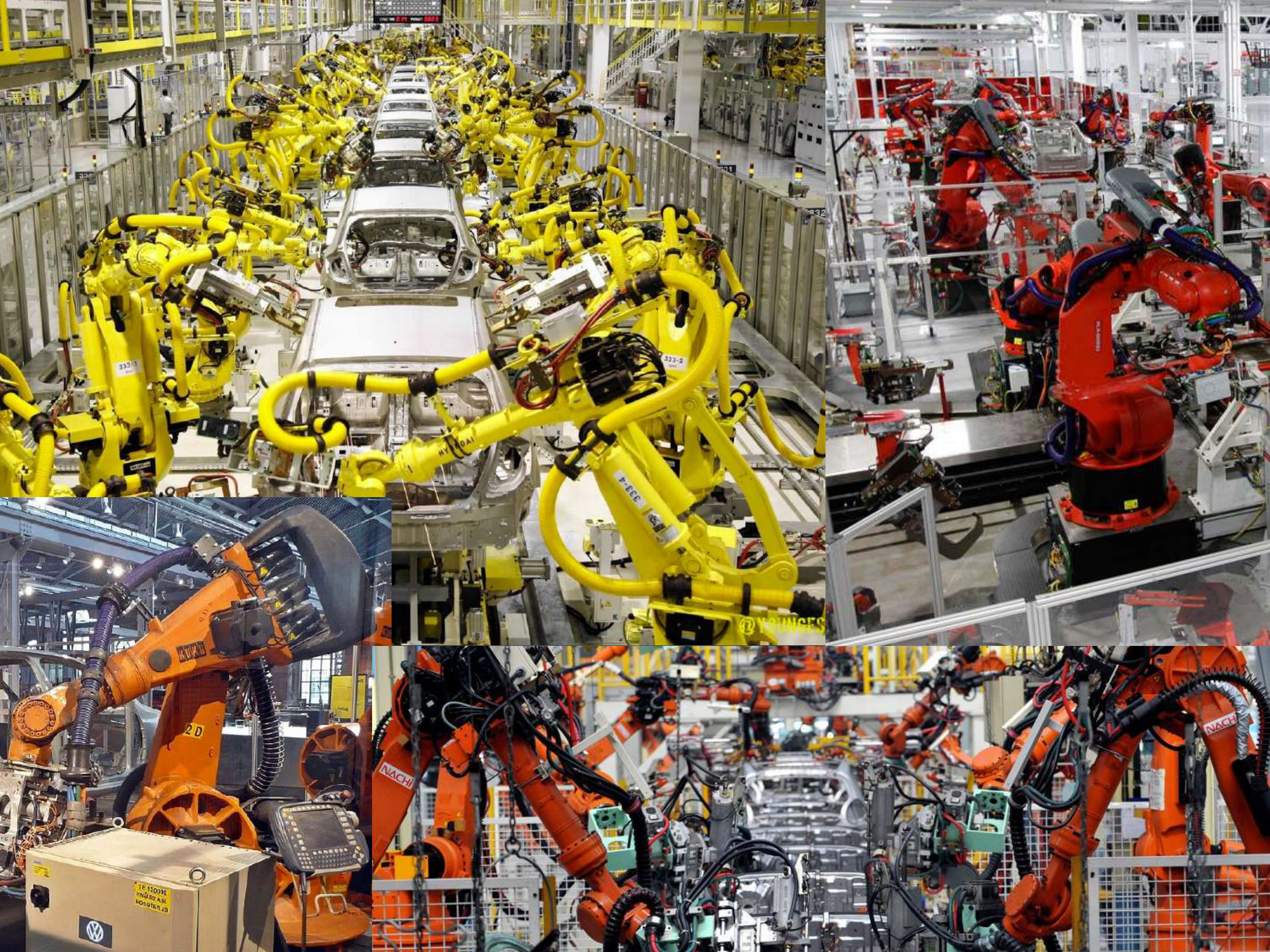
# Evolution of Robots in a Century





# Evolution of Robots in a Century




333-2
HYUNDAI
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NACHI

TASKS

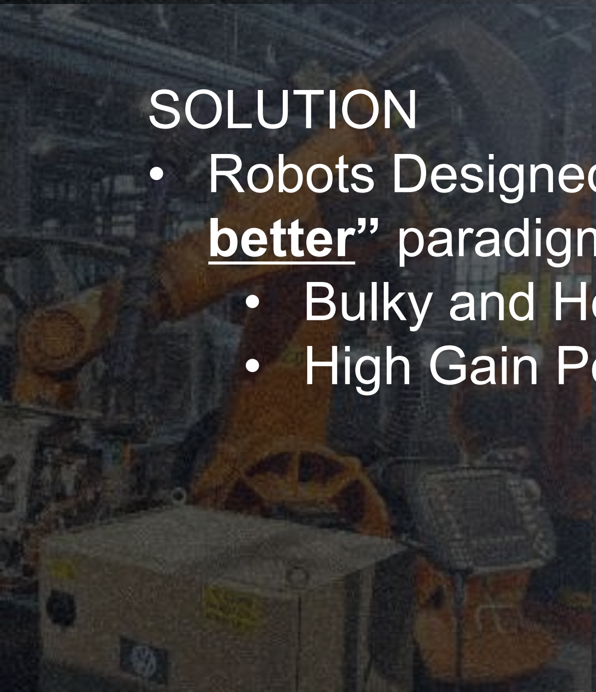
- mainly **positioning** operations in **structured environment**

GOALS

- Minimize Cost
    - Cycle Time
    - Positioning Errors

SOLUTION

- Robots Designed and Controlled according to **“the stiffer the better”** paradigm
    - Bulky and Heavy Machines
    - High Gain Position Control



HEALTHCARE

INSPECTION AND MAINTENANCE

# ROBOTICS TECHNOLOGY APPLICATION AREAS

[the good old Horizon 2020]

AGILE PRODUCTION

AGRI-FOOD

Robotics Core Technology. Call ID: ICT-10-2019-2020. Indicative Overall Budget 42 M
Deadline 28-March-2019. 91 submitted proposals


HEALTHCARE


INSPECTION


AGRI-FOOD


AGILE PRODUCTION

DIGITAL and EMERGING TECH
for the GREEN DEAL

# DESTINATIONS

[Horizon Europe]

HUMAN centred development of
DIGITAL and INDUSTRIAL TECH

[Horizon Europe – Work Programme 2021-2022 Digital, Industry, and Space]

TASKS

- **Physical Interaction** in **unstructured environments**

GOALS

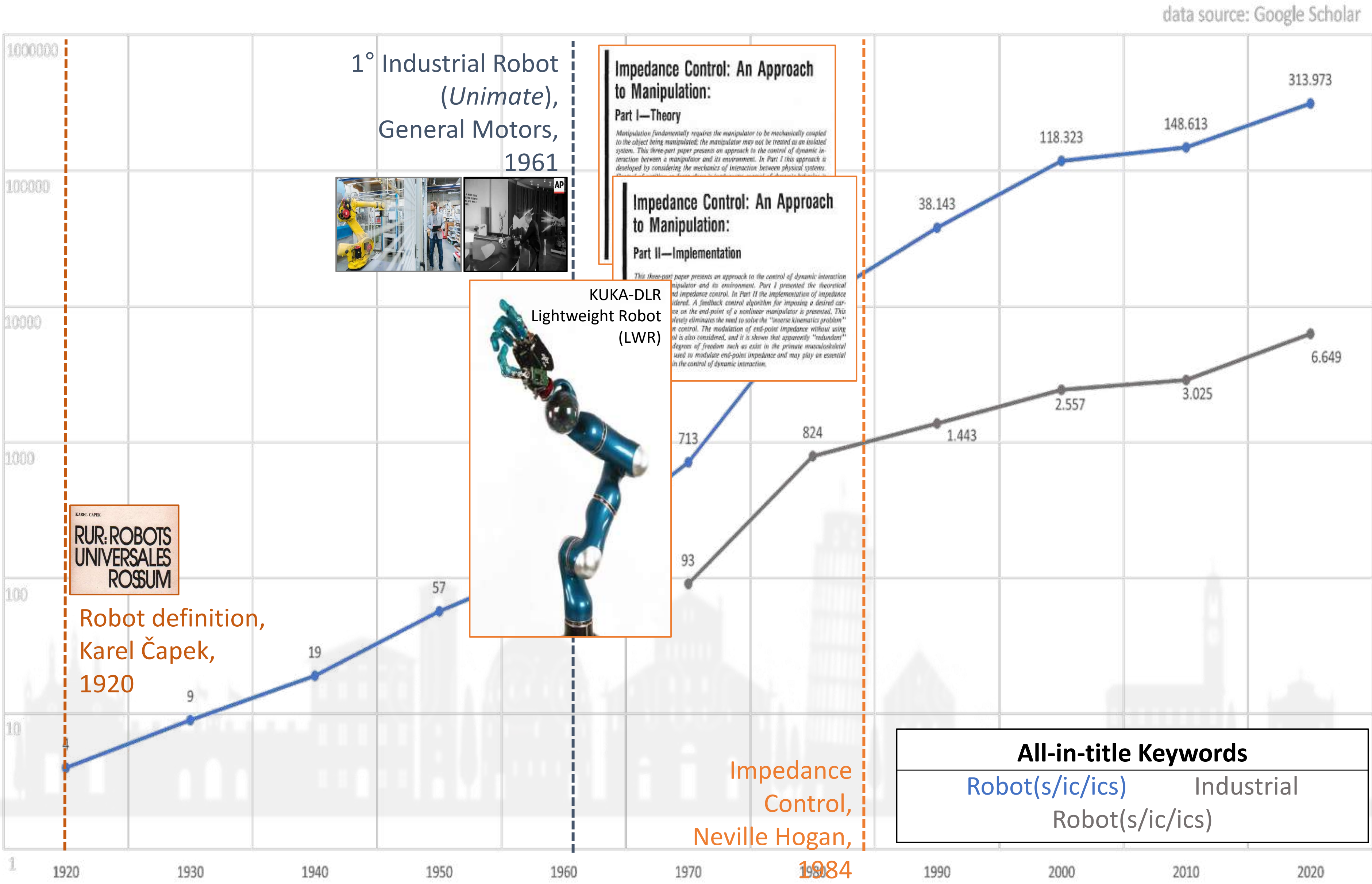
- Minimize Cost (exploiting soft dynamics)
    - Cycle time
    - Positioning error
    - Energy Consumption
    - …
- Subject to
    - **limits on interaction forces** (safety, robot resiliency, …)

NEW PROBLEM -> NEW SOLUTION

- Robots Designed and Controlled according to **“the stiffer the better”** paradigm does not work any more!

# Evolution of Robots in a Century

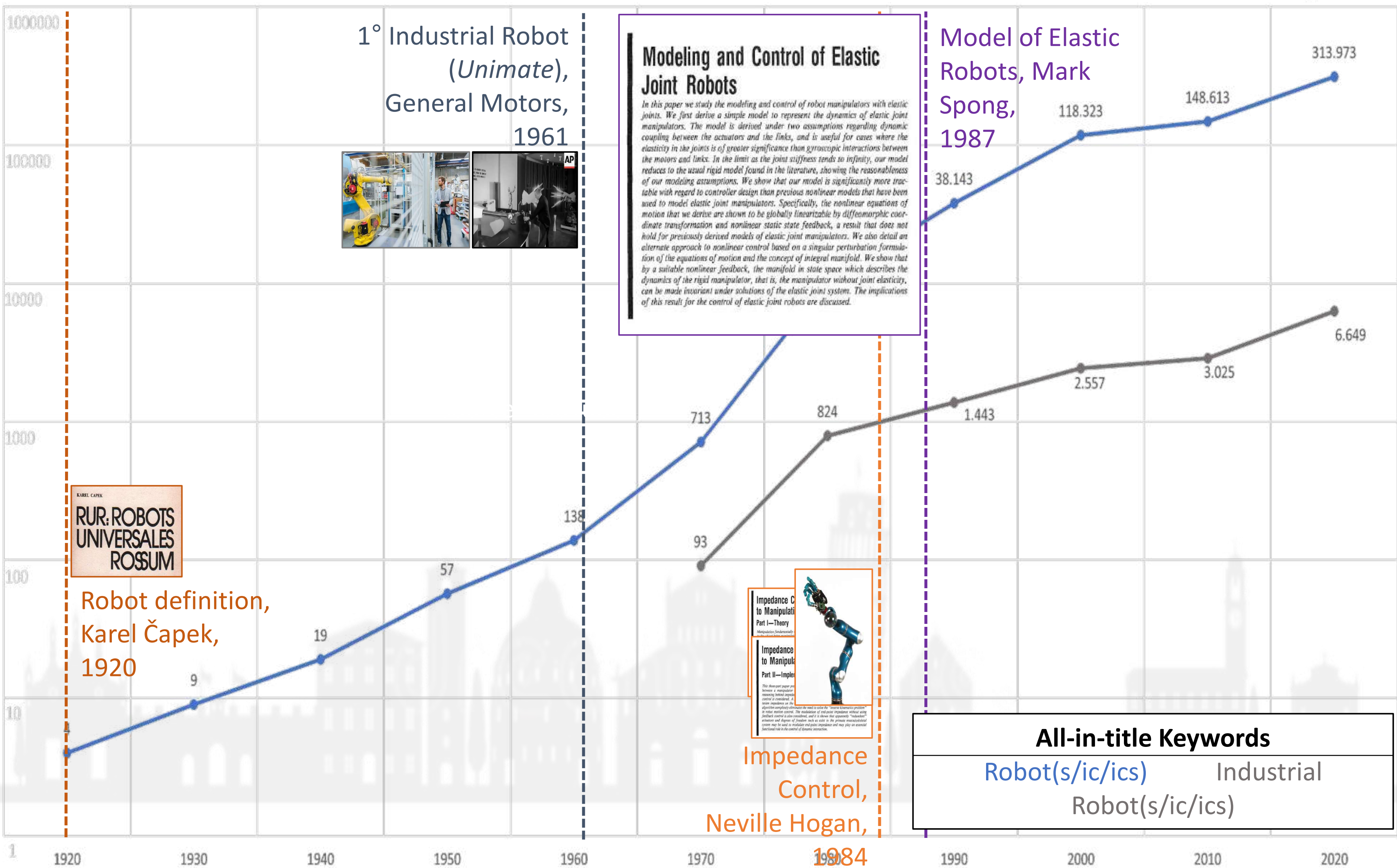




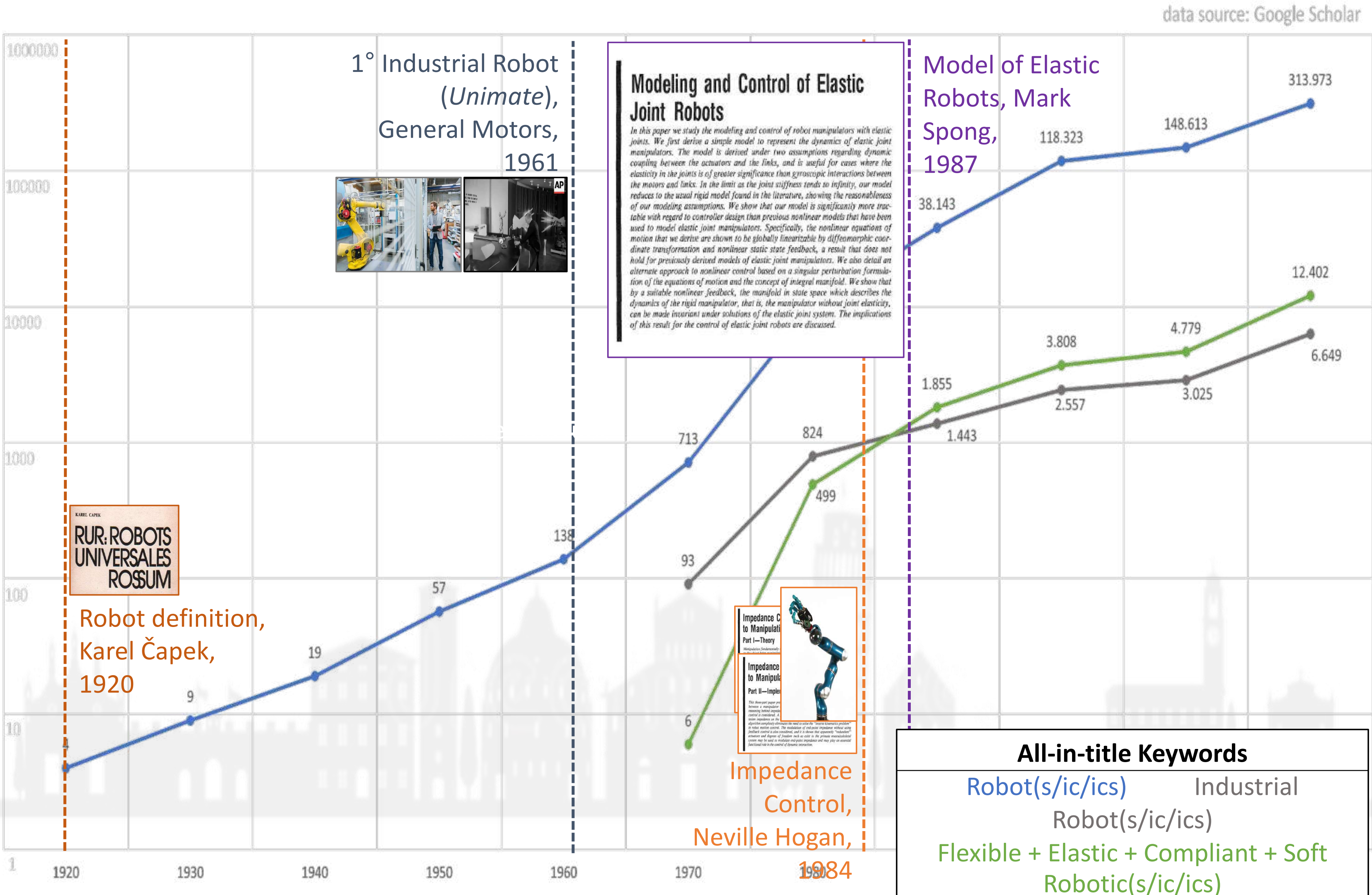
# Evolution of Robots in a Century



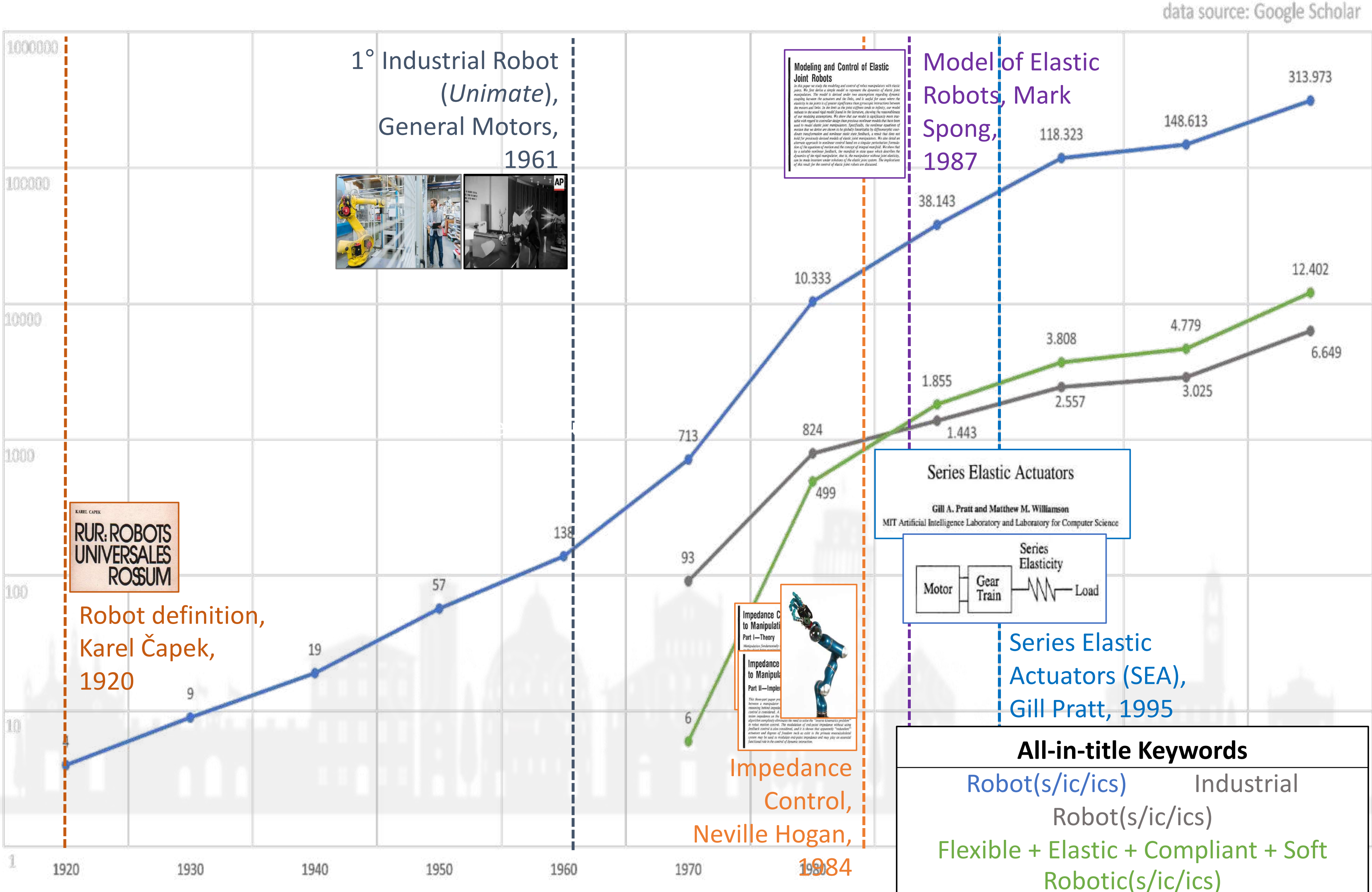
data source: Google Scholar



# Evolution of Robots in a Century 



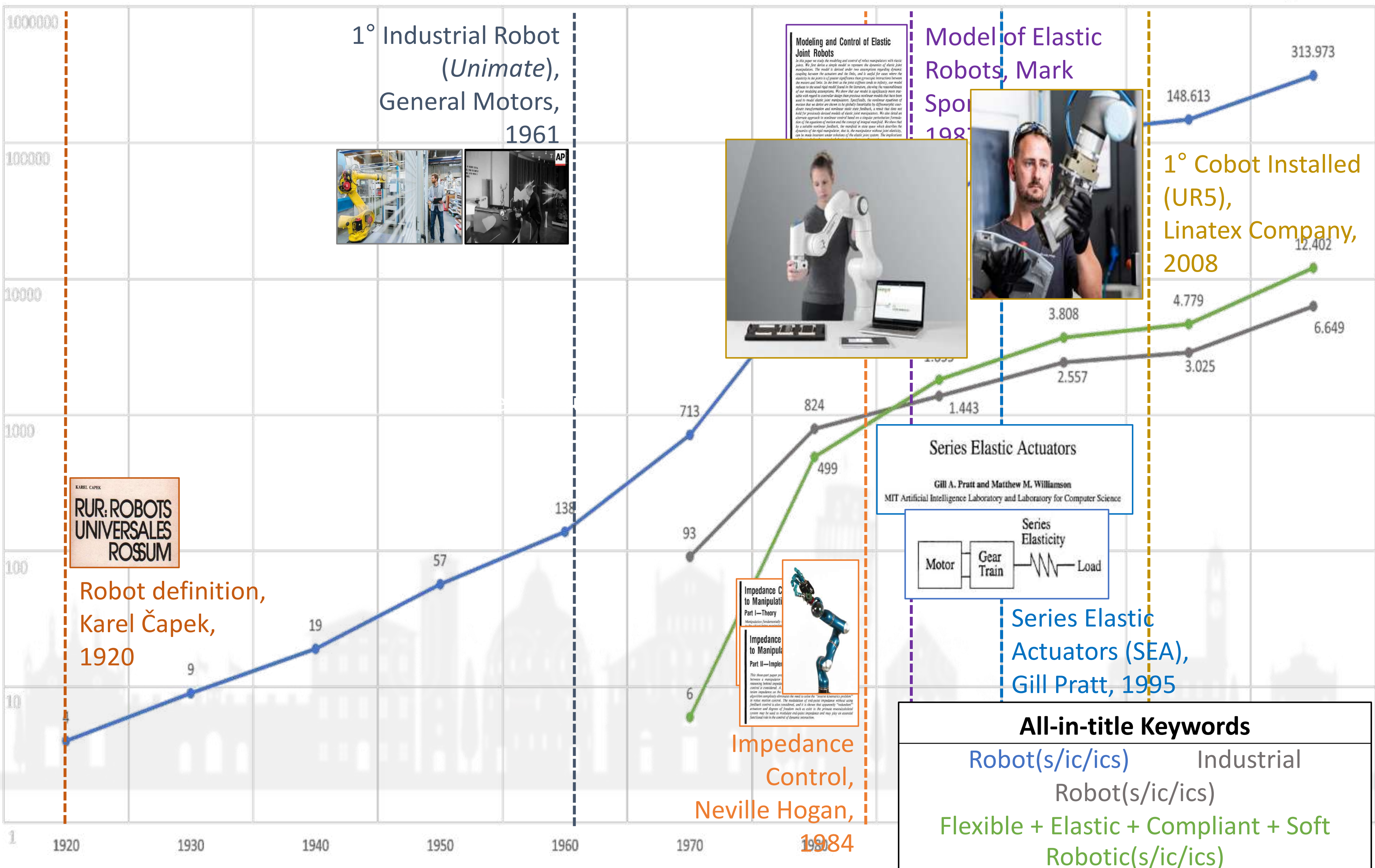
# Evolution of Robots in a Century



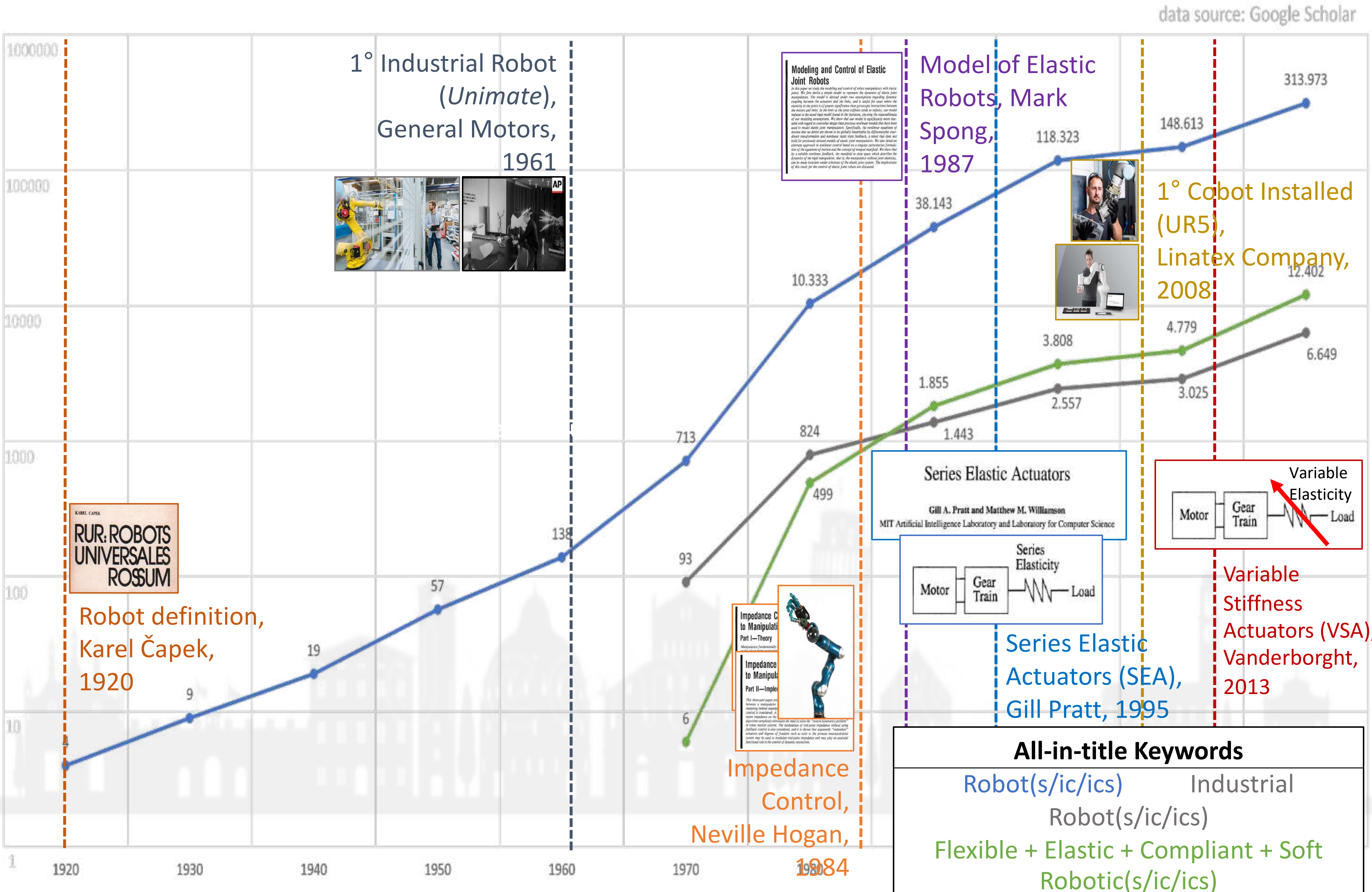
# Evolution of Robots in a Century



data source: Google Scholar

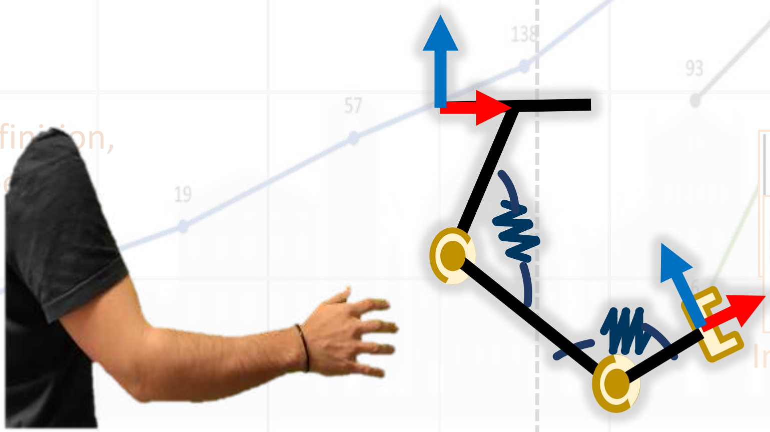


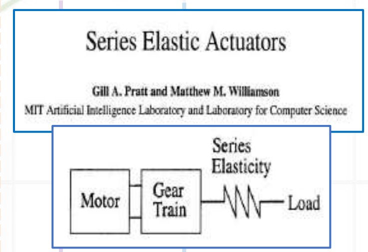
# Evolution of Robots in a Century



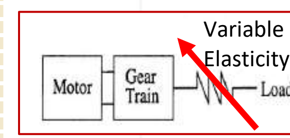
# Articulated Soft Robots






Series Elastic Actuators (SEA), Gill Pratt, 1995




Variable Stiffness Actuators (VSA), Vanderborght, 2013

[0] Della Santina, C., et al. "Soft robots." Encyclopedia of Robotics (2020).

# The potential of Soft Robotics and the many open questions about it

| Features | Soft Robot |
|---|---|
| Peak Performance | a specific design can over-perform a conventional system of the some power |
| Energy Efficiency | can be optimized in the execution of cyclic tasks |
| Robustness | Peak stress on the weaker parts can be reduced |

How to manage the additional degrees of freedom of Soft Robots to fully exploit them?

# The optimal approach

**Optimal Control** provides an absolute performance reference that **factorizes the control design out**, and allows for drawing conclusions on the intrinsic worth of the physical system.

$$\min_{x(t),u(t),p} \quad \int_0^T L(x(t),u(t))dt$$

$$\text{subject to}$$

$$g(x,\dot{x},u,p) = 0$$
$$h(x,u,p) \leq 0$$

# The optimal approach

**Optimal Control** provides an absolute performance reference that **factorizes the control design out**, and allows for drawing conclusions on the intrinsic worth of the physical system.

Analytical Solutions
- template models
- find “rules” and understand “principles”

Numerical Solutions
- realistic models
- global guarantees (and often run-time execution) for convex problems

# A (short) list of problems for soft robots that can be tackled via OC

- Minimum Time Trajectory Planning
- The Safe Brachistochrone
- Max speed
- Optimal brake
- Max Efficiency in Cyclic Tasks
- Robust Interaction
- …

# Time-Optimal Trajectory Planning for Flexible Joint Robots

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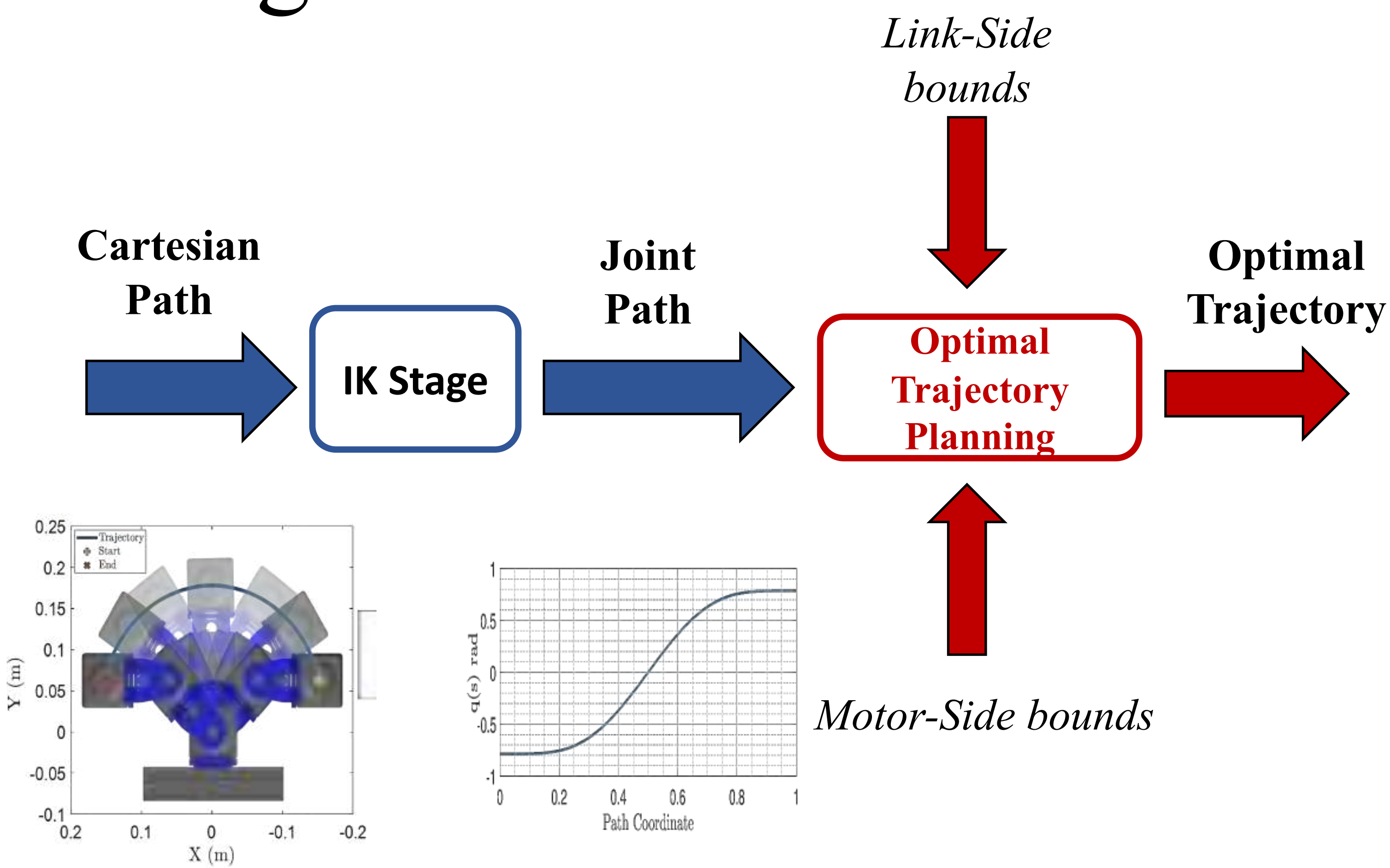
# Time-Optimal Trajectory Planning

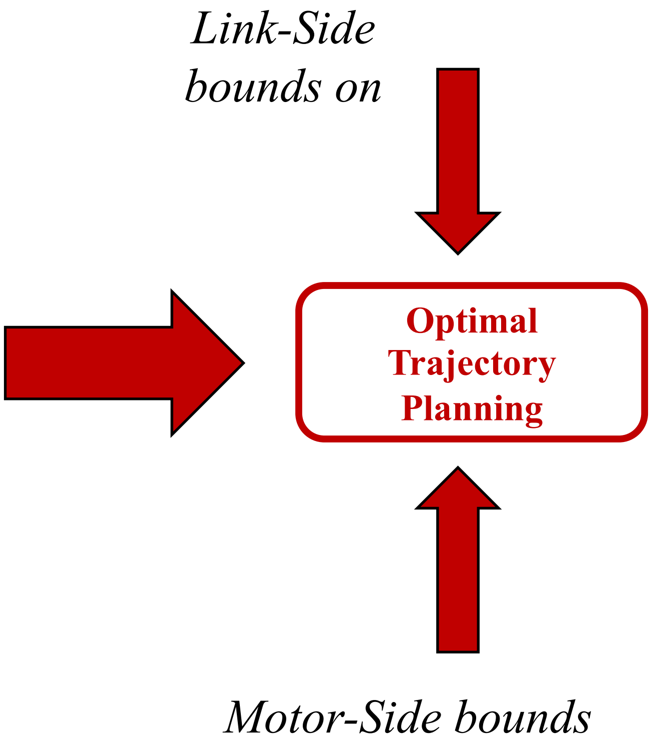


# Flexible Joint Robots

- Dynamics of a robot with flexible joints included to reformulate the constraints

$$\begin{cases} M(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + G(\mathbf{q}) + K(\mathbf{q}-\boldsymbol{\theta}) = 0 \\ B_m\ddot{\boldsymbol{\theta}} + K(\boldsymbol{\theta}-\mathbf{q}) = \boldsymbol{\tau}_m, \end{cases}$$

Dynamic model of a robot with flexible joints presented by Spong [1]




[1] - Mark W Spong - Modeling and control of elastic joint robots. - Journal of dynamic systems, measurement, and control, 109(4):310--318, 1987.

# Problem Formulation

$$
\begin{cases}
\min\limits_{s(\cdot)} \quad \int_0^T 1 \mathrm{d}t \\
\text{subject to} \\
s(0) = 0 \text{ and } s(T) = 1 \\
\dot{s}(0) = 0 \text{ and } \dot{s}(T) = 0 \\
\ddot{s}(0) = 0 \text{ and } \ddot{s}(T) = 0 \\
\dddot{s}(0) = 0 \text{ and } \dddot{s}(T) = 0 \\
\dot{s}(t) \geq 0 \\
\underline{\dot{\boldsymbol{q}}} \leq \dot{\boldsymbol{q}}\left(s(t)\right) \leq \bar{\dot{\boldsymbol{q}}} \\
\underline{\ddot{\boldsymbol{q}}} \leq \ddot{\boldsymbol{q}}\left(s(t)\right) \leq \bar{\ddot{\boldsymbol{q}}} \\
\underline{\boldsymbol{\theta}} \leq \boldsymbol{\theta}\left(s(t)\right) \leq \bar{\boldsymbol{\theta}} \\
\underline{\dot{\boldsymbol{\theta}}} \leq \dot{\boldsymbol{\theta}}\left(s(t)\right) \leq \bar{\dot{\boldsymbol{\theta}}} \\
\underline{\boldsymbol{\tau}}_{\mathrm{e}} \leq \boldsymbol{\tau}_{\mathrm{e}}\left(s(t)\right) \leq \bar{\boldsymbol{\tau}}_{\mathrm{e}} \\
\underline{\boldsymbol{\tau}}_{\mathrm{m}} \leq \boldsymbol{\tau}_{\mathrm{m}}(s(t)) \leq \bar{\boldsymbol{\tau}}_{\mathrm{m}} \\
\text{for } t \in [0, T],
\end{cases}
$$

**Nonlinear change of variables**

$$
\begin{cases}
b \triangleq \dot{s}^2, \ \alpha_1 \triangleq b' = 2\ddot{s}, \\
\alpha_2 \triangleq b'' = 2\dfrac{\dddot{s}}{\dot{s}}, \ \alpha_3 \triangleq b''' = 2\dfrac{\left(s^{(4)}\dot{s} - \ddot{s}\dddot{s}\right)}{\dot{s}^3},
\end{cases}
$$

$$\min_{b(\cdot),\beta_i(\cdot),\alpha_i(\cdot)} \int_0^1 \frac{1}{\sqrt{b}} \mathrm{d}s$$

subject to

$$\boldsymbol{\Gamma}\boldsymbol{\tau}_{\mathrm{m}} = \boldsymbol{k}_0\beta_1 + \boldsymbol{k}_1\beta_2 + \beta_3 + \boldsymbol{k}_3\beta_4 + $$
$$+ \boldsymbol{k}_4\beta_5 + \boldsymbol{k}_5\alpha_1 + \boldsymbol{k}_6\beta_6 + \boldsymbol{k}_7 b + \boldsymbol{k}_8$$

$$b(0) = 0 \text{ and } b(1) = 0$$

$$\alpha_1(0) = 0 \text{ and } \alpha_1(1) = 0$$

$$\alpha_2(0)\sqrt{b(0)} = 0 \text{ and } \alpha_2(1)\sqrt{b(1)} = 0$$

$$\alpha_1 = b', \ \alpha_2 = b'', \ \alpha_3 = b'''$$

$$\beta_1 = \alpha_3 b, \ \beta_2 = \alpha_2\alpha_1, \ \beta_3 = \alpha_2 b, \ \beta_4 = \alpha_1 b$$

$$\beta_8 = \alpha_2\beta_7, \ \beta_9 = \alpha_1\beta_7, \ \beta_{10} = b\beta_7$$

$$\beta_5 = \alpha_1^2, \ \beta_6 = b^2, \ \beta_7^2 = b$$

**Bilinear/Quadratic Constraints**

$$0 \leq b \leq \min_{i \in 1,n} \left(\bar{\dot{q}}_i\right)^2 / \left(q_i'\right)^2$$

$$\underline{\ddot{\boldsymbol{q}}} \leq \boldsymbol{q}'' b + \boldsymbol{q}'\alpha_1/2 \leq \bar{\ddot{\boldsymbol{q}}}$$

$$\underline{\boldsymbol{\theta}} - \boldsymbol{q} - \boldsymbol{K}^{-1}\boldsymbol{g} \leq \boldsymbol{k}_9\alpha_1 + \boldsymbol{k}_{10} b \leq \bar{\boldsymbol{\theta}} - \boldsymbol{q} - \boldsymbol{K}^{-1}\boldsymbol{g}$$

$$\underline{\dot{\boldsymbol{\theta}}} \leq \boldsymbol{k}_{11}\beta_7 + \boldsymbol{k}_{12}\beta_8 + \boldsymbol{k}_{13}\beta_9 + \boldsymbol{k}_{14}\beta_{10} \leq \bar{\dot{\boldsymbol{\theta}}}$$

$$\underline{\boldsymbol{\tau}}_{\mathrm{e}} \leq \boldsymbol{k}_{15}\alpha_1 + \boldsymbol{k}_{16} b \leq \bar{\boldsymbol{\tau}}_{\mathrm{e}}$$

$$\underline{\boldsymbol{\tau}}_{\mathrm{m}} \leq \boldsymbol{\tau}_{\mathrm{m}} \leq \bar{\boldsymbol{\tau}}_{\mathrm{m}}$$

for $s \in [0, 1]$.

# Problem Formulation

- Non-convex optimization problem with bilinear and quadratic constriants.
- Efficient convex relaxation for these constraints using McCormick Envelopes.
- Numerical solution using direct transcription.

$$\min_{b(\cdot),\beta_i(\cdot),\alpha_i(\cdot)} \int_0^1 \frac{1}{\sqrt{b}}\,\mathrm{d}s$$

subject to

$$\boldsymbol{\Gamma}\boldsymbol{\tau}_{\mathrm{m}} = \boldsymbol{k}_0\beta_1 + \boldsymbol{k}_1\beta_2 + \beta_3 + \boldsymbol{k}_3\beta_4 + \boldsymbol{k}_4\beta_5 + \boldsymbol{k}_5\alpha_1 + \boldsymbol{k}_6\beta_6 + \boldsymbol{k}_7 b + \boldsymbol{k}_8$$

$$b(0) = 0 \text{ and } b(1) = 0$$

$$\alpha_1(0) = 0 \text{ and } \alpha_1(1) = 0$$

$$\alpha_2(0)\sqrt{b(0)} = 0 \text{ and } \alpha_2(1)\sqrt{b(1)} = 0$$

$$\alpha_1 = b',\ \alpha_2 = b'',\ \alpha_3 = b'''$$

$$\boxed{\begin{array}{l} \beta_1 = \alpha_3 b,\ \beta_2 = \alpha_2\alpha_1,\ \beta_3 = \alpha_2 b,\ \beta_4 = \alpha_1 b \\ \beta_8 = \alpha_2\beta_7,\ \beta_9 = \alpha_1\beta_7,\ \beta_{10} = b\beta_7 \\ \beta_5 = \alpha_1^2, \beta_6 = b^2, \beta_7^2 = b \end{array}}$$

**Bilinear/Quadratic Constraints**

$$0 \le b \le \min_{i\in 1,n} (\bar{\dot{q}}_i)^2 / (q_i')^2$$

$$\underline{\ddot{\boldsymbol{q}}} \le \boldsymbol{q}'' b + \boldsymbol{q}'\alpha_1/2 \le \bar{\ddot{\boldsymbol{q}}}$$

$$\underline{\boldsymbol{\theta}} - \boldsymbol{q} - \boldsymbol{K}^{-1}\boldsymbol{g} \le \boldsymbol{k}_9\alpha_1 + \boldsymbol{k}_{10} b \le \bar{\boldsymbol{\theta}} - \boldsymbol{q} - \boldsymbol{K}^{-1}\boldsymbol{g}$$

$$\underline{\dot{\boldsymbol{\theta}}} \le \boldsymbol{k}_{11}\beta_7 + \boldsymbol{k}_{12}\beta_8 + \boldsymbol{k}_{13}\beta_9 + \boldsymbol{k}_{14}\beta_{10} \le \bar{\dot{\boldsymbol{\theta}}}$$

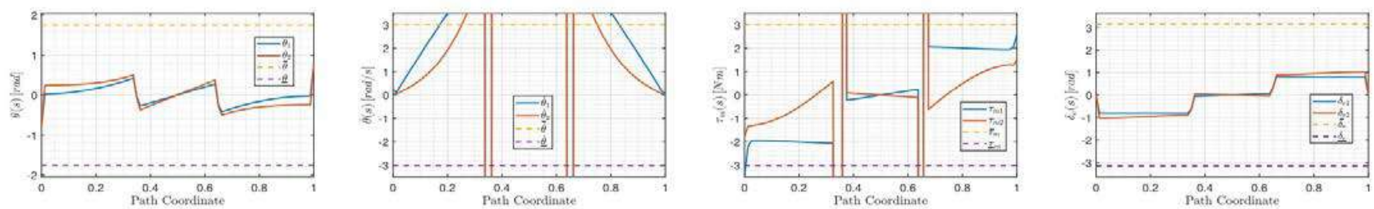
$$\underline{\boldsymbol{\tau}}_{\mathrm{e}} \le \boldsymbol{k}_{15}\alpha_1 + \boldsymbol{k}_{16} b \le \bar{\boldsymbol{\tau}}_{\mathrm{e}}$$

$$\underline{\boldsymbol{\tau}}_{\mathrm{m}} \le \boldsymbol{\tau}_{\mathrm{m}} \le \bar{\boldsymbol{\tau}}_{\mathrm{m}}$$

for $s \in [0, 1]$.

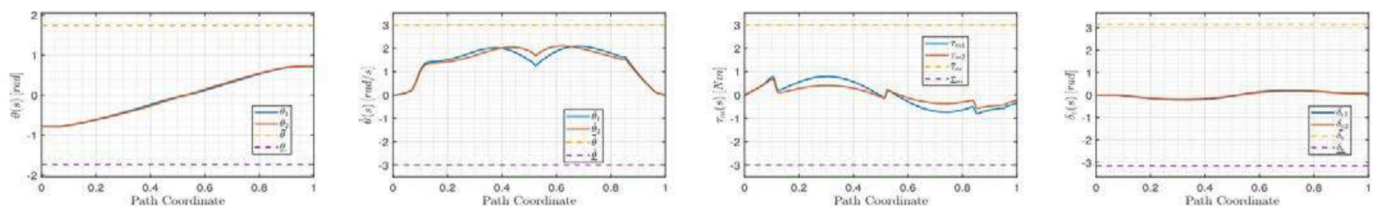
# Planner Output

Rigid



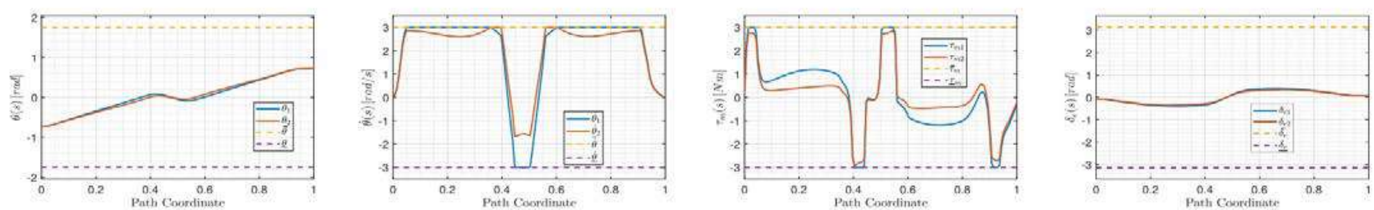

(a) Planned signals using rigid-based technique **R**.

Integrator Chain




(b) Planned signals using snap-limited technique **IC**.

Full Robot Model




(c) Planned signals using our algorithm **ED**.

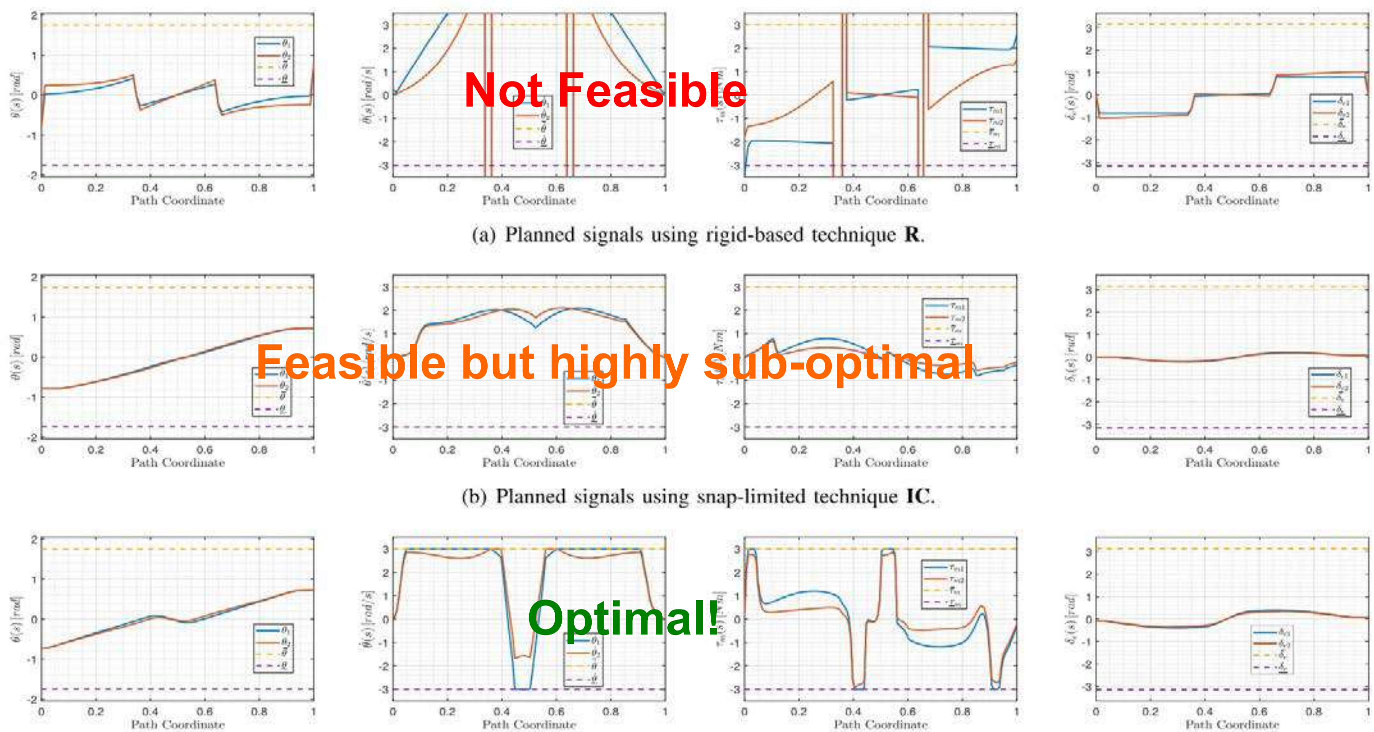
Fig. 3. From left to right: evolution of the planned motor angles, velocities, torques and of the elastic deflections for the three planning techniques **R**, **IC** and **ED**. Solid line is the signal, while the dashed lines represent the lower and upper bounds.

# Planner Output

Rigid

Integrator Chain

Full Robot Model




(a) Planned signals using rigid-based technique **R**.

(b) Planned signals using snap-limited technique **IC**.

(c) Planned signals using our algorithm **ED**.

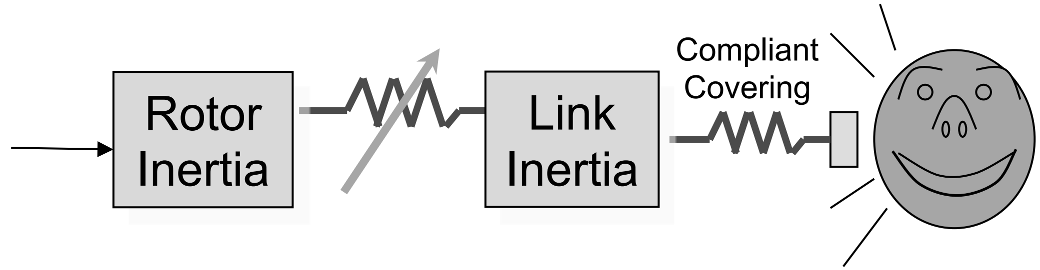
Fig. 3. From left to right: evolution of the planned motor angles, velocities, torques and of the elastic deflections for the three planning techniques **R**, **IC** and **ED**. Solid line is the signal, while the dashed lines represent the lower and upper bounds.

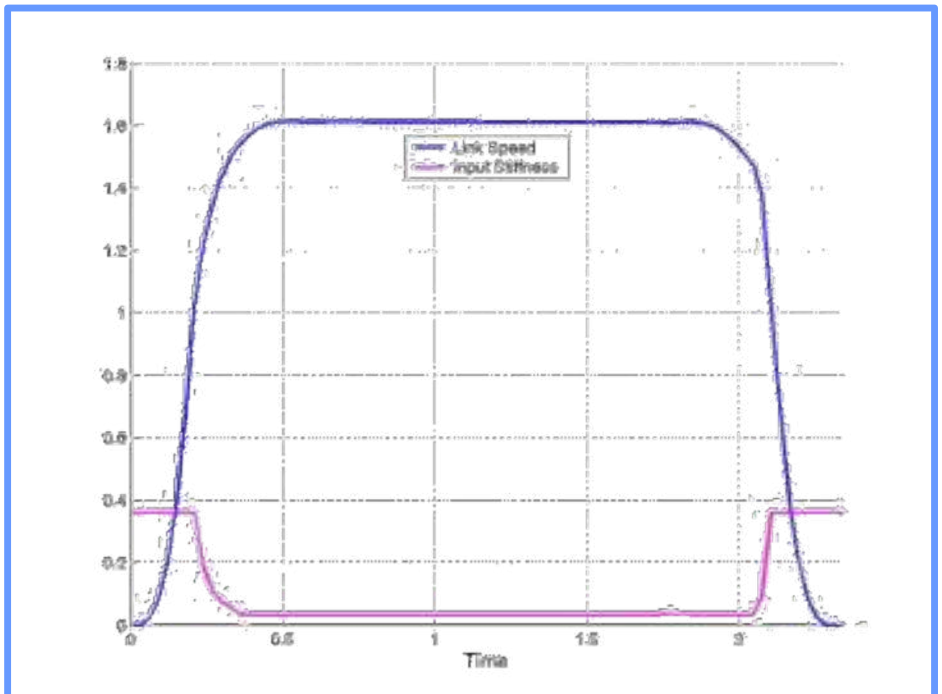
# Minimize Task Time under Safety Constraints

# Variable Stiffnes Actuators

## Optimal control policies

# Safe Brachistochrone




Minimum Time Optimal Control
in a point-to-point task
with constraints on Safety

the safety index adopted as bound is the HIC (Head Injury Criterion)

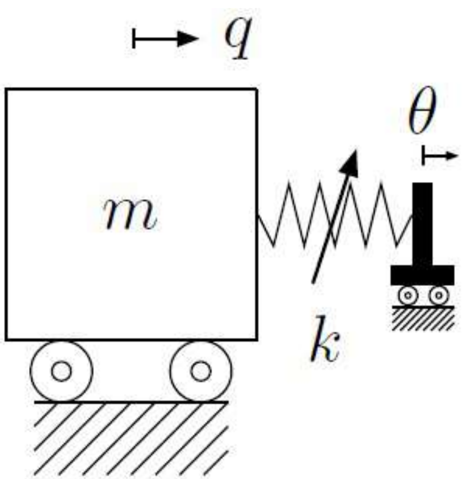
***Fast & Soft,***
***Stiff & Slow***

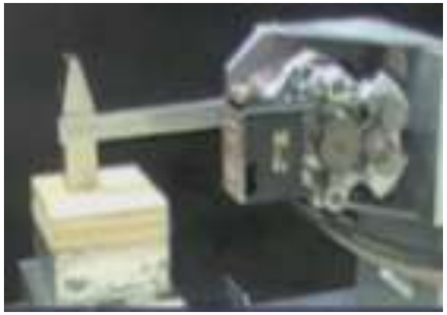
Bicchi and Tonietti, IEEE RAM 2004

# Maximize Peak Speed

# EXAMPLE: HAMMERING A NAIL

# The MaxSpeed Problem
# (The *VSA Hammer*)



$$|u| \leq u_{max}$$

Index $J = \phi(x(T)) = x_2(T) = \dot{q}(T).$

Dynamics $\dot{x} = f(x,u),$

Initial conditions $q(0) = 0$

Terminal const. $\psi(x(T)) = x_1(T) = q(T) = 0$

Hamiltonian $H(x(t),\lambda(t),u(t)) = \lambda^T(t) f(x(t),u(t))$

# PROBLEM STATEMENT- SEA

$$\begin{cases} max \;\; x_2(T) = \dot{q}(T) \\ \dot{x} = \begin{bmatrix} x_2 \\ \omega^2 (u - x_1) \end{bmatrix} \\ |u| \leq u_{max} \end{cases}$$

INDEX: TERMINAL SPEED

DYNAMICS

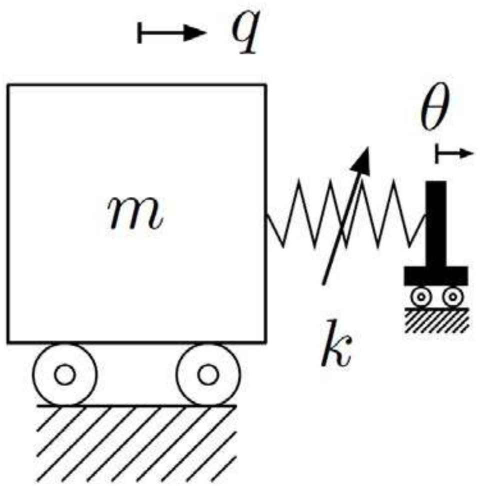
CONTROL LIMIT

INITIAL/TERMINAL CONDITIONS

$$\begin{cases} x(0) = 0 \\ x_1(T) = 0 \end{cases}$$

UNCONSTRAINED TERMINAL TIME

POSITION CONTROL (P)




$\omega = \sqrt{k/m}$ NATURAL FREQUENCY

$x^T = [q \quad \dot{q}]$ STATE

$u = \theta$ CONTROL

# PROBLEM SOLUTION - SEA

POSITION CONTROL (P)

| | |
|---|---|
| Hamiltonian | $H = \lambda_1 x_2 + \lambda_2 \omega^2 (u - x_1)$ |
| Co-state dynamics | $\dot{\lambda}^T = [\omega^2 \lambda_2 \quad -\lambda_1]$ |
| Optimal control law | $u^* = u_{max} \mathrm{sign}(\lambda_2)$ |

BANG-BANG CONTROL

Switching function

$$\lambda_2 = \cos((T-t)\omega) + \frac{\nu \sin((T-t)\omega)}{\omega}$$

$$\nu = -\omega \cot(T\omega)$$

# PROBLEM STATEMENT- SEA

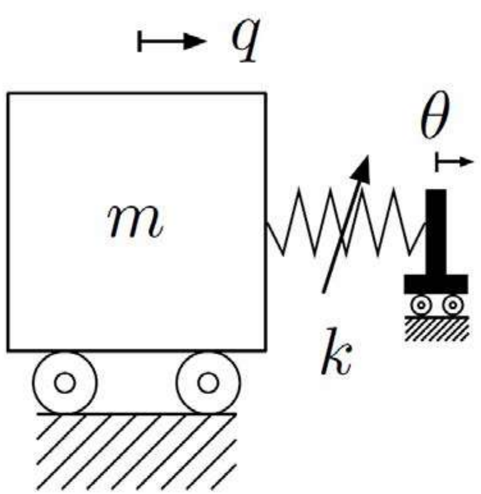
## SPEED CONTROL (S)

$$\begin{cases} max \;\; x_2(T) = \dot{q}(T) \\ \dot{x} = \begin{bmatrix} x_2 \\ \omega^2 (x_3 - x_1) \\ u \end{bmatrix} \\ |u| \leq u_{max} \end{cases}$$

INDEX: TERMINAL SPEED

LINK DYNAMIC

CONTROL LIMIT




INITIAL/TERMINAL CONDITIONS

$$\begin{cases} x(0) = 0 \\ x_1(T) = 0 \end{cases}$$

$\omega = \sqrt{k/m}$ NATURAL FREQUENCY

$x^T = [q \quad \dot{q} \quad \theta]$ STATE

$u = \dot{\theta}$ CONTROL

## PROBLEM STATEMENT- SEA

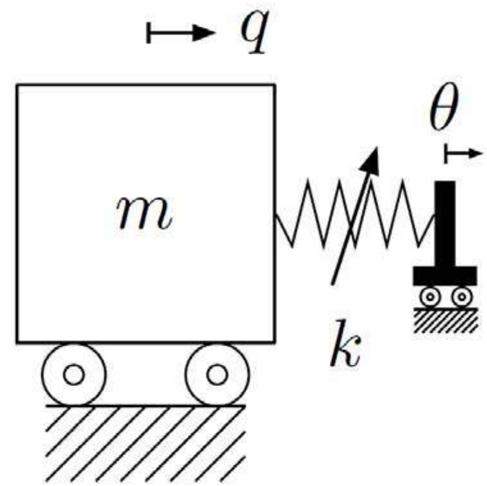
ACCELERATIONCONTROL (A)

$$\begin{cases} max \;\; x_2(T) = \dot{q}(T) \\ \dot{x} = \begin{bmatrix} x_2 \\ \omega^2 (x_3 - x_1) \\ x_4 \\ u \end{bmatrix} \\ |u| \leq u_{max} \end{cases}$$

INDEX: TERMINAL SPEED

LINK DYNAMIC

CONTROL LIMIT




INITIAL/TERMINAL CONDITIONS

$$\begin{cases} x(0) = 0 \\ x_1(T) = 0 \end{cases}$$

$\omega = \sqrt{k/m}$ NATURAL FREQUENCY

$x^T = \begin{bmatrix} q & \dot{q} & \theta & \dot{\theta} \end{bmatrix}$ STATE

$u = \ddot{\theta}$ CONTROL

## PROBLEM SOLUTION - SEA

TERMINAL LINK SPEED

(P) $v_{max} = 2\sqrt{2}u_{max}\omega$ THE STIFFER THE BETTER

(S) $v_{max} = 4u_{max}$ STIFFNESS INDEPENDENT

(A) $v_{max} = 5.74\frac{u_{max}}{\omega}$ THE SOFTER THE BETTER

where $\omega = \sqrt{k/m}$

# PROBLEM STATEMENT- SEA

REALISTIC MODEL (A)+(S)+(P)

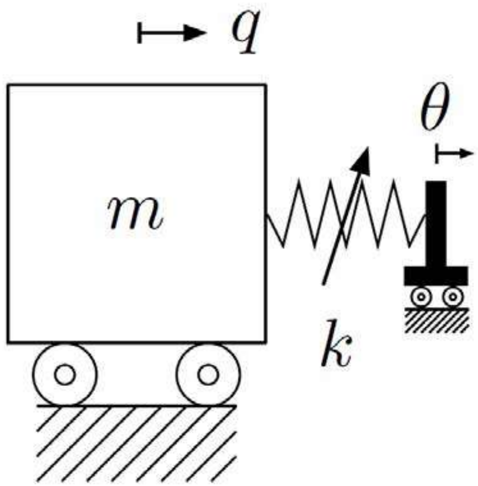
$$\begin{cases} max \;\; x_2(T) = \dot{q}(T) \\ \dot{x} = \begin{bmatrix} x_2 \\ \omega^2 (x_3 - x_1) \\ x_4 \\ u \end{bmatrix} \\ |u| \leq u_{max} \\ |x_3| < \theta_{max} \text{ and } |x_4| < \dot{\theta}_{max} \end{cases}$$

INDEX: TERMINAL SPEED

LINK DYNAMIC

CONTROL LIMIT

STATE CONSTRAINTS




INITIAL/TERMINAL CONDITIONS

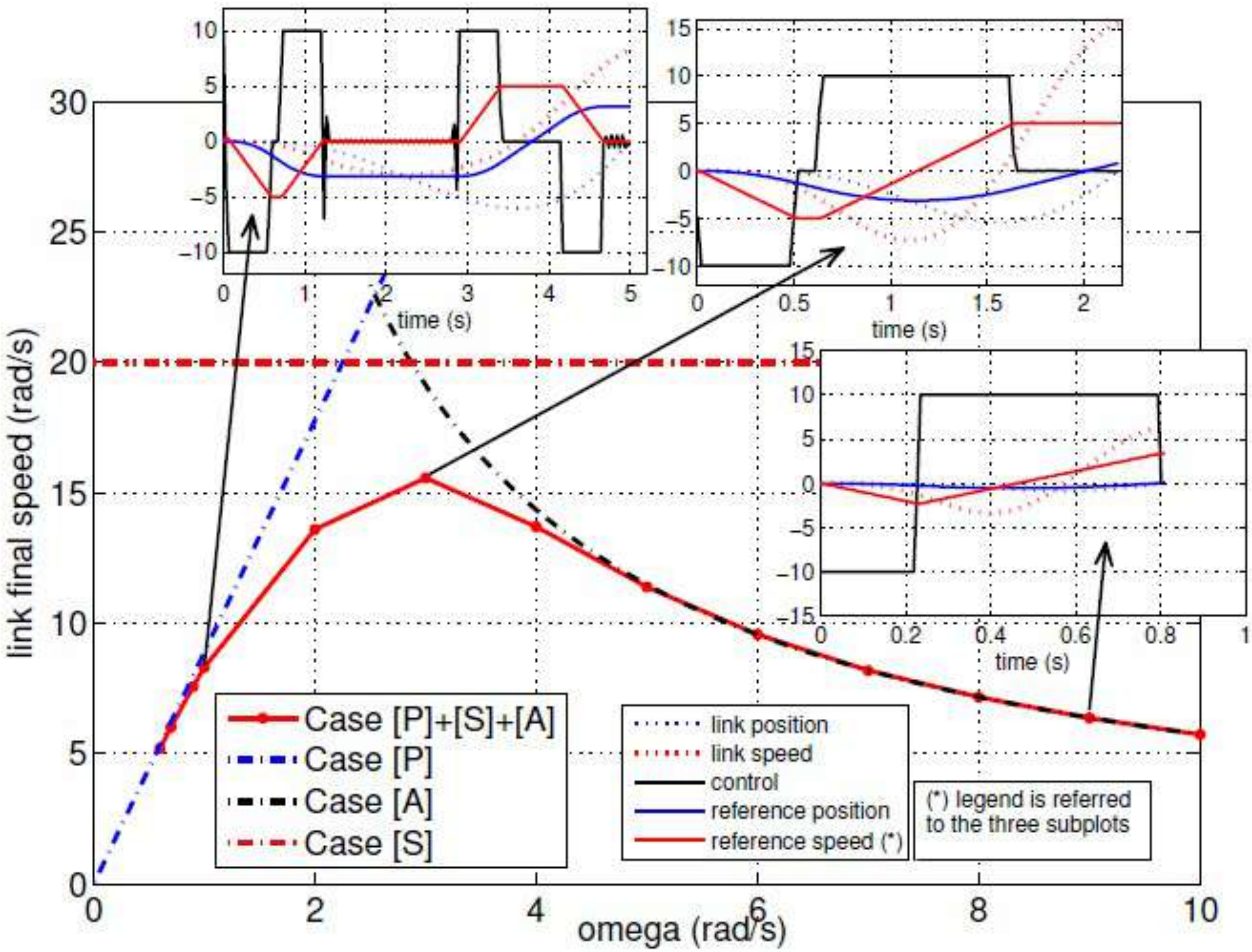
$$\begin{cases} x(0) = 0 \\ x_1(T) = 0 \end{cases}$$

$$\omega = \sqrt{k/m}$$

$$x^T = \begin{bmatrix} q & \dot{q} & \theta & \dot{\theta} \end{bmatrix}$$ STATE

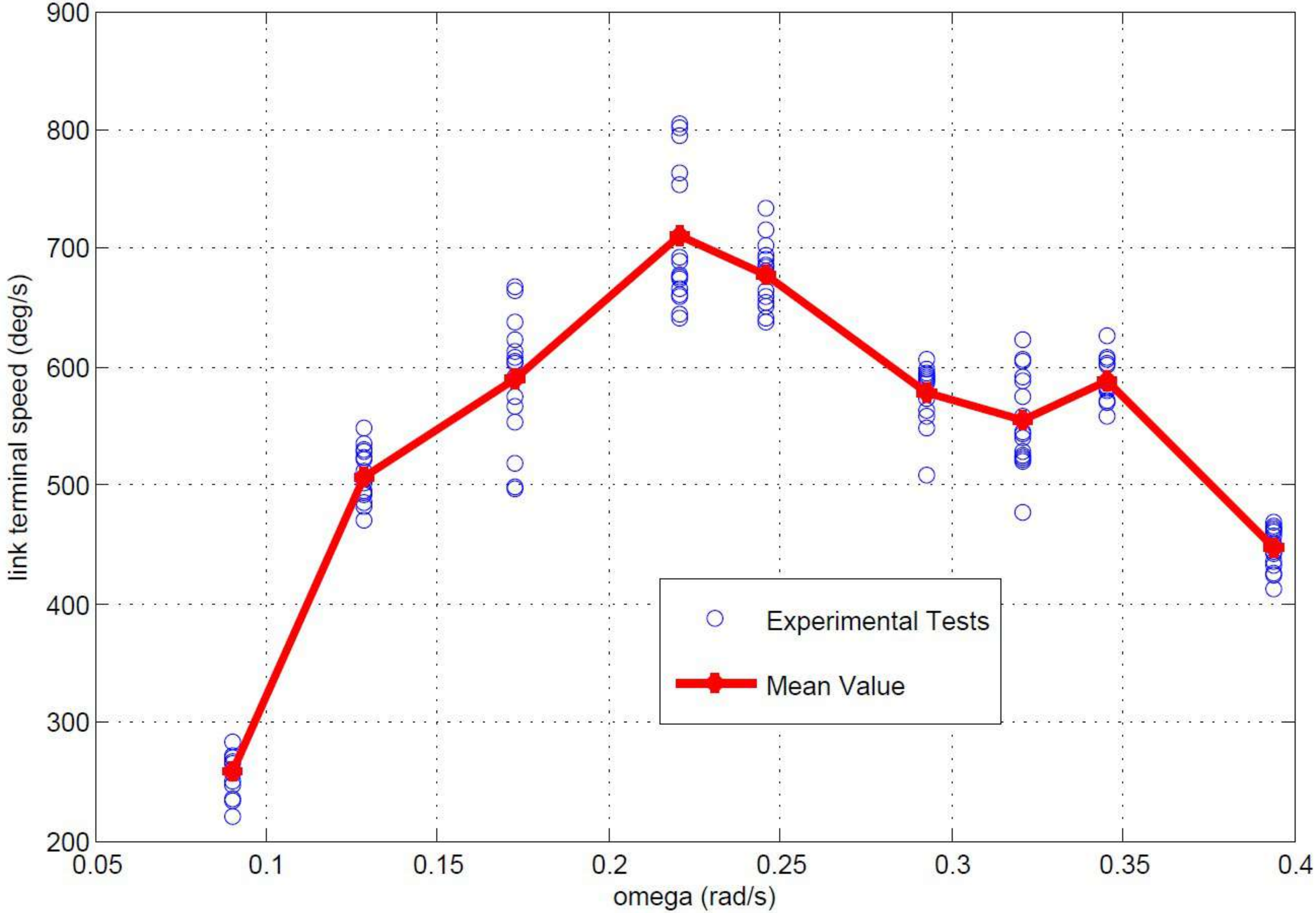
$$u = \ddot{\theta}$$

# SEA - Analytical and Numerical Results

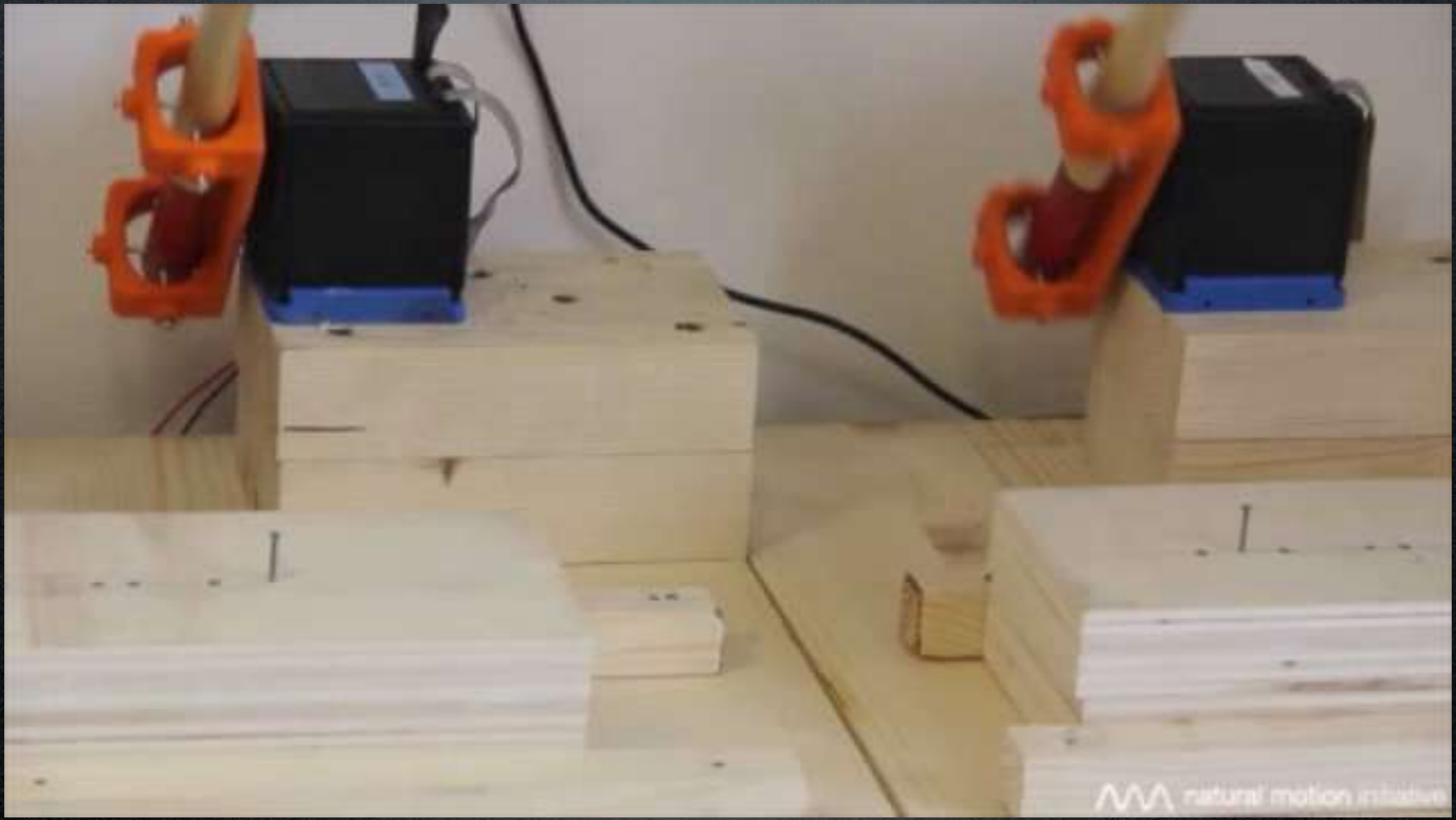



There exist an optimal stiffness that depends on link inertia and motor constraints!

# SEA - Experimental Results

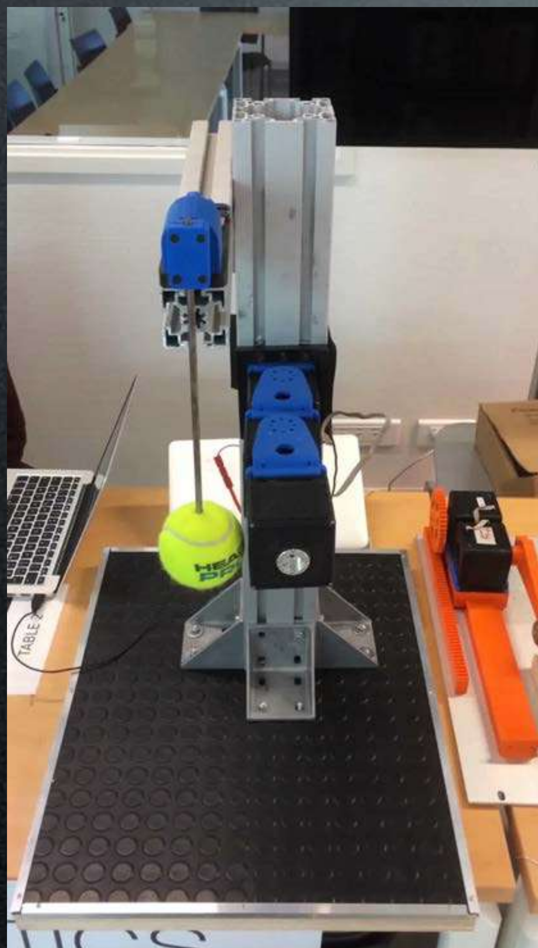


# the *qb* hammer



# 2DoF hitting

Stiff



Soft

# PROBLEM STATEMENT- VSA

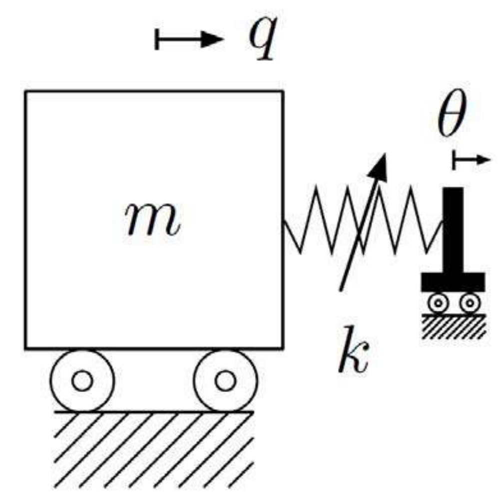
## POSITION AND STIFFNESS CONTROL (P)

$$\begin{cases} max \;\; x_2(T) = \dot{q}(T) \\ \dot{x} = \begin{bmatrix} x_2 \\ \frac{u_2}{m}(u_1 - x_1) \end{bmatrix} \\ u_{min} < u < u_{max} \end{cases}$$

INDEX: TERMINAL SPEED

LINK DYNAMIC

CONTROL LIMITS




INITIAL/TERMINAL CONDITIONS

$$\begin{cases} x(0) = 0 \\ x_1(T) = 0 \end{cases}$$

$x^T = [q \quad \dot{q}]$ STATE

$u^T = [\theta \quad k]$ CONTROLS

UNCONSTRAINED TERMINAL TIME

# PROBLEM SOLUTION - VSA

## POSITION AND STIFFNESS CONTROL

Hamiltonian $$H = \lambda_1 x_2 - \lambda_2 \frac{u_2}{m}(x_1 - u_1)$$

---

Co-State dynamics $$\begin{cases} \dot{\lambda}^T = \begin{bmatrix} \frac{u_2}{m}\lambda_2 & -\lambda_1 \end{bmatrix} \\ \lambda(T)^T = [0 \quad 1] \end{cases}$$

---

Optimal control law

$$u_1^* = \begin{cases} u_{1,max} & \text{if } \lambda_2 > 0 \\ -u_{1,max} & \text{if } \lambda_2 < 0 \end{cases}$$

$$u_2^* = \begin{cases} u_{2,max} & \text{if } \lambda_2 (u_1 - x_1) > 0 \\ u_{2,min} & \text{if } \lambda_2 (u_1 - x_1) < 0 \end{cases}$$

BANG-BANG CONTROL

# VSA – THEORETICAL RESULTS

## POSITION AND STIFFNESS CONTROL

THE OPTIMAL CONTROL POLICY CAN BE SUMMARIZED

$$u_2 = \begin{cases} u_{2,\max} & \text{if } \dot{q}\ddot{q} > 0 \\ u_{2,\min} & \text{if } \dot{q}\ddot{q} < 0 \end{cases}$$

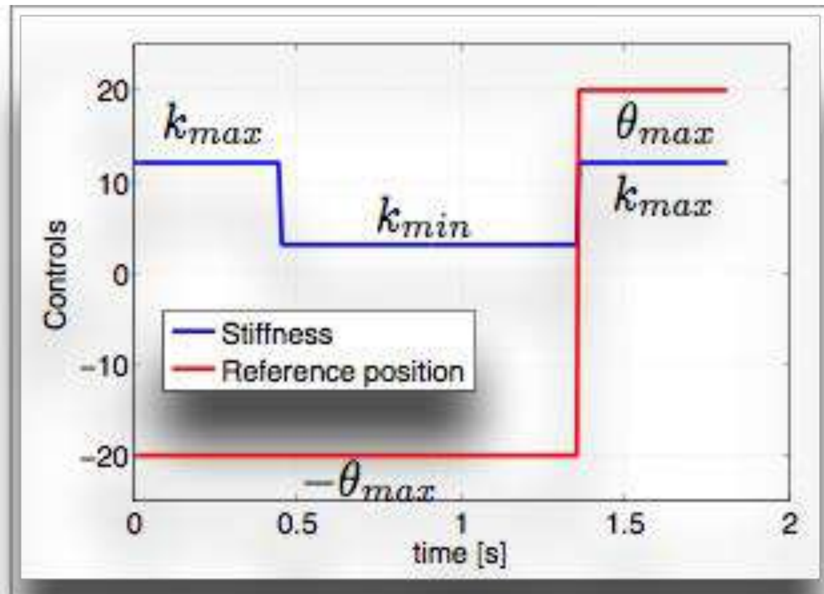
SPEED UP AND STIFF, SLOW DOWN AND SOFT

# VSA – THEORETICAL RESULTS

## POSITION AND STIFFNESS CONTROL

minimum theoretical improvement of VSA w.r.t. SEA

$$\frac{v_{VSA}}{v_{SEA}} = \frac{1 + \sqrt{k_{max}/k_{min}}}{2}$$



# VSA – Experimental Tests

| case | type | inertia | boundary | $\dot{q}(T)$ |
|---|---|---|---|---|
| (a) | SEA-like | $m = 0.1$kg | $q_{D,max}$ | $\simeq 207$deg/s |
| (b) | SEA-like | $m = 0.1$kg | $q_{D,min}$ | $\simeq 242$deg/s |
| (c) | VSA | $m = 0.1$kg | $q_{S,min,max}$ | $\simeq 276$deg/s |
| (d) | SEA-like | $m = 1$kg | $q_{D,max}$ | $\simeq 341$deg/s |
| (e) | SEA-like | $m = 1$kg | $q_{D,mid}$ | $\simeq 374$deg/s |
| (f) | VSA | $m = 1$kg | $q_{S,min,max}$ | $\simeq 499$deg/s |

TABLE III

FIRST AND SECOND COLUMNS DEAL WITH THE SIX EXPERIMENTS' CASES. THE THIRD AND THE FOURTH SHOW THE FIXED PARAMETERS. FINALLY, IN THE FIFTH COLUMN ARE SHOWN THE MEASURED VALUES OF LINK SPEED AT HIT TIME

For a proper inertia improvement of 30%

# MAXIMUM FINAL SPEED ONE POSITION SWITCHING

## Fixed Terminal Time

EXAMPLE: First Time Kick

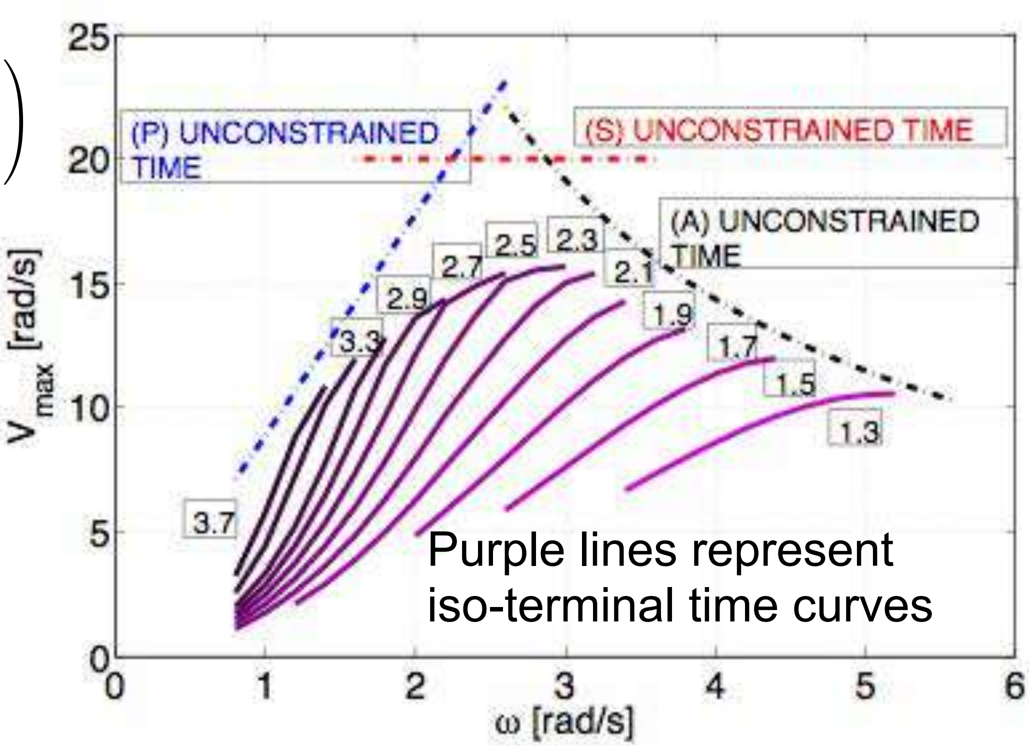
# SEA - Analytical and Numerical Results

Position Control Max Speed

$$v_{max} = u_{max}\omega \left( 2\sqrt{1-\cos\left(\frac{T\omega}{2}\right)}^{4} - \sin\left(T\omega\right) \right)$$

Analytical solution for (P), (S), and (A) cases

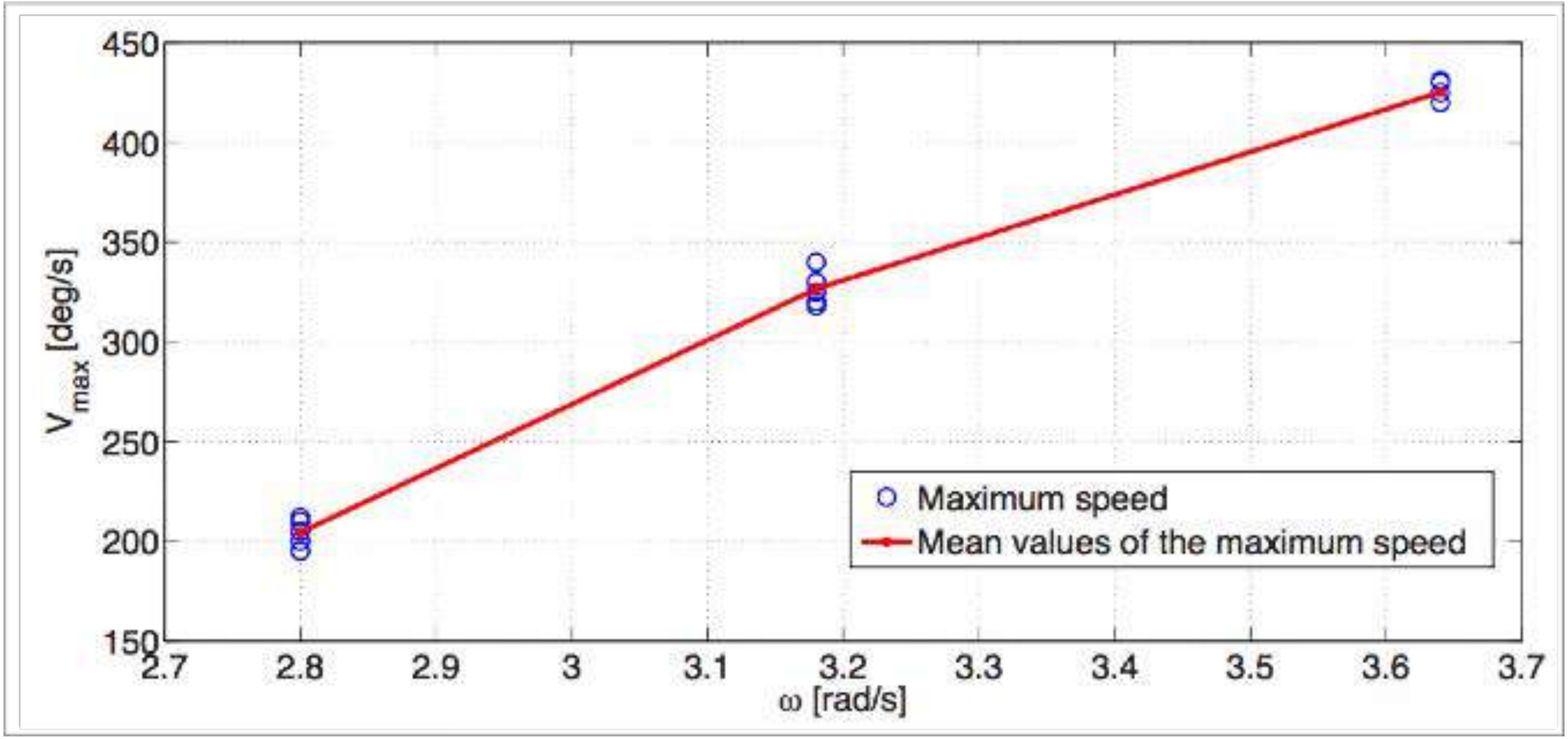
Numerical solution for (P) + (S) + (A) case




There exist an **optimal stiffness** that depends on:

- link inertia
- terminal time (opt. stiffness increases if T decreases)
- motor constraints

# SEA - Experimental Results




Experimental results for a SEA prototype with different stiffness at same terminal time

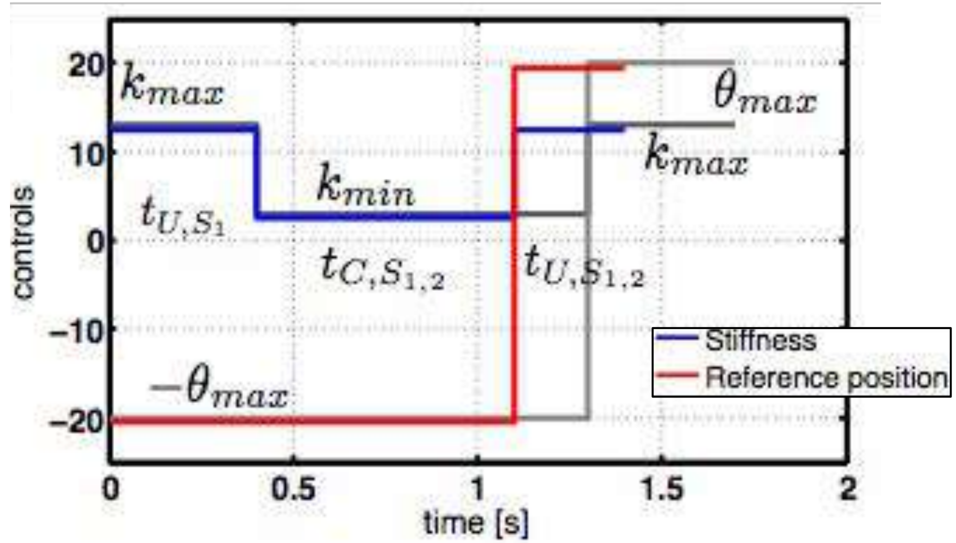
# VSA – Theoretical Results

Fixed terminal time – Position and Stiffness control

How does the optimal control policy change w.r.t. the free terminal time case?

Free terminal time switching intervals

$$t_{U,S_1} = \pi/2\sqrt{m/k_{max}}$$
$$t_{U,S_{1,2}} = \pi/2\sqrt{m/k_{min}}$$




Fixed terminal time switching intervals

$$t_{C,S_1} = t_{U,S_1}$$
$$t_{C,S_{1,2}} \leq t_{U,S_{1,2}}$$

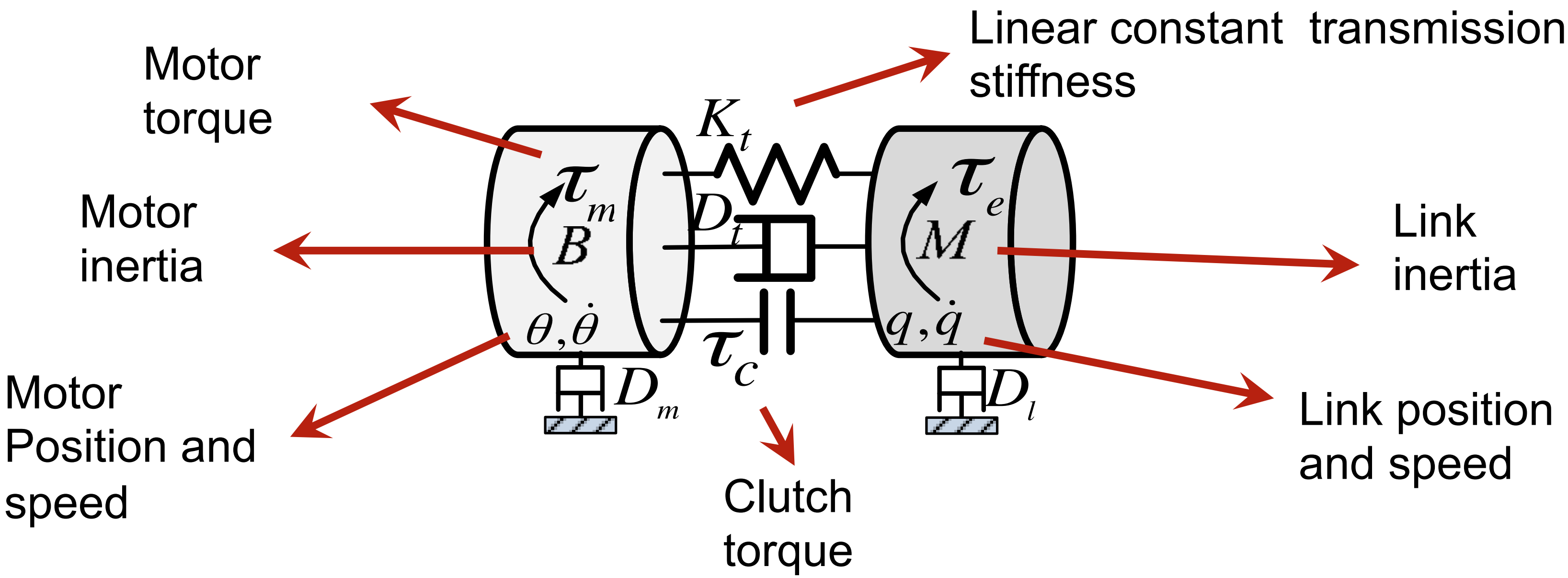
Terminal time threshold below which VSA is not convenient

$$T_t = \pi\sqrt{m/k_{max}} \quad \begin{cases} T > T_t \rightarrow \text{VSA} \\ T \leq T_t \rightarrow \text{SEA} \end{cases}$$

# MaxSpeed problem for compliant Variable Damping Actuators

M. Laffranchi, N. Tsagarakis, and D. G. Caldwell, "A compact compliant actuator (CompActTM) with variable physical damping," ICRA, 2011

# Compact™ Actuator
## The model




$$M\ddot{q} + D_l\dot{q} + D_t(\dot{q} - \dot{\theta}) + K_t(q - \theta) + \tau_c = \tau_e$$
$$B\ddot{\theta} + D_m\dot{\theta} - D_t(\dot{q} - \dot{\theta}) - K_t(q - \theta) - \tau_c = \tau_m$$

L. Chen, **M. Garabini**, M. Laffranchi, N. Kashiri, N. Tsagarakis, A. Bicchi, and D. Caldwell**. Optimal Control for Maximizing Velocity of the CompAct Compliant Actuator**. In International Conference of Robotics and Automation - **ICRA 2013**

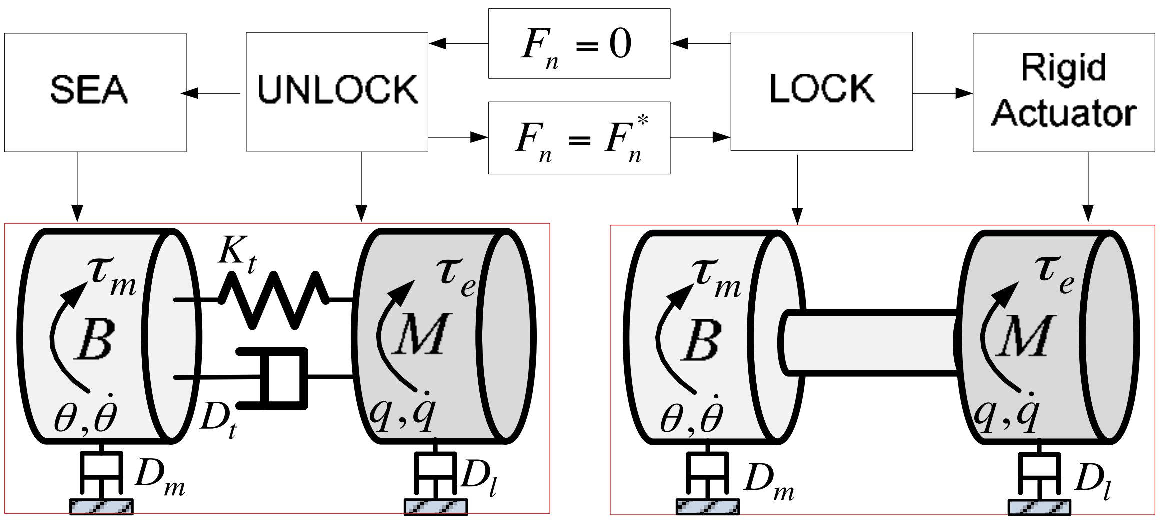
# MaxSpeed for the Compact™ Actuator

## Problem Solution: First Step

**From**
**Maximum Pontryagin Principle**

$$\tau_c^* = \begin{cases} \tau_{c,max} \text{ if } f(x,\lambda) > 0 \\ \tau_{c,min} \text{ if } f(x,\lambda) < 0 \end{cases}$$

*f*: switching function depending on state and co-state



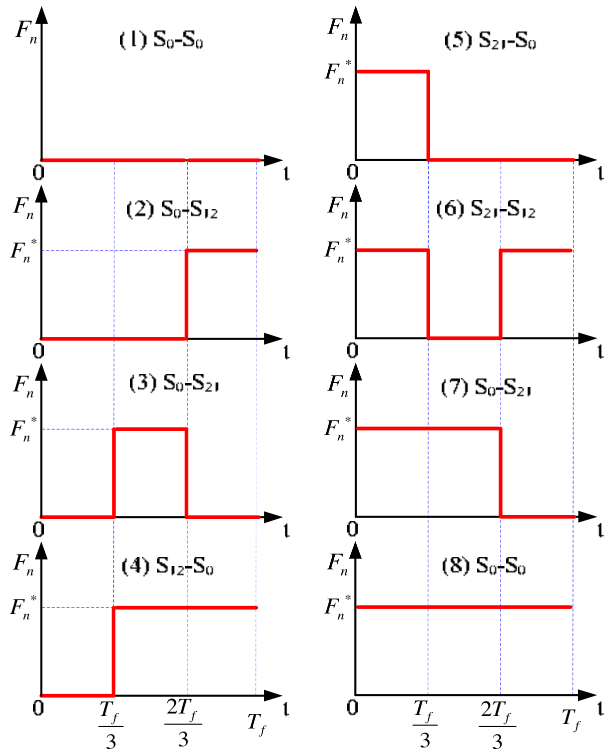

Two dynamic systems have to be considered in the following with one control input: the motor torque


L. Chen, **M. Garabini**, M. Laffranchi, N. Kashiri, N. Tsagarakis, A. Bicchi, and D. Caldwell**. Optimal Control for Maximizing Velocity of the CompAct Compliant Actuator**. In International Conference of Robotics and Automation - **ICRA 2013**

# MaxSpeed for the Compact™ Actuator
## Problem Solution: Second Step

The optimal control problem can be translated in several convex optimization problems

**Given a maximum number of switchings we derive all possible the SEA / RIGID sequences**




Possible sequences of the clutch force for a maximum number of switching of 3

L. Chen, **M. Garabini**, M. Laffranchi, N. Kashiri, N. Tsagarakis, A. Bicchi, and D. Caldwell**. Optimal Control for Maximizing Velocity of the CompAct Compliant Actuator**. In International Conference of Robotics and Automation - **ICRA 2013**

# MaxSpeed for the Compact$^{TM}$ Actuator

## Problem Solution: Second Step

The optimal control problem can be translated in several convex optimization problems

**For each sequence we can derive the system dynamic for the whole task**

| *Switching Group* \ *Condition & Time* | *Operating Conditions* | | |
|---|---|---|---|
| | *$0 < t < T_f/3$* | *$T_f/3 < t < 2T_f/3$* | *$2T_f/3 < t < T_f$* |
| (1) $S_0$-$S_0$ | $C_1$ | $C_1$ | $C_1$ |
| (2) $S_0$-$S_{12}$ | $C_1$ | $C_1$ | $C_2$ |
| (3) $S_0$-$S_{21}$ | $C_1$ | $C_2$ | $C_1$ |
| (4) $S_{12}$-$S_0$ | $C_1$ | $C_2$ | $C_2$ |
| (5) $S_{21}$-$S_0$ | $C_2$ | $C_1$ | $C_1$ |
| (6) $S_{21}$-$S_{12}$ | $C_2$ | $C_1$ | $C_2$ |
| (7) $S_0$-$S_{21}$ | $C_2$ | $C_2$ | $C_1$ |
| (8) $S_0$-$S_0$ | $C_2$ | $C_2$ | $C_2$ |

$$\boldsymbol{x}^T = [q \quad \theta \quad \dot{q} \quad \dot{\theta}]$$

$$\boldsymbol{x}(n) = \left(\boldsymbol{I} + \Delta t \cdot \boldsymbol{A}\right) \cdot \boldsymbol{Q} \cdot \boldsymbol{x}(n-1) + \Delta t \cdot \boldsymbol{B} \cdot u(n)$$

$$C_1 \to C_2 : Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{M}{M+B} & \frac{B}{M+B} \\ 0 & 0 & \frac{M}{M+B} & \frac{B}{M+B} \end{bmatrix} \quad \boldsymbol{A} = \boldsymbol{A_2} \quad \boldsymbol{B} = \boldsymbol{B_2}$$

$$C_1 \to C_1 \, and \, C_2 \to C_1 : \boldsymbol{Q} = \boldsymbol{I} \quad \boldsymbol{A} = \boldsymbol{A_1} \quad \boldsymbol{B} = \boldsymbol{B_1}$$

$$C_2 \to C_2 : \boldsymbol{Q} = \boldsymbol{I} \quad \boldsymbol{A} = \boldsymbol{A_2} \quad \boldsymbol{B} = \boldsymbol{B_2}$$

L. Chen, **M. Garabini**, M. Laffranchi, N. Kashiri, N. Tsagarakis, A. Bicchi, and D. Caldwell**. Optimal Control for Maximizing Velocity of the CompAct Compliant Actuator**. In International Conference of Robotics and Automation - **ICRA 2013**

# MaxSpeed for the Compact™ Actuator

## Problem Solution: Second Step

The optimal control problem can be translated in several convex optimization problems

**Since the dynamic is linear and constraints are convex the Optimal Control Problem (for each switching sequence) can be converted into a Convex Optimization Problem**

$$J = f_3\left(\boldsymbol{x}(0), u, \Delta t\right) = \dot{q}(N)$$

subject to

$$q(N) = q_f, \theta(N) = \theta_f$$

Terminal Constraints

$$\dot{\theta}_{\min} \leq \dot{\theta}(n) \leq \dot{\theta}_{\max}, \ \theta_t(n) \in \left[\theta_{t\min}, \theta_{t\max}\right]$$

Motor speed lmits

$$u(n) \in \left[-\tau_{m,\max}, \tau_{m,\max}\right], n = 1, \cdots, N$$

Motor torque lmits


L. Chen, **M. Garabini**, M. Laffranchi, N. Kashiri, N. Tsagarakis, A. Bicchi, and D. Caldwell**. Optimal Control for Maximizing Velocity of the CompAct Compliant Actuator**. In International Conference of Robotics and Automation - **ICRA 2013**

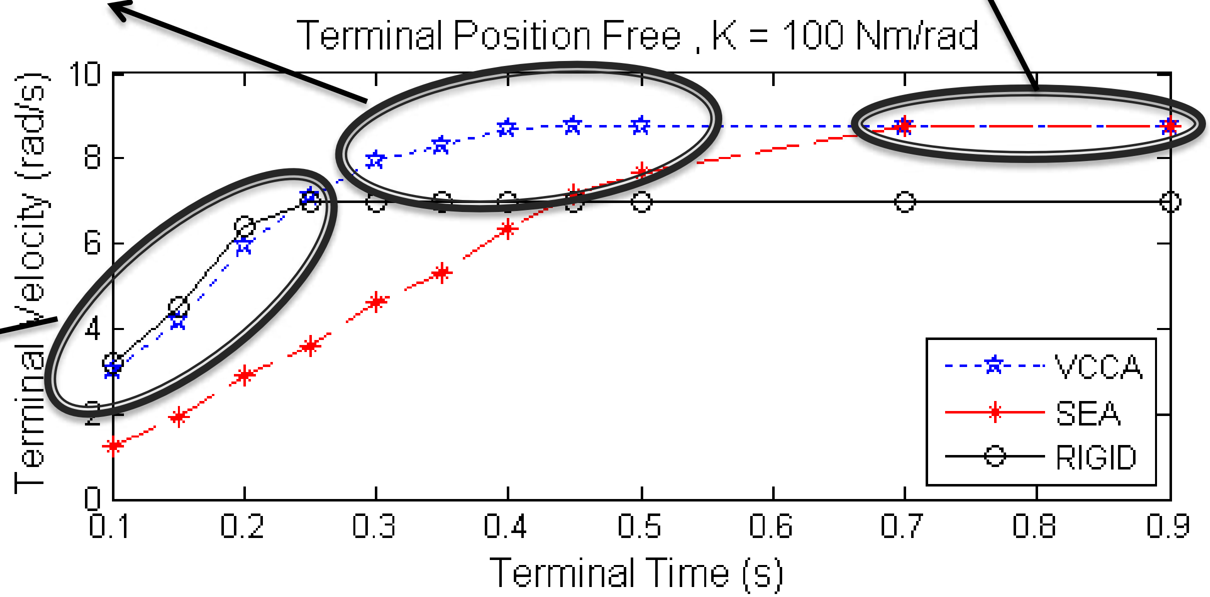
# MaxSpeed for the Compact™ Actuator
## Main Simulation Results

**The maximum link speed obtainable with the Compact™ (VCCA) is larger or equal than the speed obtainable with both rigid and SEA**

Intermediate terminal times Compact better than rigid and SEA

Large terminal times Compact as good as the SEA

Short terminal times Compact as good as the rigid actuator




L. Chen, **M. Garabini**, M. Laffranchi, N. Kashiri, N. Tsagarakis, A. Bicchi, and D. Caldwell**. Optimal Control for Maximizing Velocity of the CompAct Compliant Actuator**. In International Conference of Robotics and Automation - **ICRA 2013**

# Maximize Jumping Height

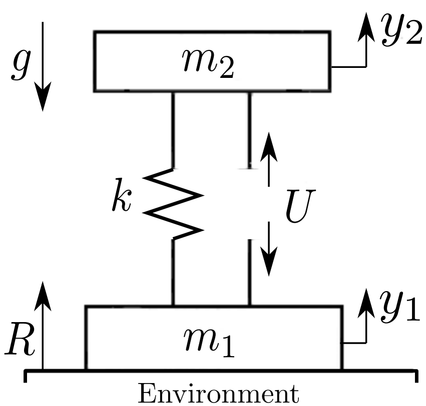
R. Incaini, L. Sestini, **M. Garabini**, M. Catalano, G. Grioli, and A. Bicchi**. Optimal Control and Design Guidelines for Soft Jumping Robots: Series Elastic Actuation and Parallel Elastic Actuation in comparison**. **ICRA 2013**

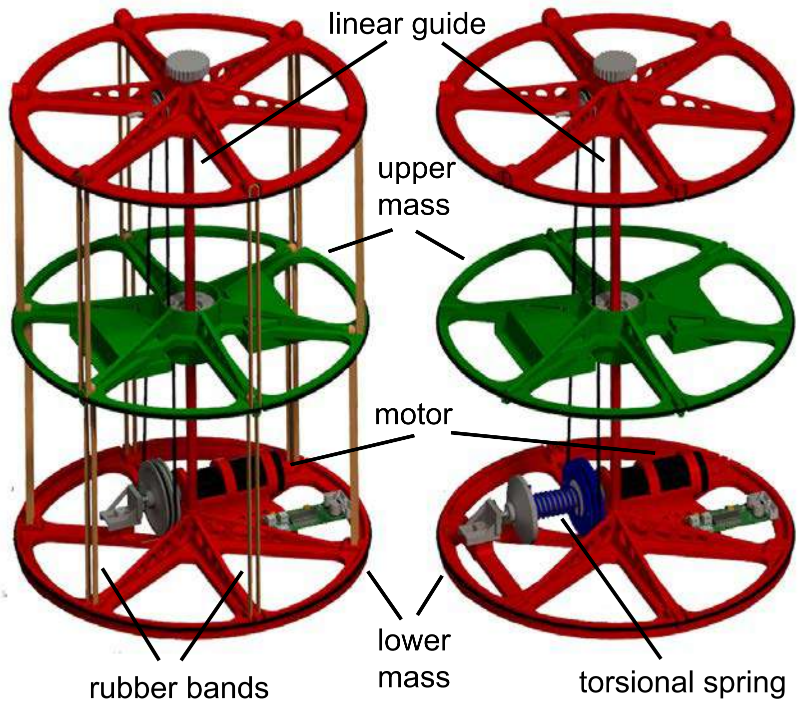
# Maximize Jumping Height

## SEA and PEA in comparison

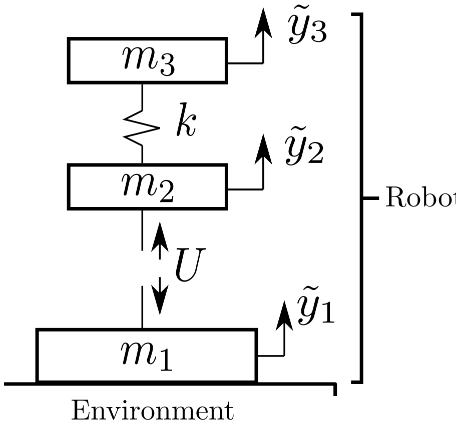
- Optimal control as a mean for comparing two different soft actuator designs when performing the jumping task

**Parallel Elastic Actuation (PEA)**




**Series Elastic Actuation (SEA)**





R. Incaini, L. Sestini, **M. Garabini**, M. Catalano, G. Grioli, and A. Bicchi. **Optimal Control and Design Guidelines for Soft Jumping Robots: Series Elastic Actuation and Parallel Elastic Actuation in comparison. ICRA 2013**

# Maximize Jumping Height

## It can be translated into the MaxSpeed problem

**Target**

We want to maximize the highest reachable point of the center of gravity of the robot

$$\max \quad h_{G,max}$$

Our optimization variable is the force exerted by the motor $U$.
Using the conservation of mechanical energy principle, we derive an estimation of $h_{G,max}$ depending on the state of the robot soon after the take-off instant $T$

$$h_{G,max} = h_G(T^+) + \frac{V_G(T^+)^2}{2g}$$

where $h_G(T^+)$ and $V_G(T^+)$ are the height and the speed, respectively, of the center of gravity of the robot soon after the take-off instant.

We can assume that the variation of $h_G(t)$ during the stance phase is negligible if compared to the variation of $V_G(t)^2/(2g)$:

$$\max \quad h_{G,max} \approx \max \quad V_G(T^+) = \begin{cases} \frac{m_2}{m_1+m_2} z_2(T^-) & \text{(PEA)} \\ \\ \frac{m_3}{m_1+m_2+m_3} \tilde{z}_4(T^-) & \text{(SEA)} \end{cases}$$

# Maximize Jumping Height

## Problem Statement

More realistic models taking into account path state and control constraints

The input *u* is the motor torque

The robot has to jump at the time T (contact constraints)

Bi-linear Torque-Speed motor characteristic

The upper mass must not touch the ground (spring deflection constraints)

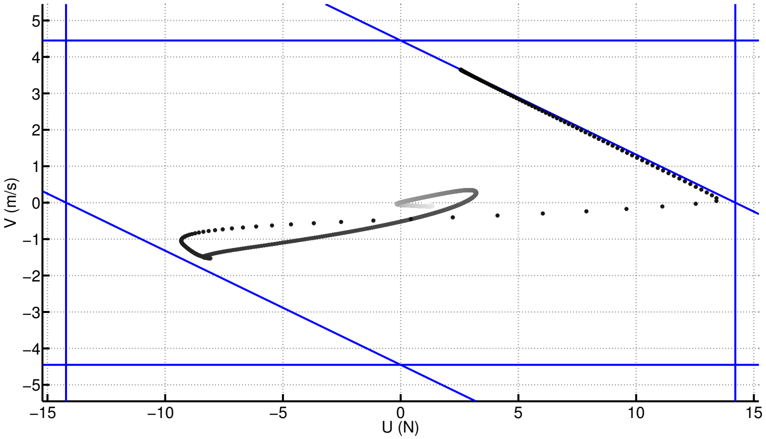
| **PEA** | **SEA** | |
|---|---|---|
| maximize $z_2(T)$ | maximize $\widetilde{z}_4(T)$ | (a) |
| subject to | subject to | |
| $\dot{z}(t) = Az(t) + Bu(t)$ | $\dot{\widetilde{z}}(t) = \widetilde{A}\widetilde{z}(t) + \widetilde{B}u(t)$ | (b) |
| $-kz_1(t) + U(t) + m_1 g > 0$ | $U(t) + m_1 g > 0$ | (c) |
| $-kz_1(T) + U(T) + m_1 g = 0$ | $U(T) + m_1 g = 0$ | (d) |
| $U_{min} \leq U(t) \leq U_{max}$ | $U_{min} \leq U(t) \leq U_{max}$ | (e) |
| $V_{min} \leq z_2(t) \leq V_{max}$ | $V_{min} \leq \widetilde{z}_3(t) \leq V_{max}$ | (f) |
| $V_{min} - \frac{V_{max}}{U_{max}} U(t) \leq z_2(t)$ | $V_{min} - \frac{V_{max}}{U_{max}} U(t) \leq \widetilde{z}_3(t)$ | (g) |
| $z_2(t) \leq V_{max} - \frac{V_{max}}{U_{max}} U(t)$ | $\widetilde{z}_3(t) \leq V_{max} - \frac{V_{max}}{U_{max}} U(t)$ | (h) |
| $\delta_{min} \leq z_1(t) \leq \delta_{max}$ | $\delta_{min} \leq \widetilde{z}_2(t) \leq \delta_{max}$ | (i) |
| | $\phi_{min} \leq \widetilde{z}_2(t) - \widetilde{z}_1(t) \leq \phi_{max}$ | (l) |

# Maximize Jumping Height

## Results

**Since the dynamic is linear and constraints are convex the Optimal Control Problem can be converted into a Convex Optimization Problem**

A typical evolution of the working point of the motor subject to the considered Torque-Speed constraint




**Also in a more realistic context there exists an optimal stiffness value (for both PEA and SEA) that maximize the peak speed and it depends on the task and robot parameters:**

- **Terminal time**
- **Inertia**
- **Motor Characteristics (Max Torque, Max Speed, Gear Ratio)**
- **Deflection Constraints**

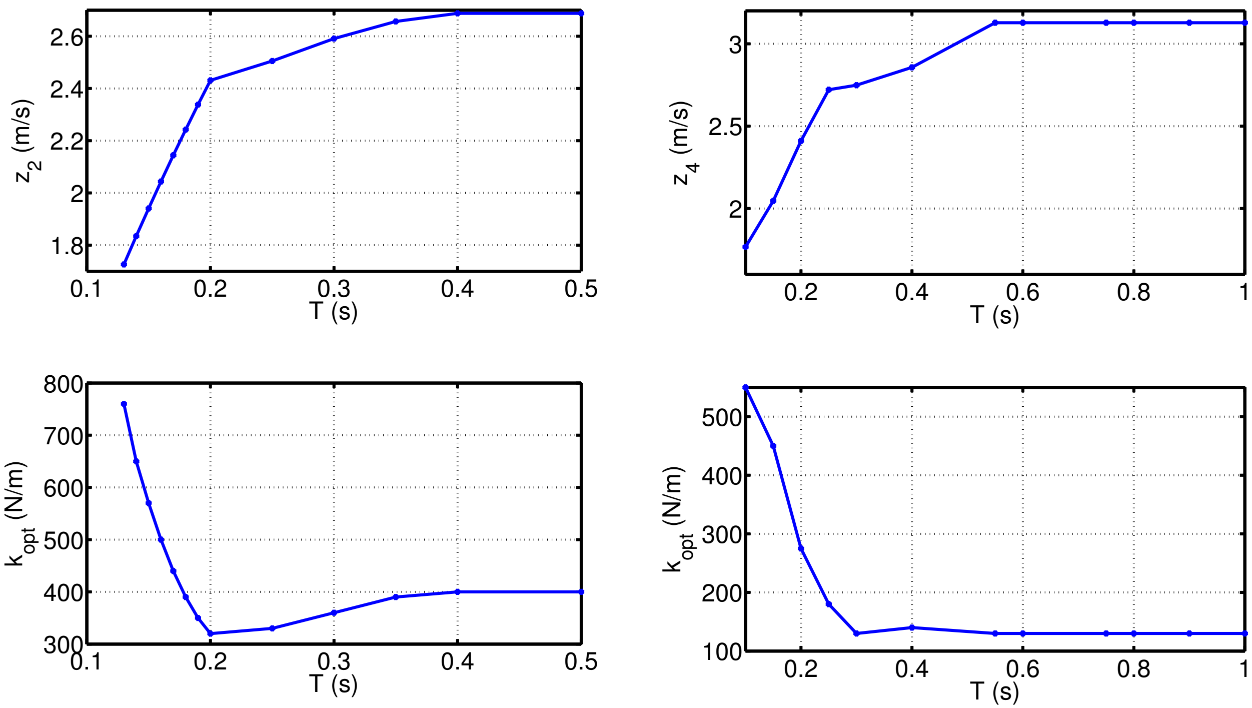
# Maximize Jumping Height
## Optimal Stiffness and Terminal Time

the performance and the optimal stiffness depend on the terminal time T:

- The speed increases with T until a limit
- The optimal stiffness globally decreases with T

Max Speed

Opt. k




PEA

SEA

The best performance:

- Max Speed SEA 20% higher than PEA
- Max Speed SEA 50% higher than rgid

# Optimal Control For Soft Robots

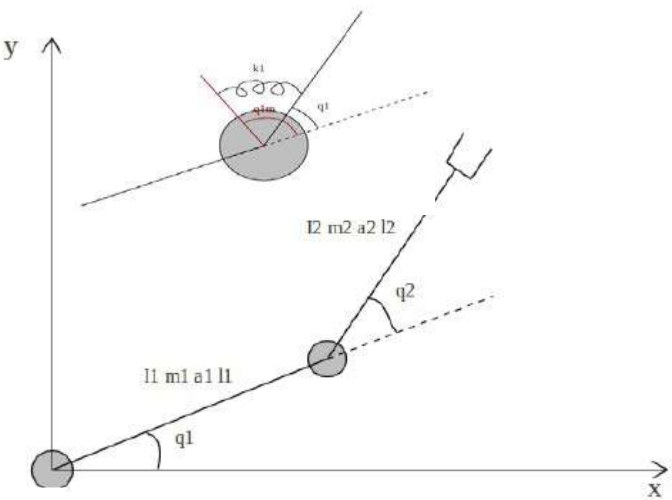
## Maximize peak speed for a multi-DoF soft robot



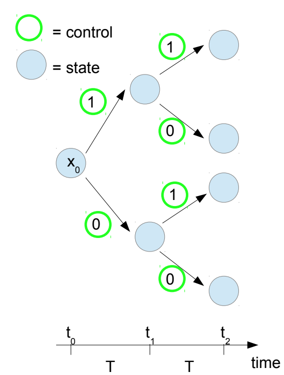
- PROBLEM
  - Maximize peak speed at a given terminal time and given terminal position of the end-effector of a multi-dof soft robot

$$V_X(q,\dot q) = -a_1 \dot q_1^{MAX} \sin(q_1) - a_2(\dot q_1^{MAX} + \dot q_2^{MAX}) \sin(q_1+q_2)$$
$$V_Y(q,\dot q) = a_1 \dot q_1^{MAX} \cos(q_1) + a_2(\dot q_1^{MAX} + \dot q_2^{MAX}) \cos(q_1+q_2)$$

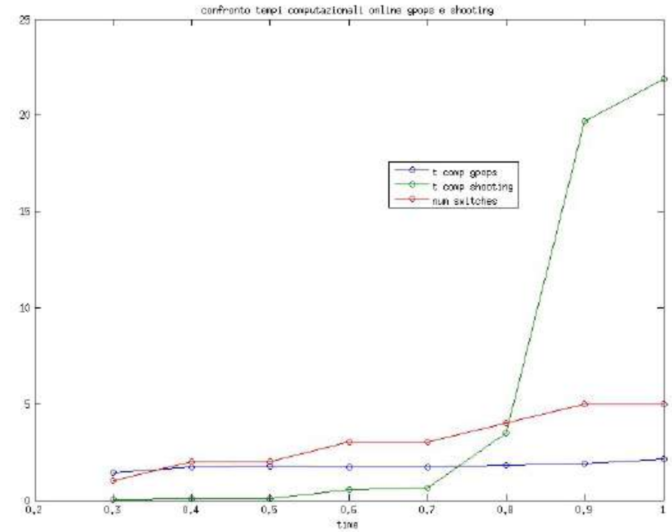
$$V_{EE}(q,\dot q) = -V_X \sin(q_1+q_2) + V_Y \cos(q_1+q_2)$$

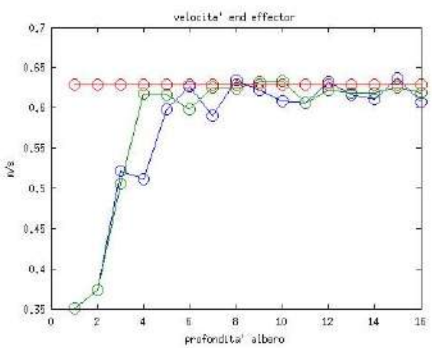


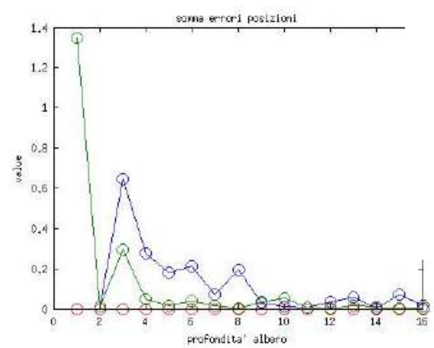

- SOLUTION METHOD KEY IDEAS
  - Exploiting Bang-Bang control solution coming from classical OC theory
  - 2-stage optimization of switching time
    - Off-line rough map between switching instants and terminal time and position
    - On-line local optimization of the switching instants for given terminal time and position

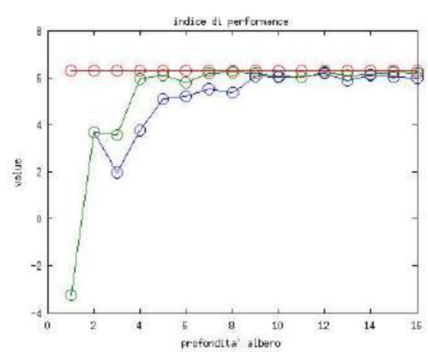


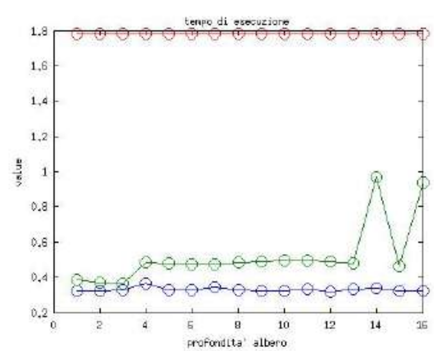
# Numerical Tests on the “goodness” of the solution



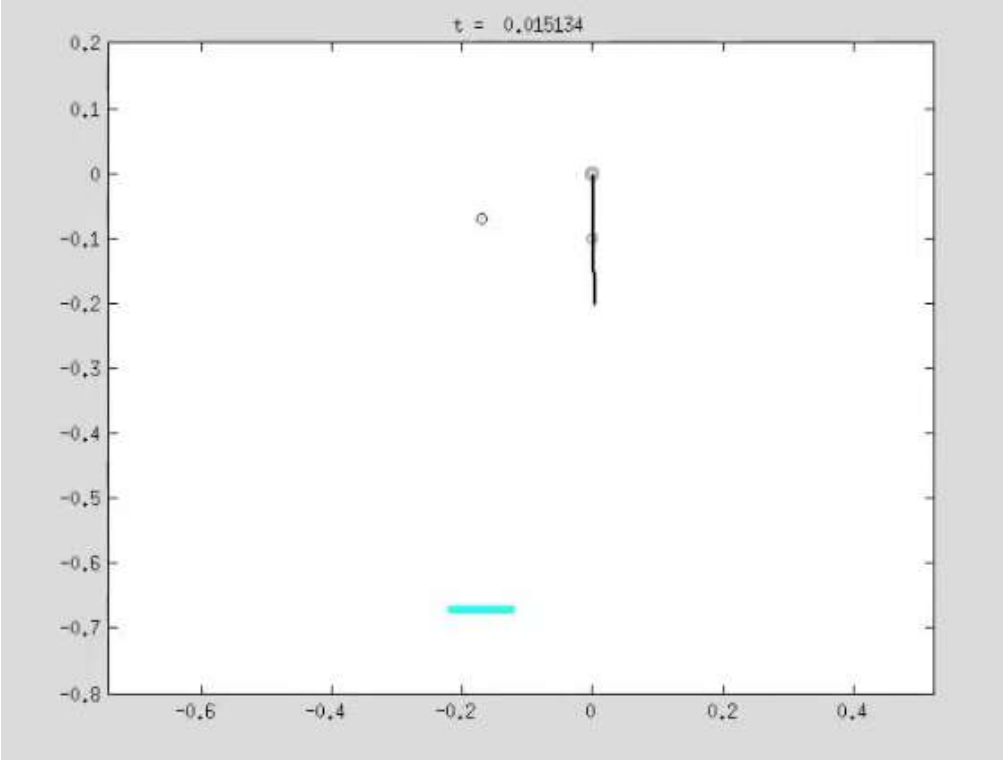



# Optimal Control For Soft Robots

## Maximize peak speed for a multi-DoF soft robot

Link length: 10 cm
Target distance: 60 cm
Target width: 10 cm




Terminal time errors
Max: 0.95 ms
Min: 0.26 ms
Mean: 0.62 ms

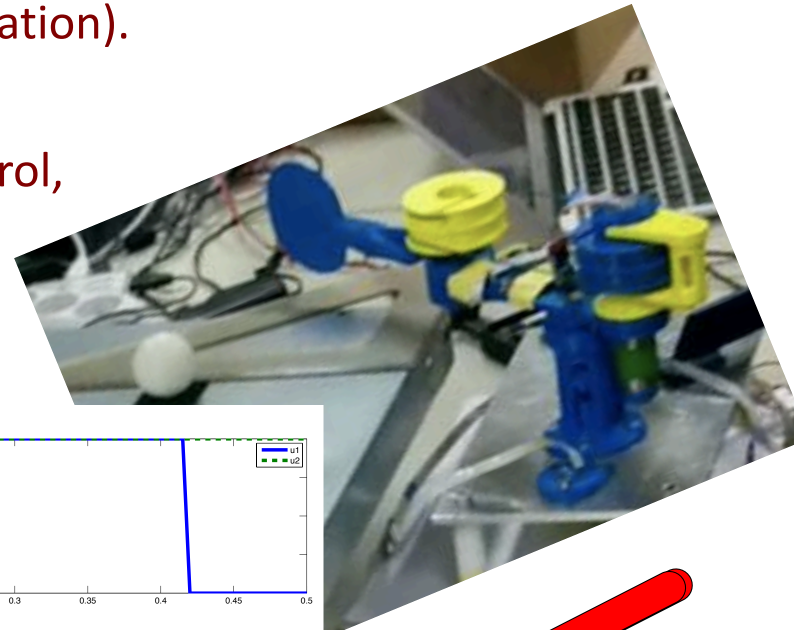
Target error
Max: 8.4 mm
Min: 0.1 mm
Mean: 3.2 mm

# Real-Time Optimal Control for Soft Robots

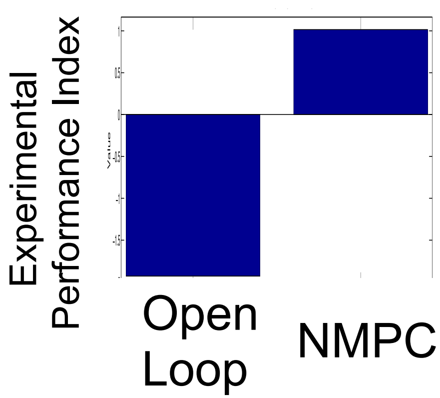
## Maximize Speed at give terminal time and position for a multi-DOF robot

- Real-Time optimal control for a two-dof soft robotic arm is achieved by exploiting Bang-Bang optimal control solution coming from the analysis of the system via classical OC theory combined with a two stage optimization of the switching instants (1-Stage off-line to obtain a rough map between switching instants and terminal position and time. 2-Stage on-line local optimization).

- Experimental tests, conducted combining this approach with Nonlinear Model Predictive Control, show that the final velocity can be more than doubled w.r.t. the one of the rigid arm

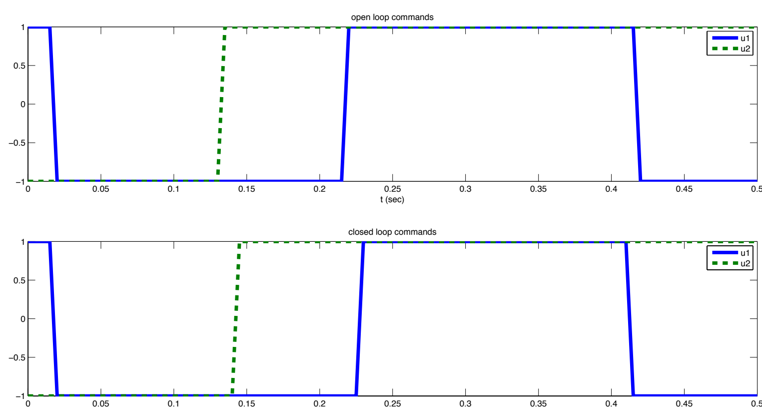


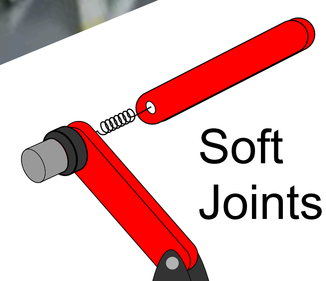
Open Loop Vs NMPC




Open Loop

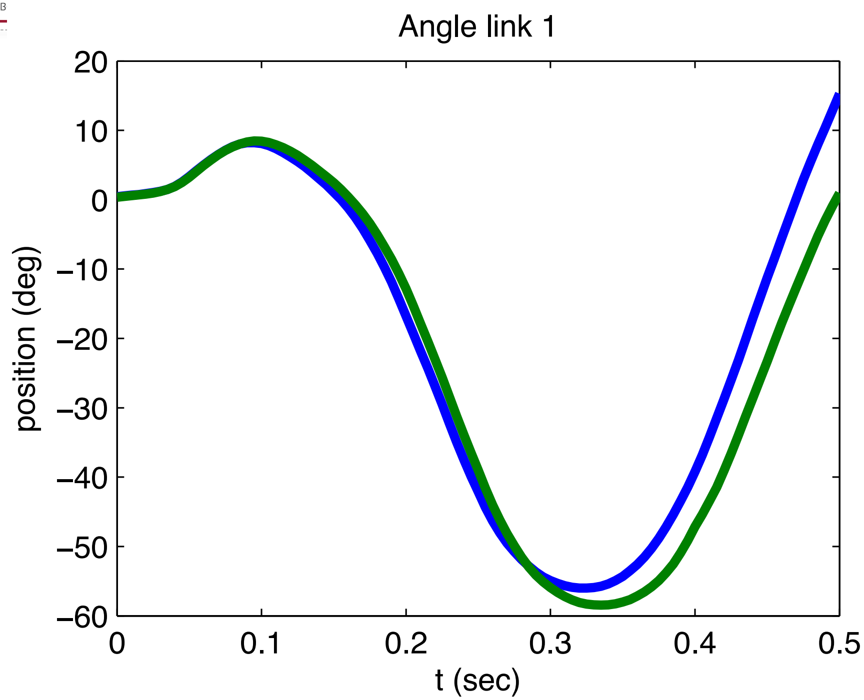
NMPC

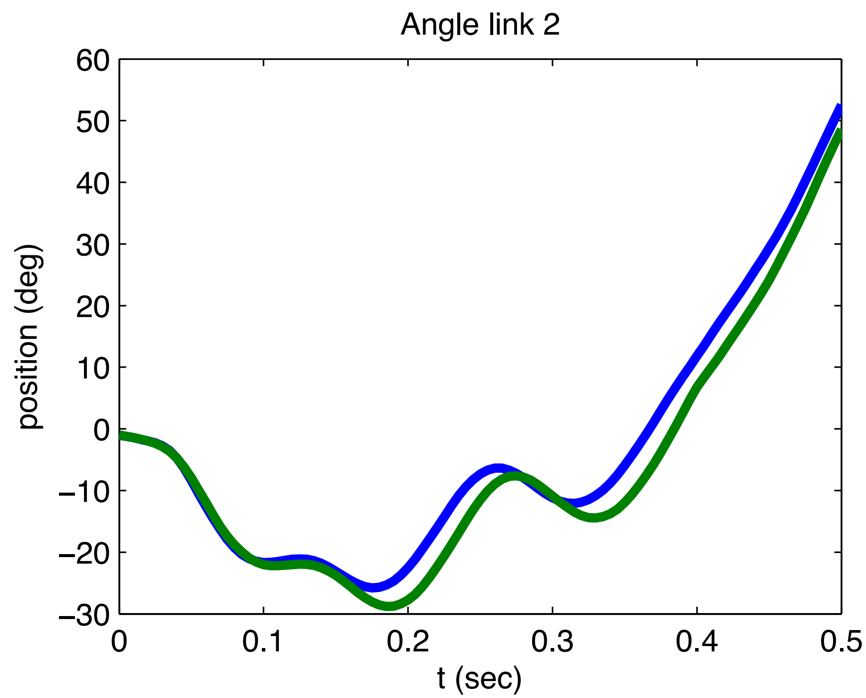


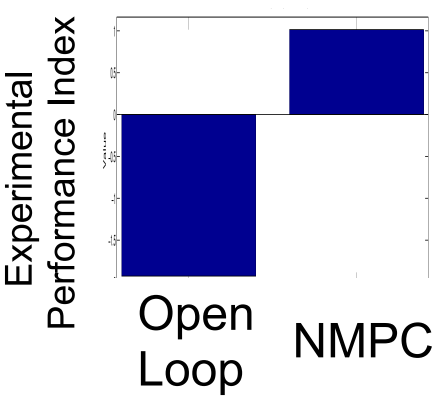



# Experimental Results

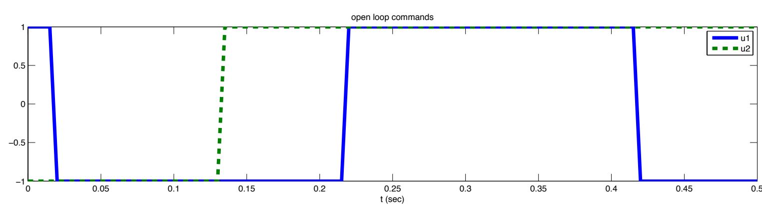
## Open Loop Control Vs NMPC



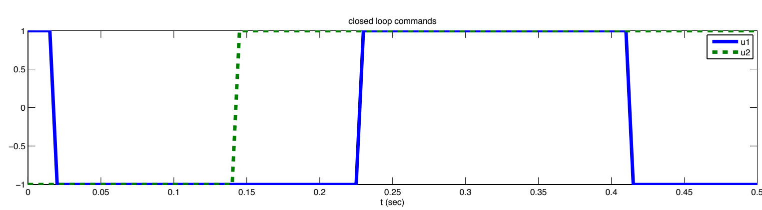




Open Loop




NMPC



# Real-Time Op

**Maximize Speed at g**  **F robot**

# Variable Stiffness Control for Oscillation Damping

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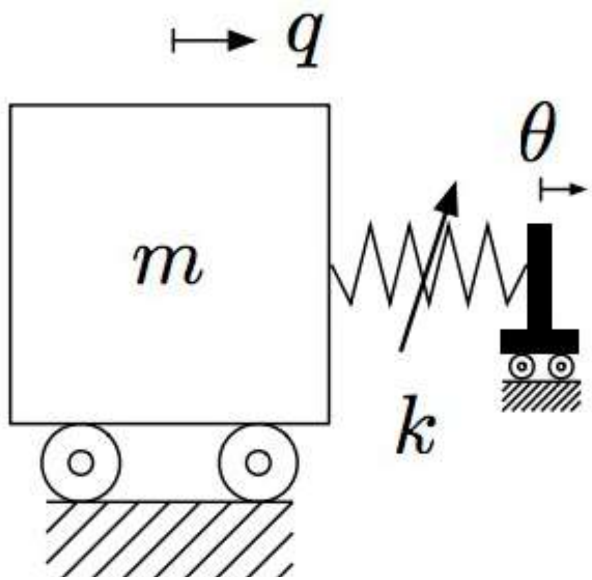
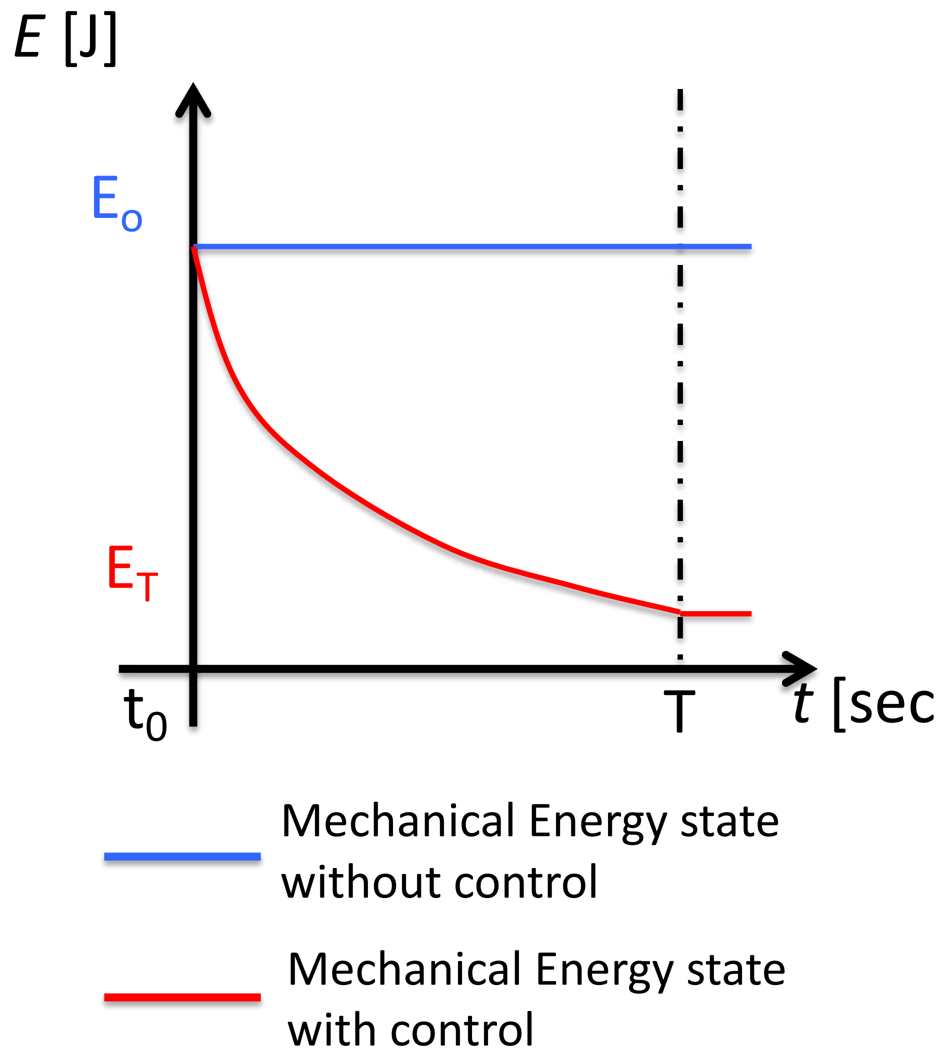
# Problem Definition

1 DoF Variable Stiffness Actuator




$$\ddot{q} + w^2(q - \theta) = 0$$
$$w = \sqrt{k / m}$$
$$u(t) = k$$

Objective



State

$$x(t) = [x_1, x_2]^T = [q - \theta, \dot{q}]^T$$

State Dynamic

$$\dot{x}(t) = \begin{bmatrix} x_2 \\ -w^2 x_1 \end{bmatrix}$$

Fixed terminal time

$$T$$

Terminal Mechanical Energy

$$\phi(x(T))$$

Control input

$$u(t)$$

Optimal Control Problem

$$\phi\left(x\left(T\right)\right) = \frac{x_1^2(T)}{2} + \frac{x_2^2(T)}{2}$$

$$\begin{aligned}
&\min_{u(t)} \phi\left(x\left(T\right)\right) \\
&\dot{x}(t) = f(x, u) \\
&x(0) = x_0 \\
&0 \le k_{min} \le u(t) \le k_{max}
\end{aligned}$$

Costate

$$\lambda(t) = [\lambda_1, \lambda_2]^T$$

Costate Dynamic

$$\dot{\lambda}(t) = \begin{bmatrix} w^2 \lambda_2(t) \\ -\lambda_1(t) \end{bmatrix}$$

Hamiltonian function

$$H = \lambda^T f(x,u) = \lambda_1(t) x_2(t) - \frac{u(t)}{m} \lambda_2(t) x_1(t)$$

From Pontryagin Maximum Principle (PMP) necessary conditions on the optimal control u(t) can be derived by minimising the Hamiltionian.
It follows:

Switching function

$$\sigma(t) = \lambda_2(t) x_1(t)$$

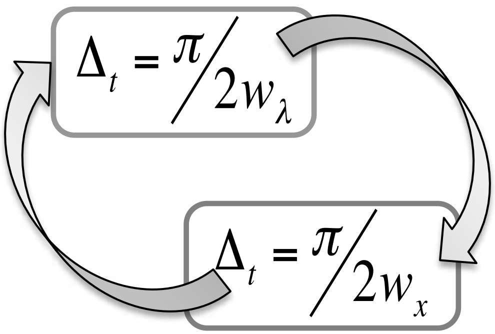
Switches conditions

$$t_s \in (0,T) \quad \begin{matrix} x_1(t_s) = 0 \\ \lambda_2(t_s) = 0 \end{matrix}$$

Bang-Bang Control

$$u(t) = \begin{cases} k_{\max} & \text{when} \quad \sigma(t) < 0 \\ k_{\min} & \text{when} \quad \sigma(t) > 0 \end{cases}$$

The switching intervals do not depend from model parameter




From calculus can be showed that :

$$\lambda_2 = h(x_2, w_x, w_\lambda)$$

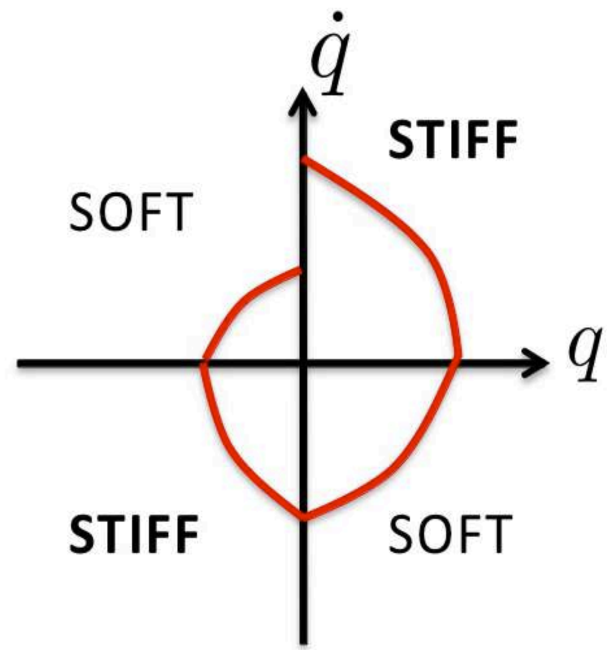
The new switching fucntion:

$$\sigma(t) = x_1(t) x_2(t) p(w_x, w_y)$$
$$p(w_x, w_y) > 0$$

## Optimal Control law

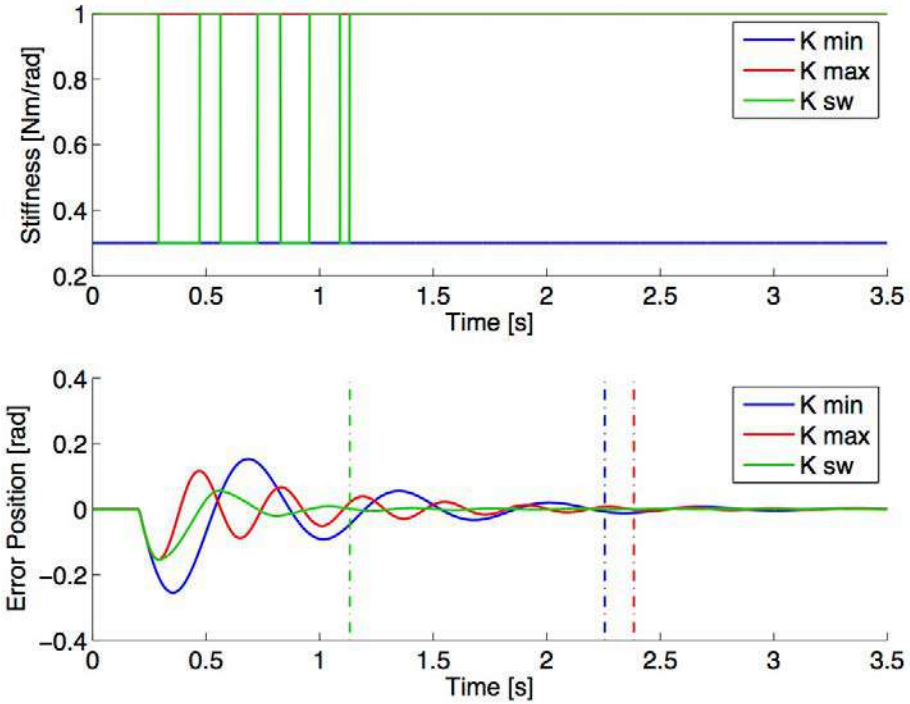
$$u(t) = \begin{cases} k_{\max} & x_1(t) x_2(t) < 0 \\ k_{\min} & x_1(t) x_2(t) > 0 \end{cases}$$

**Model Free**

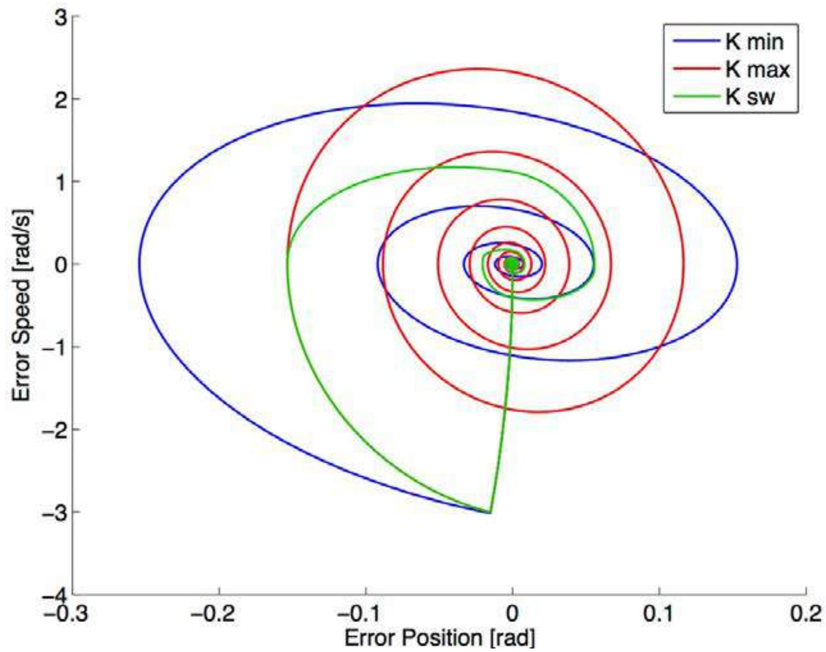


# Simulation 1 Dof case with damping

Stiffness profile and Error evolution




Phase space



# Results 2 DoF

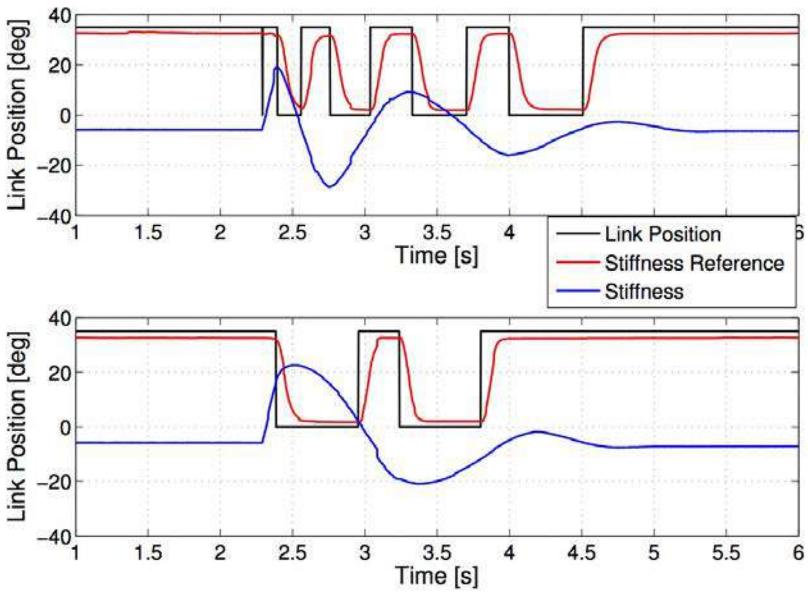



Fig.4

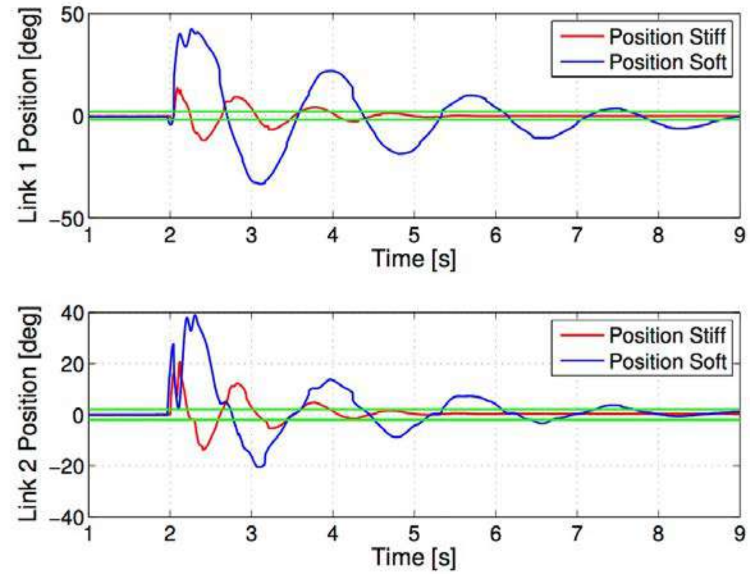



Fig.5

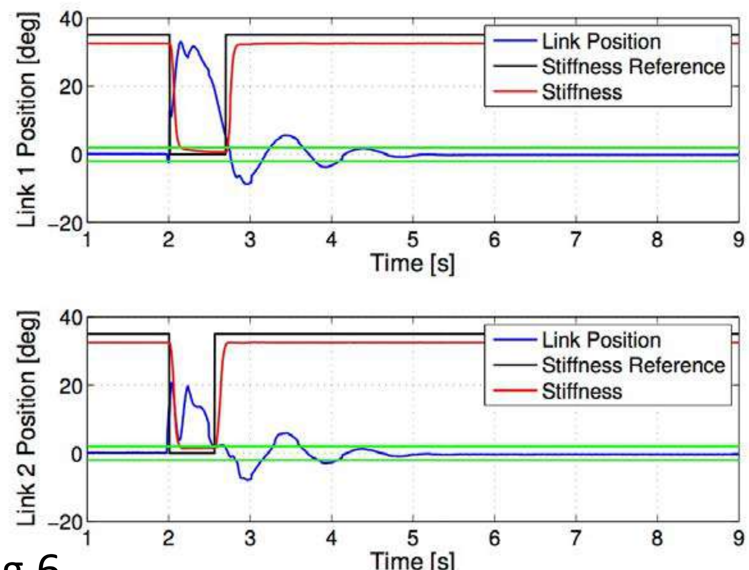



Fig.6

Results 2 DoF case:

Fig. 4 Link position and stiffness evolution (ideal case, no thresholds)

With thresholds

Fig. 5 Link position constant stiffness (Max, Min)

Fig. 6 Link position with optimal switching control

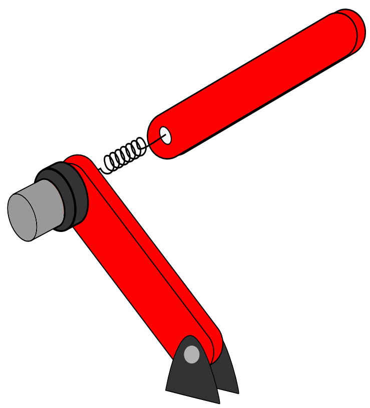
# Summing up

- Optimal Control gives insight in physical properties of embodied behaviors, by factorizing control and physical performance
- For safe&fast motion
  - STIFF should go with SLOW, and SOFT with FAST
- For max speed
  - STIFF goes with SPEED-UP, and SOFT with SLOW-DOWN
- For optimal brake
  - STIFF goes with SLOW-DOWN and SOFT with SPEED-UP

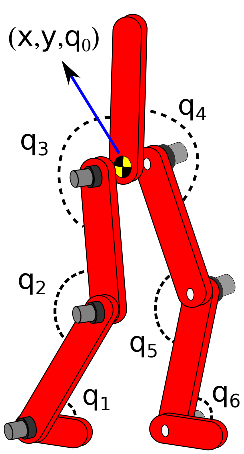
# Minimize Energy Consumption in Cyclic Tasks

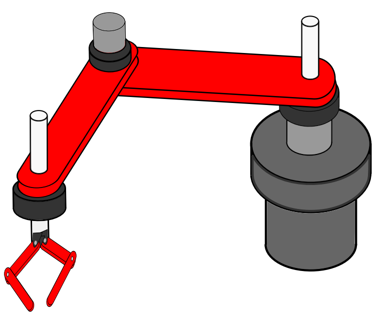
# Minimize Energy Consumption

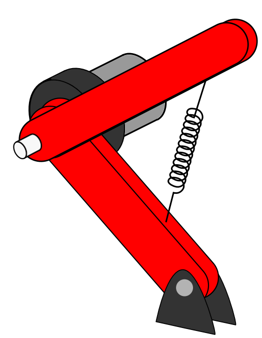
What role does soft actuation play in the reduction of energy cost for mechanical systems that perform cyclic motions?



Joint with SEA





Joint with PEA

- Hypothesis: $K = \mathrm{diag}[K_1, \ldots, K_n]$

The problem can be decoupled

- Optimize actuation parameters
  SEA: Stiffness (K)
- Then, optimize trajectory:

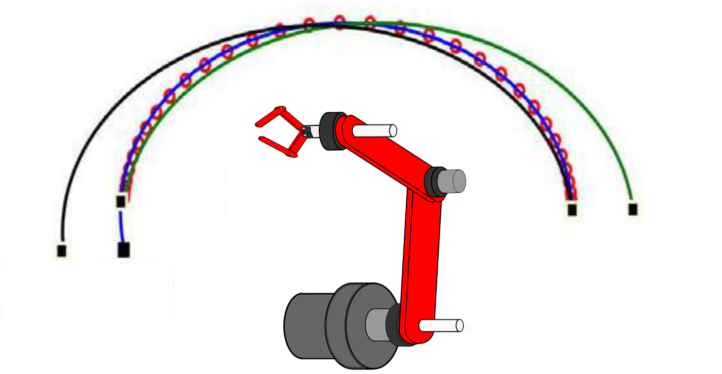
$$q(t) = q_d(t),\ \dot{q}(t) = \dot{q}_d(t)\ ,\ \ddot{q}(t) = \ddot{q}_d(t)$$

# Minimize Energy Consumption
## Analytical Results

| | $J_1$ | $J_2$ |
|---|---|---|
| SEA | A Solution of $4\,A_{S,j} + 3K_j B_{S,j} + 2C_{S,j} K_j^2 + D_{S,j} K_j^3 = 0$ | $\hat{K}_j = -2\frac{F_{S,j}}{G_{S,j}}$ |
| PEA | $\hat{K}_j = \frac{B_{P,j}E_{P,j} - 2C_{P,j}D_{P,j}}{4D_{P,j}F_{P,j} - E_{P,j}^2}$ <br> $\hat{q}_{e,j} = \frac{C_{P,j}E_{P,j} - 2B_{P,j}F_{P,j}}{2C_{P,j}D_{P,j} - B_{P,j}E_{P,j}}$ | $\hat{K}_j = \frac{\hat{q}_{e,j}H_{P,j} - I_{P,j}}{2(\hat{q}_{e,j}^2 T + L_{P,j} - \hat{q}_{e,j}M_{P,j})}$ <br> $\hat{q}_{e,j} = \frac{H_{P,j} + \hat{K}_j M_{P,j}}{2\hat{K}_j T}$ |

A, B, C, D, F, G, E, H, I, M, L are integrals that depend on the trajectories and on the mechanical model

# Minimize Energy Consumption
## The SEA Pick and Place Example: Simulations



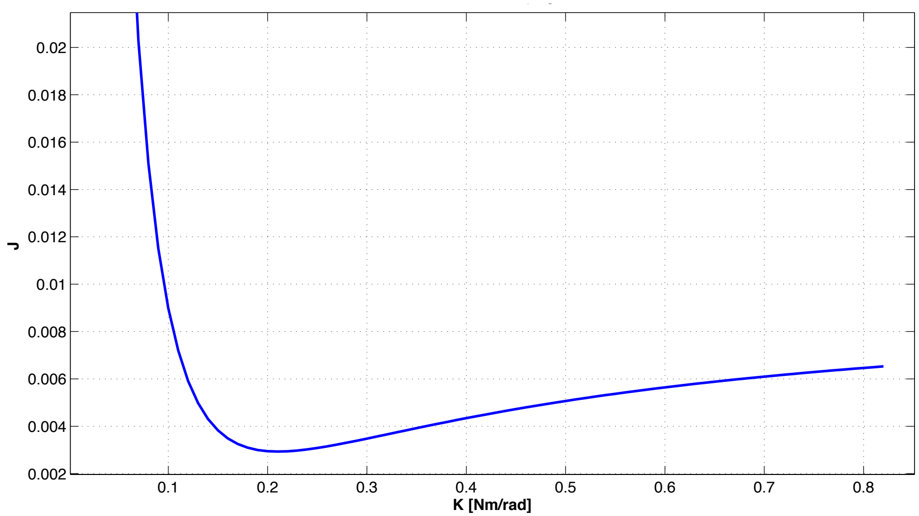
- 2 DoF Manipulator
- Sinusoidal joint trajectories
- Given Pick and Place Points

Theoretical
Optimal stiffness

Available
Optimal
stiffness

| | Joint 1 $q_1(t)$ | Joint 2 $q_2(t)$ | TOTAL |
|---|---|---|---|
| $\hat{K}$ $[Nm/rad]$ | 0.2 | 0.09 | |
| $J_1$ | 0.003 | 0.07 | 0.073 |
| $K^*$ $[Nm/rad]$ | 0.22 | 0.1 | |
| $J_1$ | 0.003 | 0.075 | 0.078 |
| $K_{min}$ $[Nm/rad]$ | 0.05 | 0.05 | |
| $J_1$ | 0.04 | 0.35 | 0.39 |
| $K_{max}$ $[Nm/rad]$ | 0.82 | 0.82 | |
| $J_1$ | 0.07 | 0.1 | 0.17 |
| Stiff | $K \to \infty$ | $K \to \infty$ | |
| $J_1$ | 0.09 | 0.1 | 0.19 |

A typical performance index VS stiffness curve



# Parametric Trajectory Libraries for Online Motion Planning with Application to Soft Robots

Tobia Marcucci, Manolo Garabini,
Gian Maria Gasparri, Alessio Artoni,
Marco Gabiccini, and Antonio Bicchi

# GOAL: Efficient Pick and Place

Object positions are known at the very beginning of each task

**Energy Efficiency IFF Planning is Real Time**

# The Problem

to combine optimality
and short computation times

# The Idea

## Online Motion Planning via Parametric Trajectory Library

**Offline Phase**

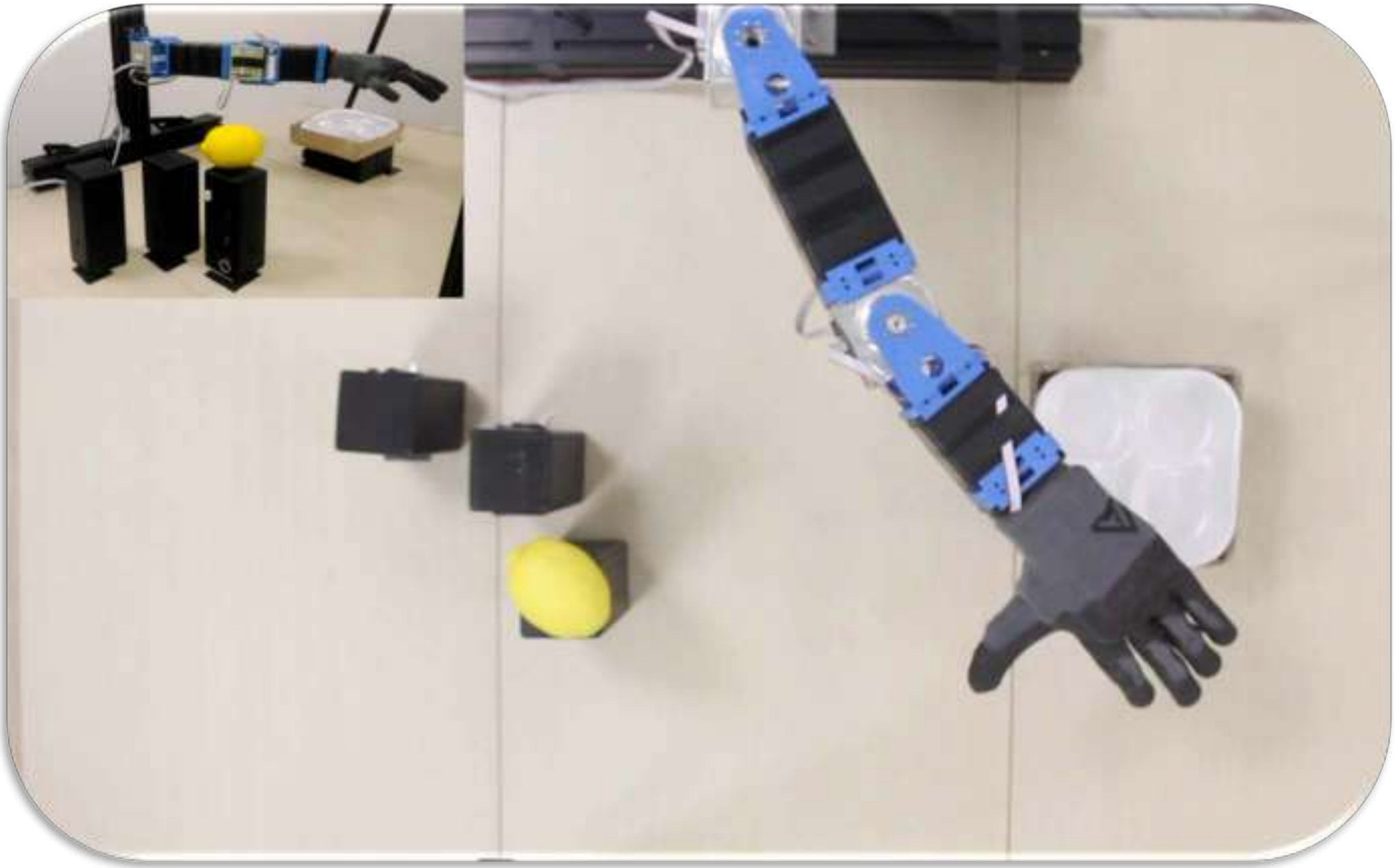
- Parametric library of optimal trajectories via nonlinear programming
- Sensitivity analysis
- Preconditioned tangent Quadratic Program (QP)

**Online Phase**

- Search of the best candidate trajectory in the library
- Trajectory refinement via online QP solution

Average Planning Time < 10 ms

Task time > 1 s



# Exploiting Joint Elasticity in Efficient Walking and Running via Numerical Optimization

G. M. Gasparri, S. Manara, D. Caporale, G. Averta, M. Catalano, G. Grioli, M. Bianchi, A. Bicchi, and M. Garabini

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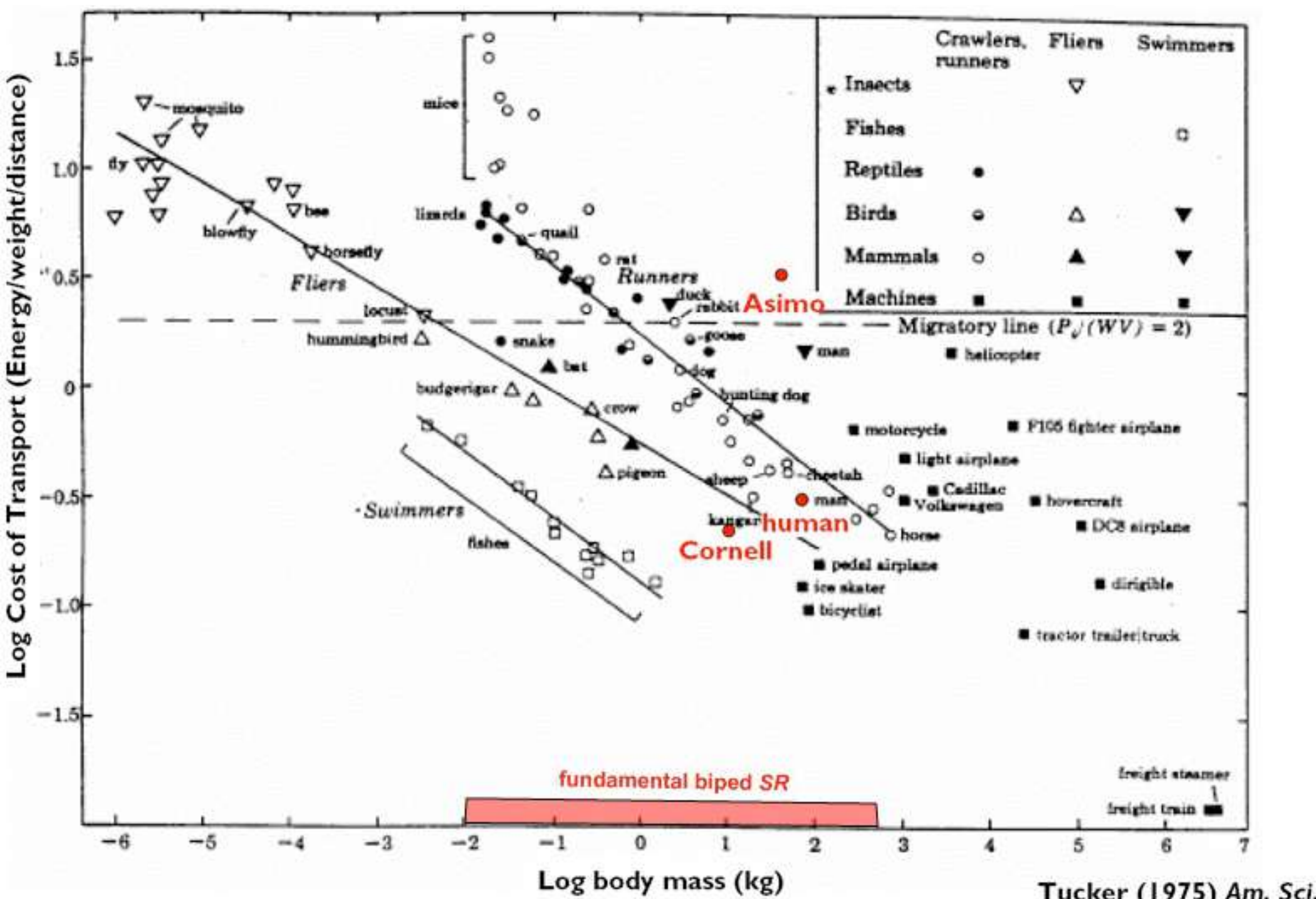
Centro E. Piaggio Università di Pisa

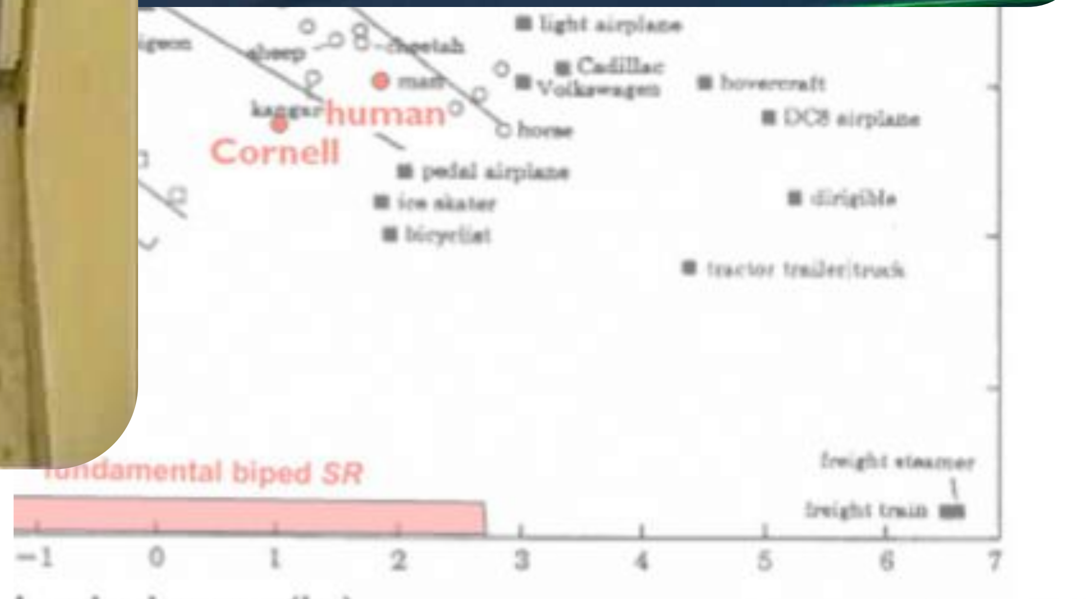
UNIVERSITÀ DI PISA

# The Energetic Cost of Moving About




[www.wprize.org/TheChallenge, updating The Energetic Cost of Moving About, Tucker 1975]

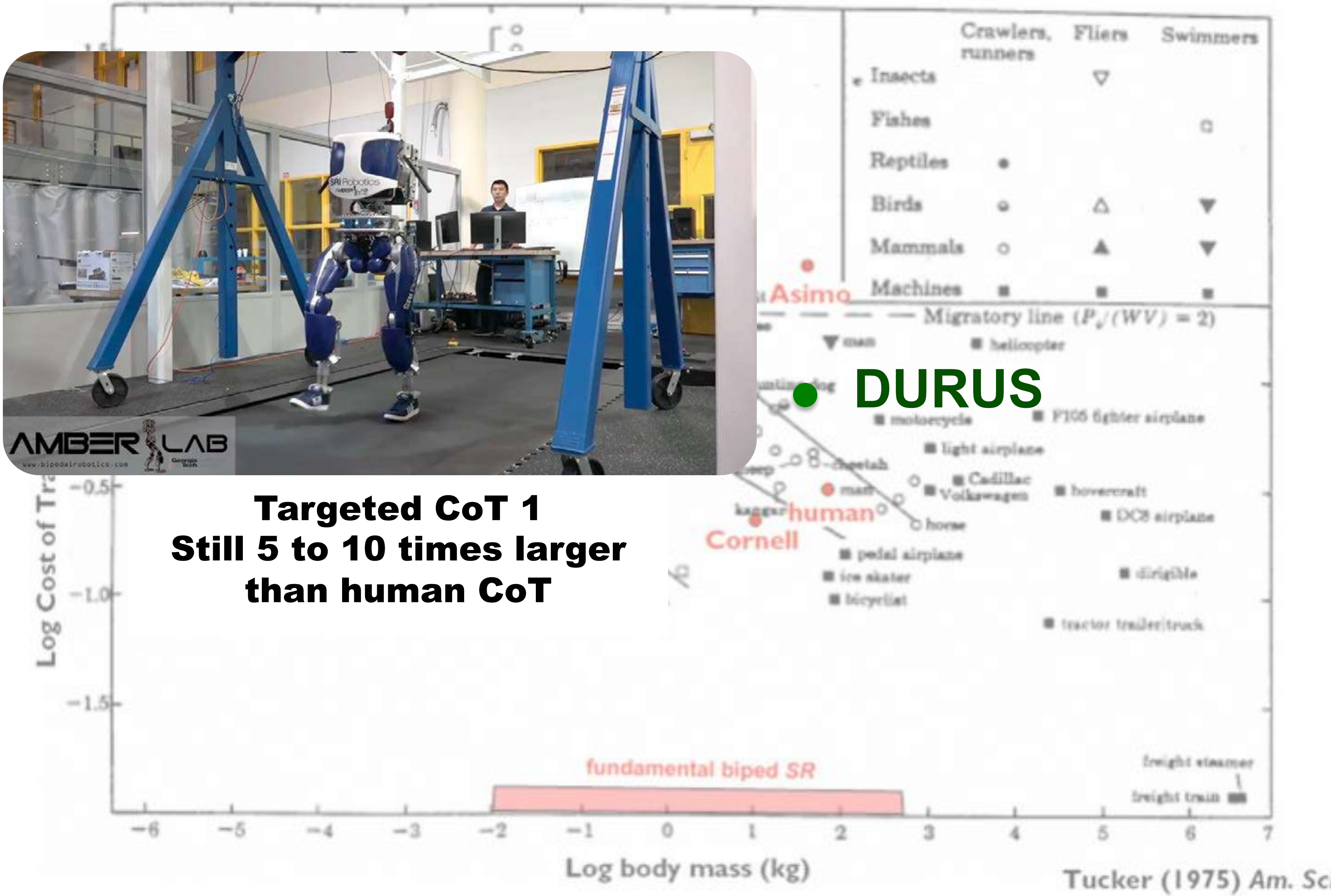
**Eliud Kipchoge run a "Marathon" in 2:00:23**


Tucker (1975) *Am. Sci.*

**Passive Walkers beat humans**
**- Carefully tuned dynamics**
**- Limited Flexibility**

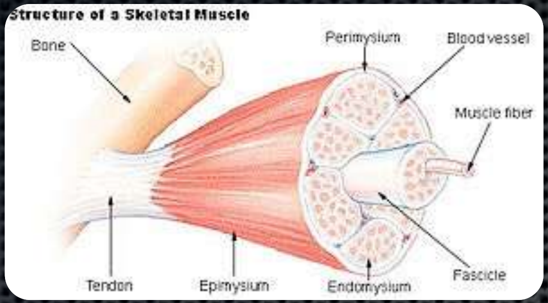
[A Three-Dimensional Passive-Dynamic Walking Robot with Two Legs and Knees, Collins et al. ]




**Targeted CoT 1**
**Still 5 to 10 times larger than human CoT**

[DURUS: SRI's Ultra-Efficient Walking Humanoid Robot, IEEE SPECTRUM]

# Which are the main limitations of robots?



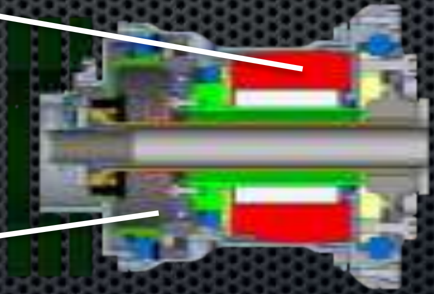

Muscle Efficiency

**< 20%**

[Mechanical efficiency of pure positive and pure negative work with special reference to the work intensity, Aura and Komi 1986, Int. Journal of Sports Medicine]

## Prime Movers

Electric Motor



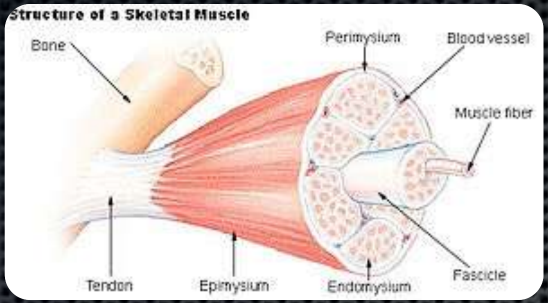
Gearbox

Robot Drive Efficiency

Motor Eff. X Gearbox Eff.

**> 40%**

# Which are the main limitations of robots?

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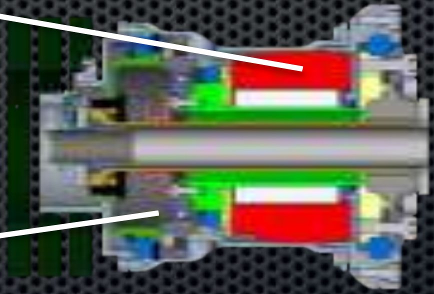


Robot Drive Efficiency

Motor Eff. X Gearbox Eff.

**> 40%**

Robot drives efficiency is at least twice than muscle efficiency

# Which are the main limitations of robots?

Dynamic Richness

Musculo-skeletal system
- Redundancy
- Compliance, Damping…
- ..that are adjusted based on tasks
- Variable Recruiting Mechanisms

Robot joints and bodies


# Which are the main limitations of robots?

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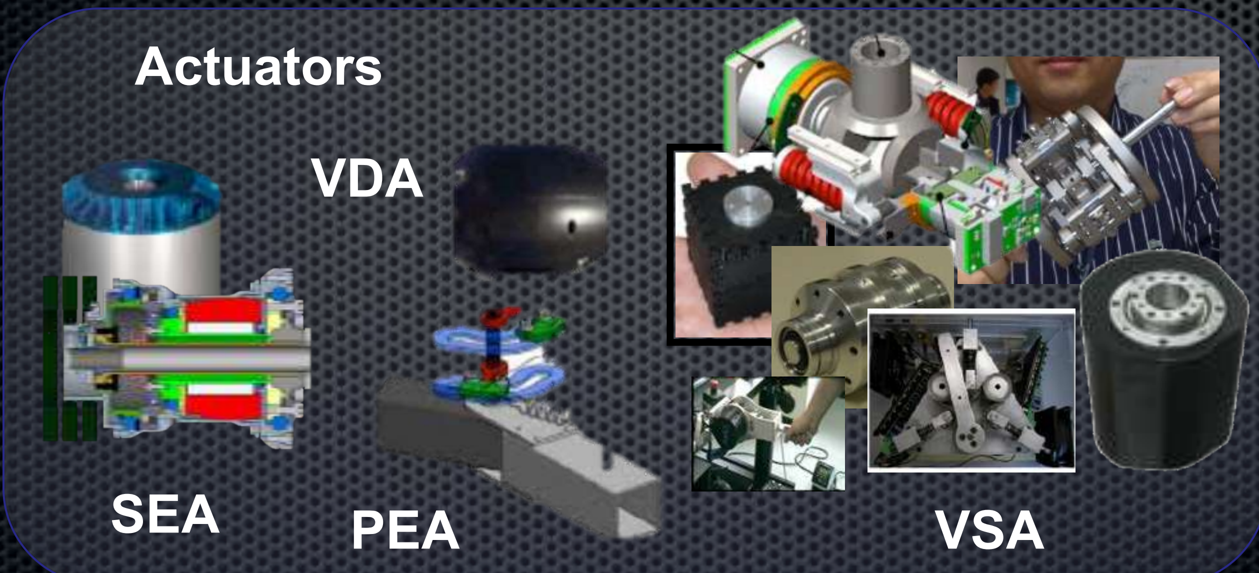
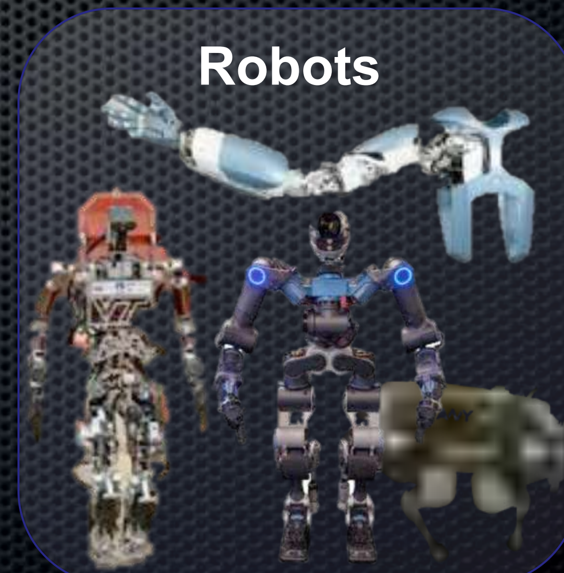
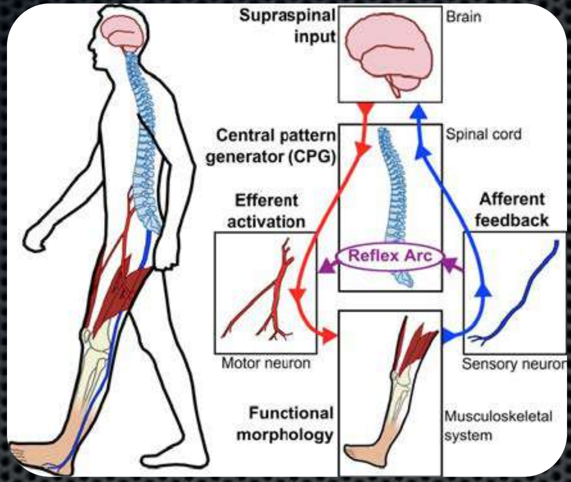


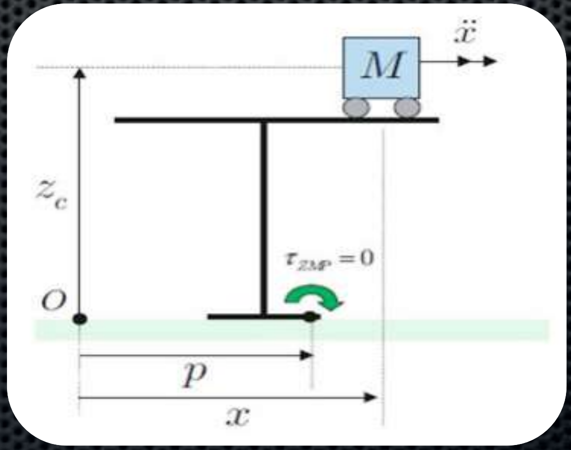

new-gen robots are approaching the variety of biologic behaviors but they did not shown large efficiency improvements yet

# Which are the main limitations of robots?

Movement Generation




<u>Human Motor Control</u>
- CPG [Sherrington]
- Synergies [Bizzi]
- Learning [Kawato]
- Peripheral & Central Loops [Grillner]
- Reflexes
- …




<u>Robot Motion Control</u>
- Motion generation based on simple models (cart-table, IP, SLIP …) to guarantee online feasibility

The problem:

Locomotion generation via classical models can not exploit the full robot dynamics to obtain efficient motions

A possible solution:

**numerical optimization** on the **full dynamic model** of the robot to determine robot **trajectories (and possibly dynamic params)** that **minimize the Energetic Cost of locomotion**

# Locomotion via Numerical Optimization

The optimal control problem

$$\min_{x(t),u(t),p} \int_0^T L(x(t),u(t))dt$$

$$\text{subject to}$$

$$g(x,\dot{x},u,p) = 0$$
$$h(x,u,p) \leq 0$$

# Locomotion via Numerical Optimization

The optimal control problem

Minimize the Cost of Transport

$$\min_{x(t),u(t),p} \int_0^T L(x(t), u(t))dt$$

$$\text{subject to}$$

$$g(x, \dot{x}, u, p) = 0$$
$$h(x, u, p) \leq 0$$

# Locomotion via Numerical Optimization

The optimal control problem

$$\min_{x(t),u(t),p} \int_0^T L(x(t),u(t))dt$$

$$\text{subject to}$$

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Robot dynamics through different contact phases

- State/control bounds
- Discontinuities at phase changes
- Periodicity
- Unilateral Contact

# Locomotion via Numerical Optimization

The optimal control problem

$$\min_{x(t),u(t),p} \int_0^T L(x(t), u(t))dt$$

$$\text{subject to}$$

$$g(x, \dot{x}, u, p) = 0$$
$$h(x, u, p) \leq 0$$

- **States (x)**
- **Controls (u)**
- **Dynamic parameters (p)**

**(e.g. transmission stiffness)**

# Locomotion via Numerical Optimization

The optimal control problem

$$\min_{x(t),u(t),p} \int_0^T L(x(t),u(t))dt$$

$$\text{subject to}$$

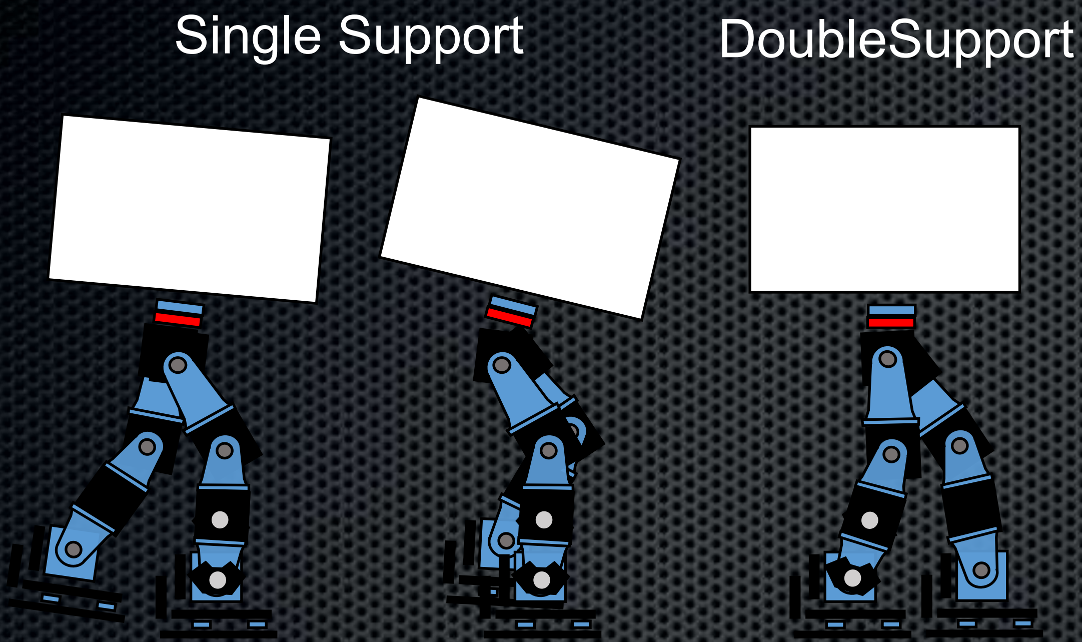
$$g(x,\dot{x},u,p) = 0$$

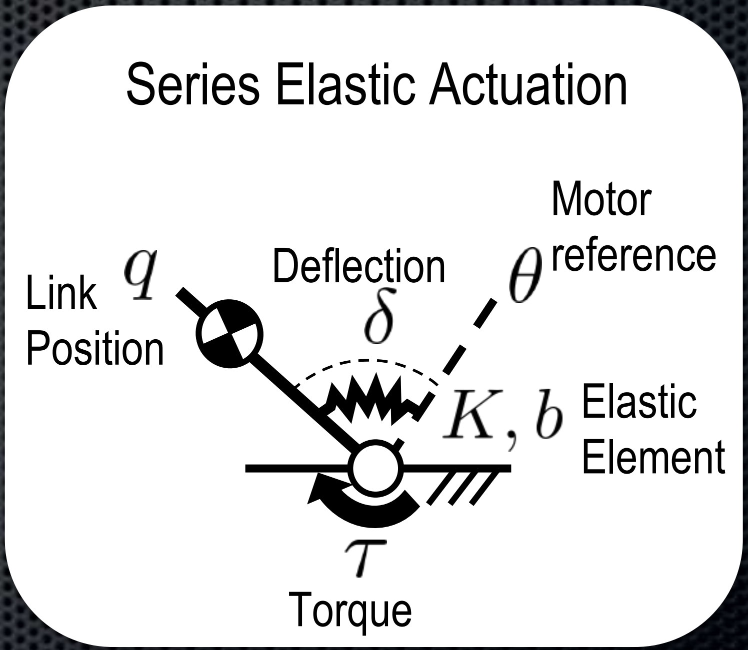
$$h(x,u,p) \leq 0$$

Solution Approach: first discretize then optimize

- Direct Collocation
- Algoritmic Differentiation
- Interior Point Method

# Case Study: a Planar 6DoF Soft Biped



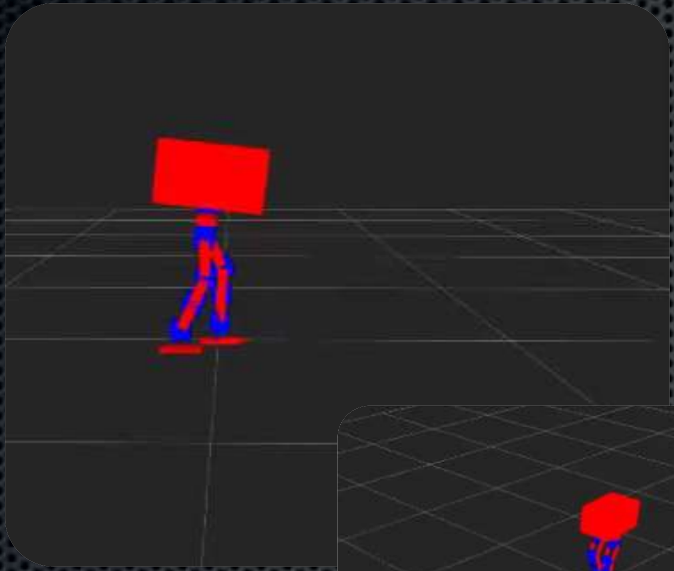



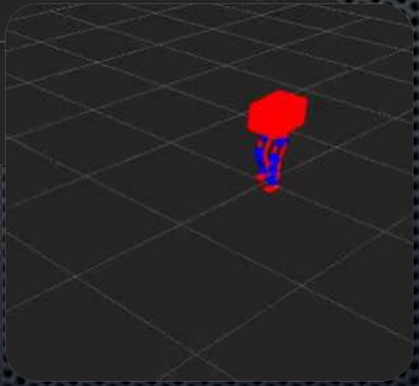

Constraints

- Link Position Limits
- Motor Torque and Speed Limits
- Swing Foot Height

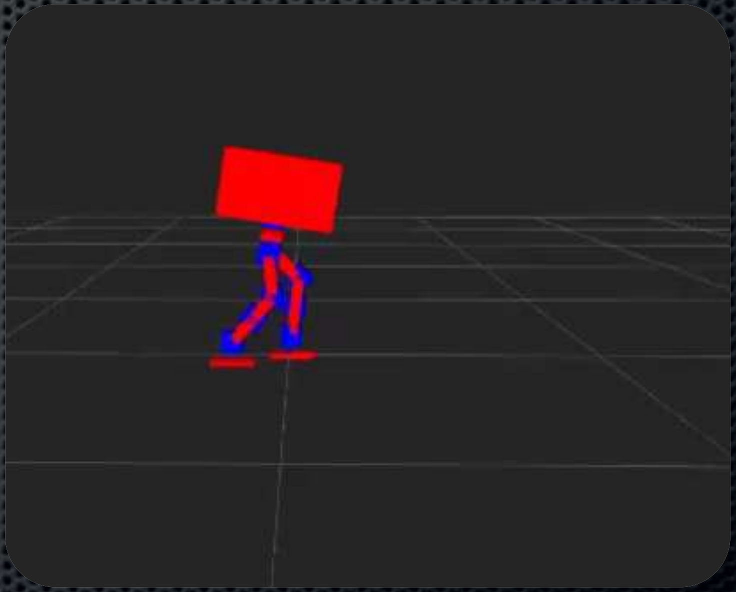
# Walking

Speed 0.2 m/s

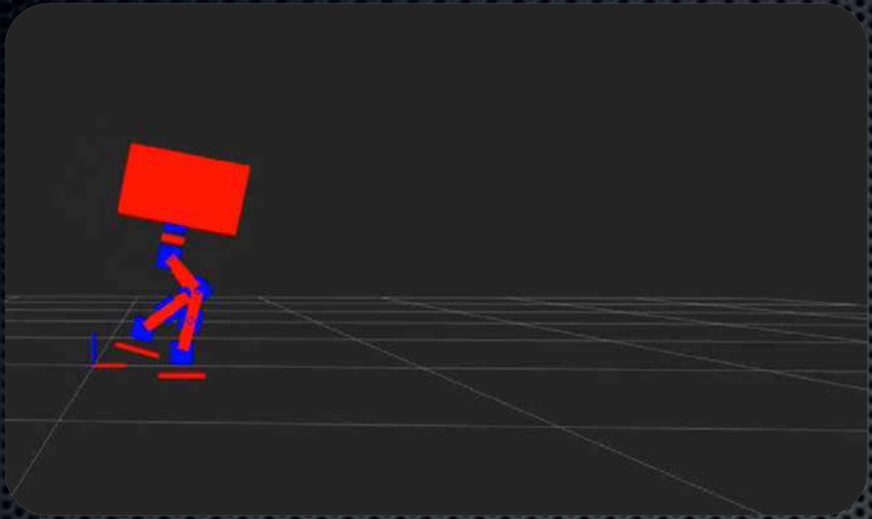




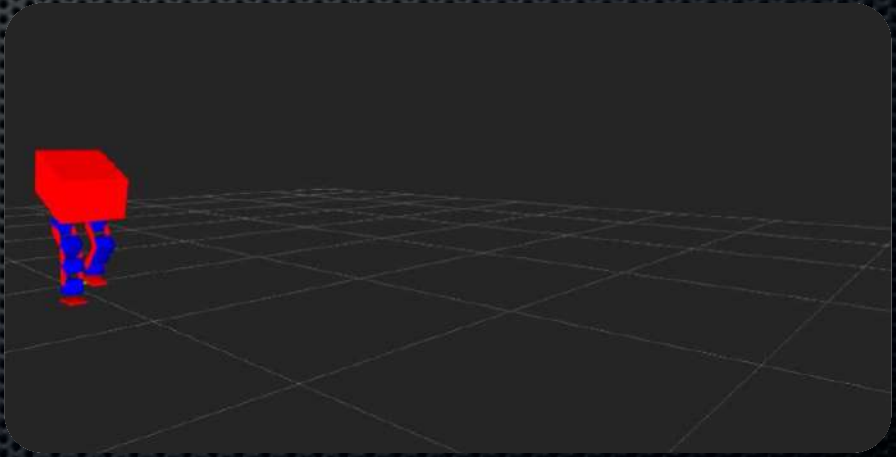
Speed 0.4 m/s



# Running

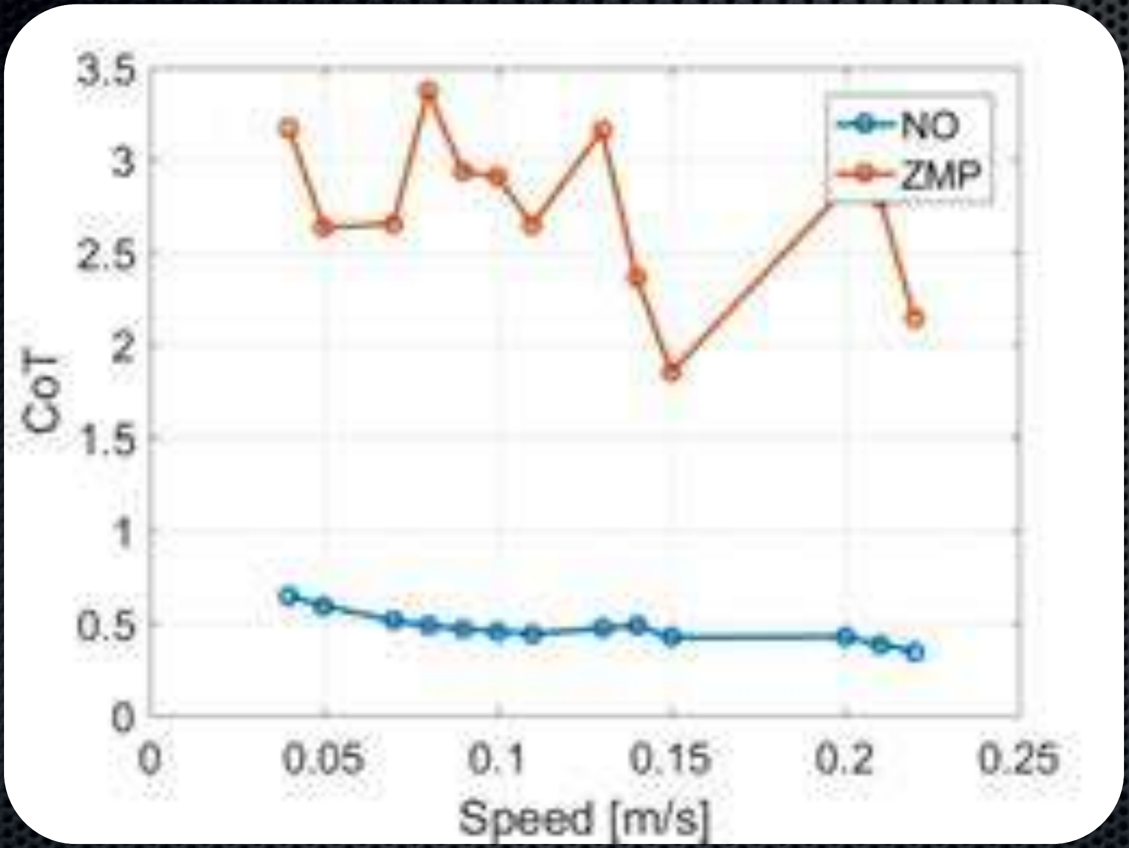


Speed 0.5 m/s



# Classical Locomotion Vs Optimized Locomotion

One Step CoT Vs Forward Speed

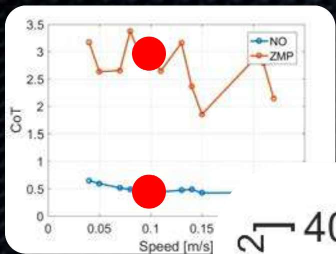



ZMP-based (ZMP)
- Given gait params
- Cart-table model to determine COM traj.
- determine joint traj.

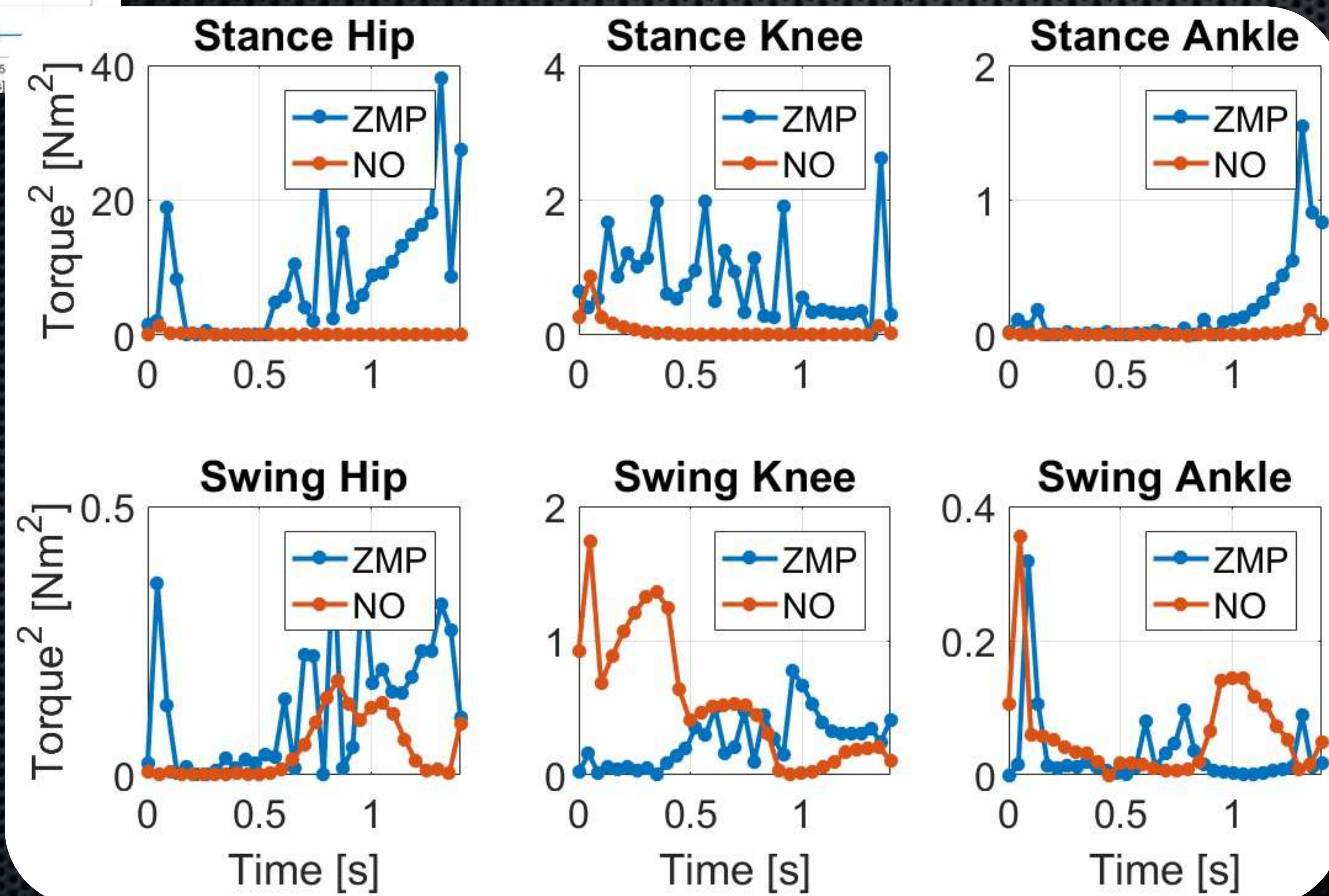
Optimization-based (NO)
- Given gait params
- Numerical Optimization to determine joint traj. That minimize CoT

Optimization based locomotion has a Cost of Transport 5 times lower than a ZMP based locomotion

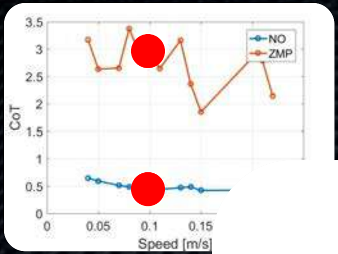
# Classical Locomotion Vs Optimized Locomotion




Forward Speed 0.1 m/s



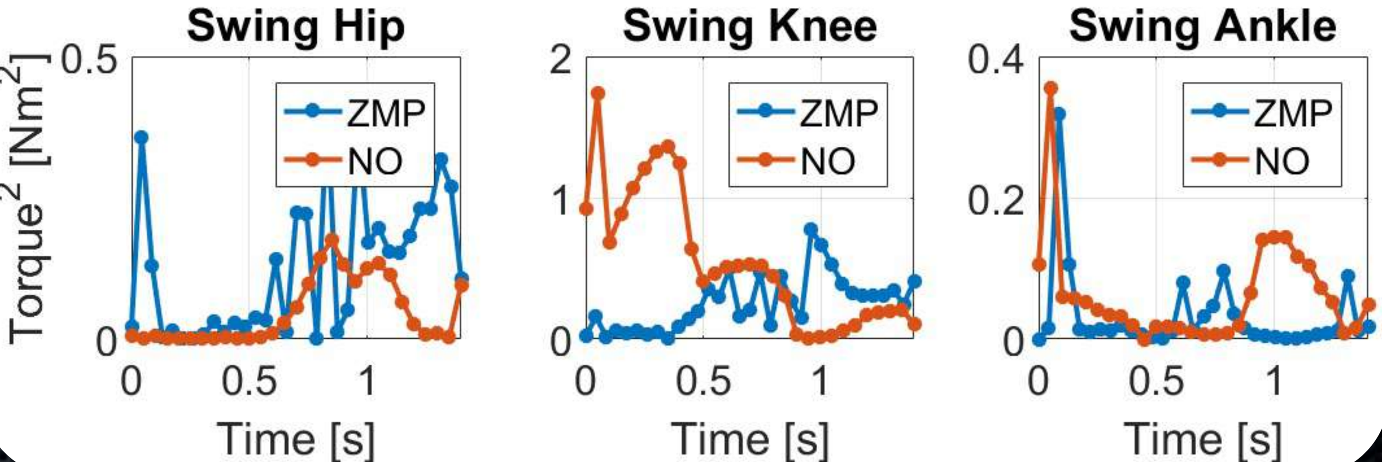
# Classical Locomotion Vs Optimized Locomotion



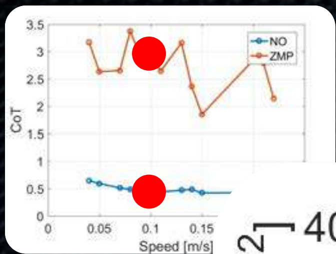

Forward Speed 0.1 m/s

<u>SWING LEG</u>

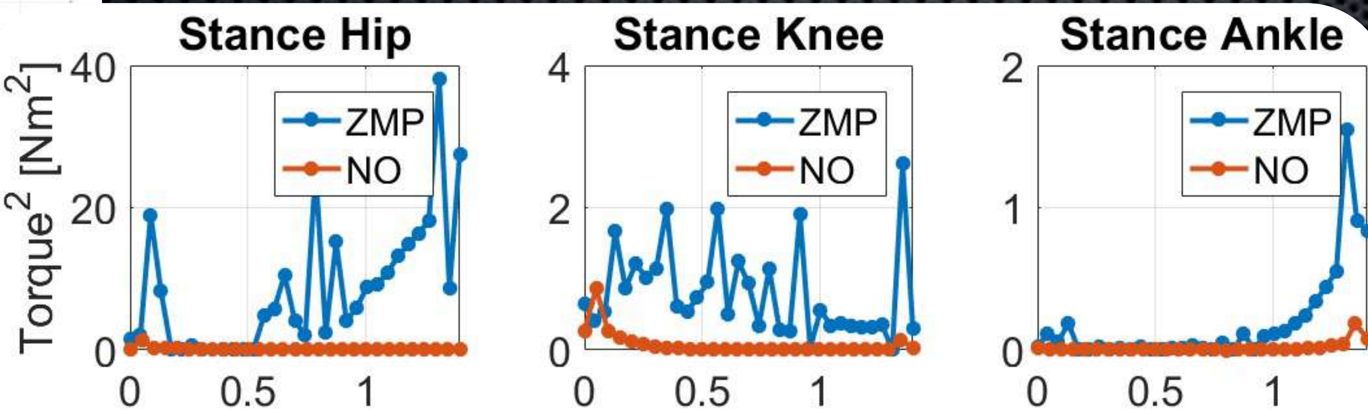
The two techniques show **comparable values**



# Classical Locomotion Vs Optimized Locomotion




Forward Speed 0.1 m/s




<u>STANCE LEG</u>

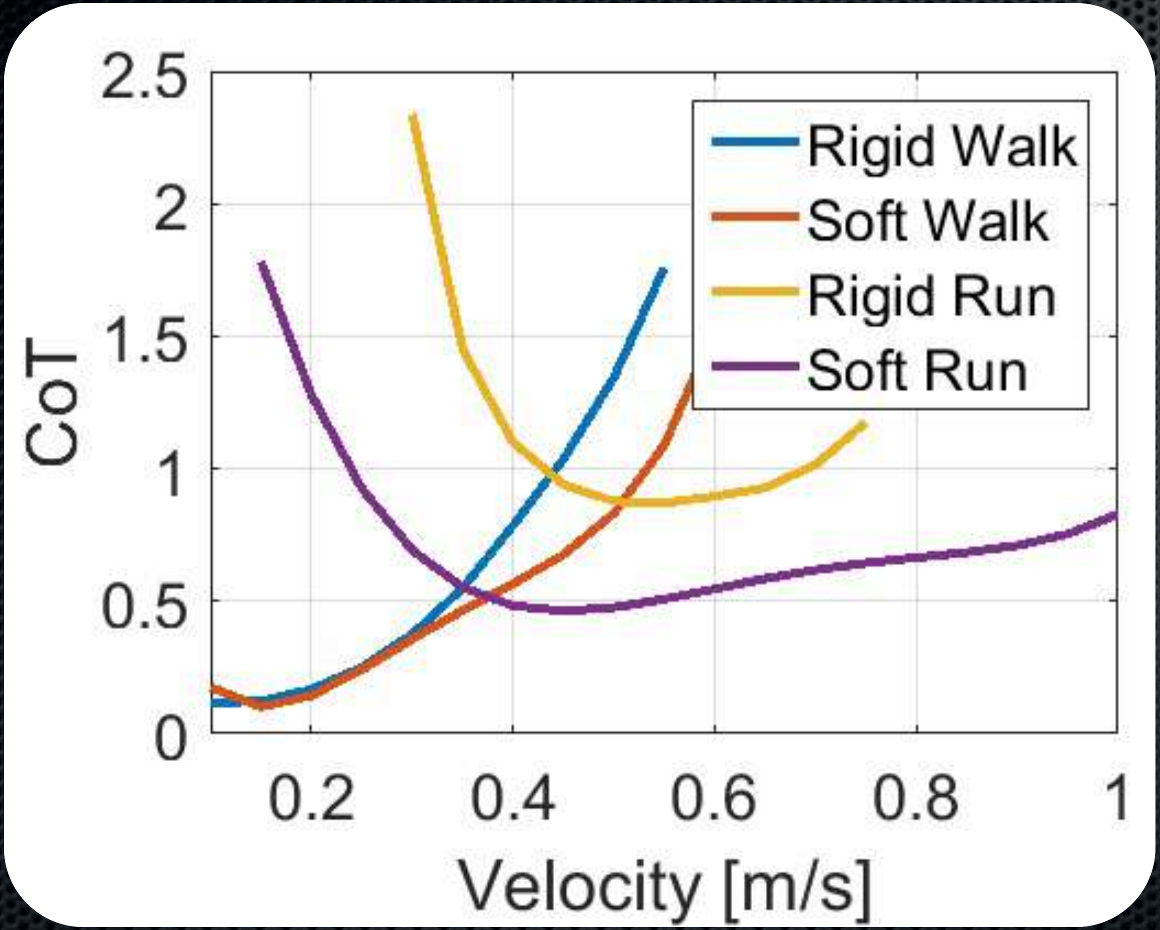
- Optimized Locomotion

**Torques ≈ 0** (except at the very beginning and the very end)

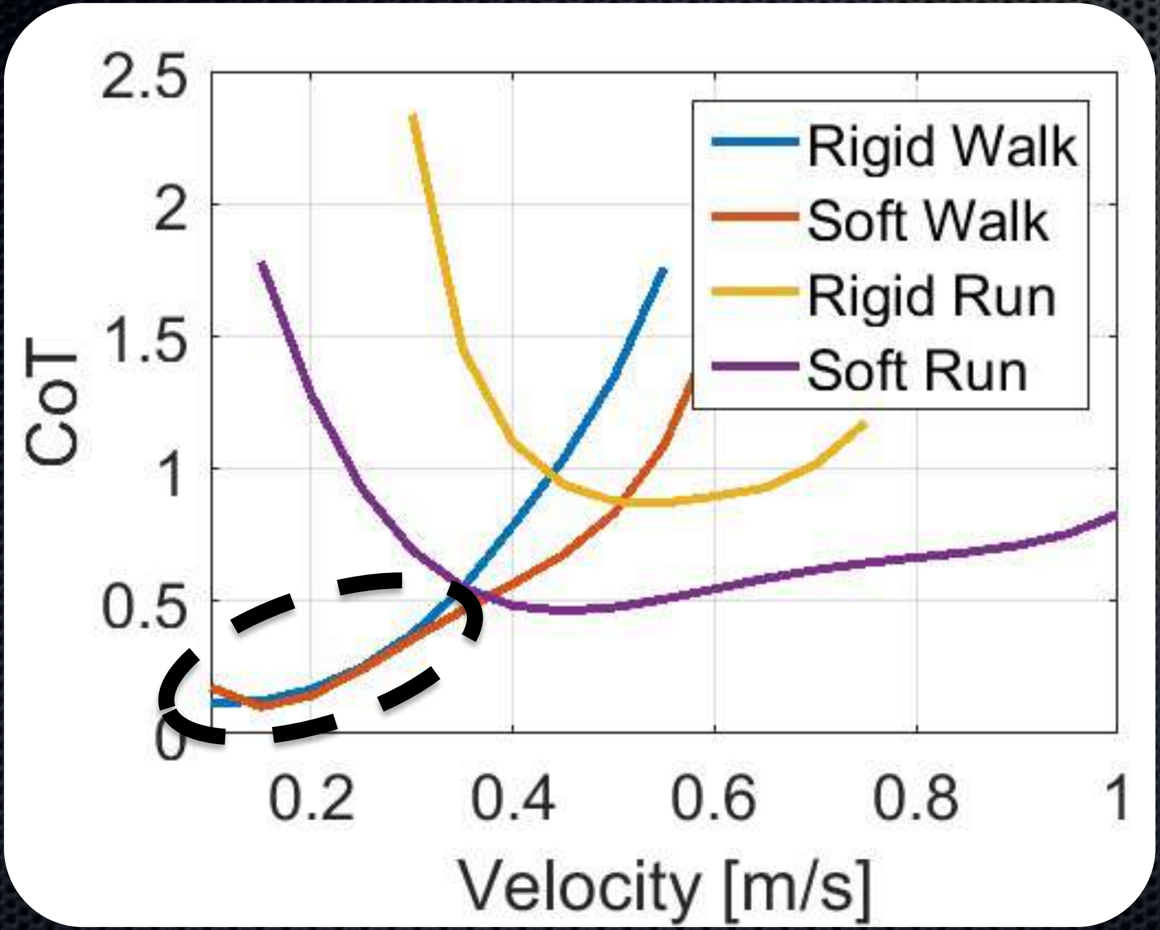
- ZMP-based Locomotion

Hip, Knee (especially) and Ankle torques are substantial due to the **non-straight leg configuration**

# Opt. Rigid Locomotion Vs Opt. Soft Locomotion



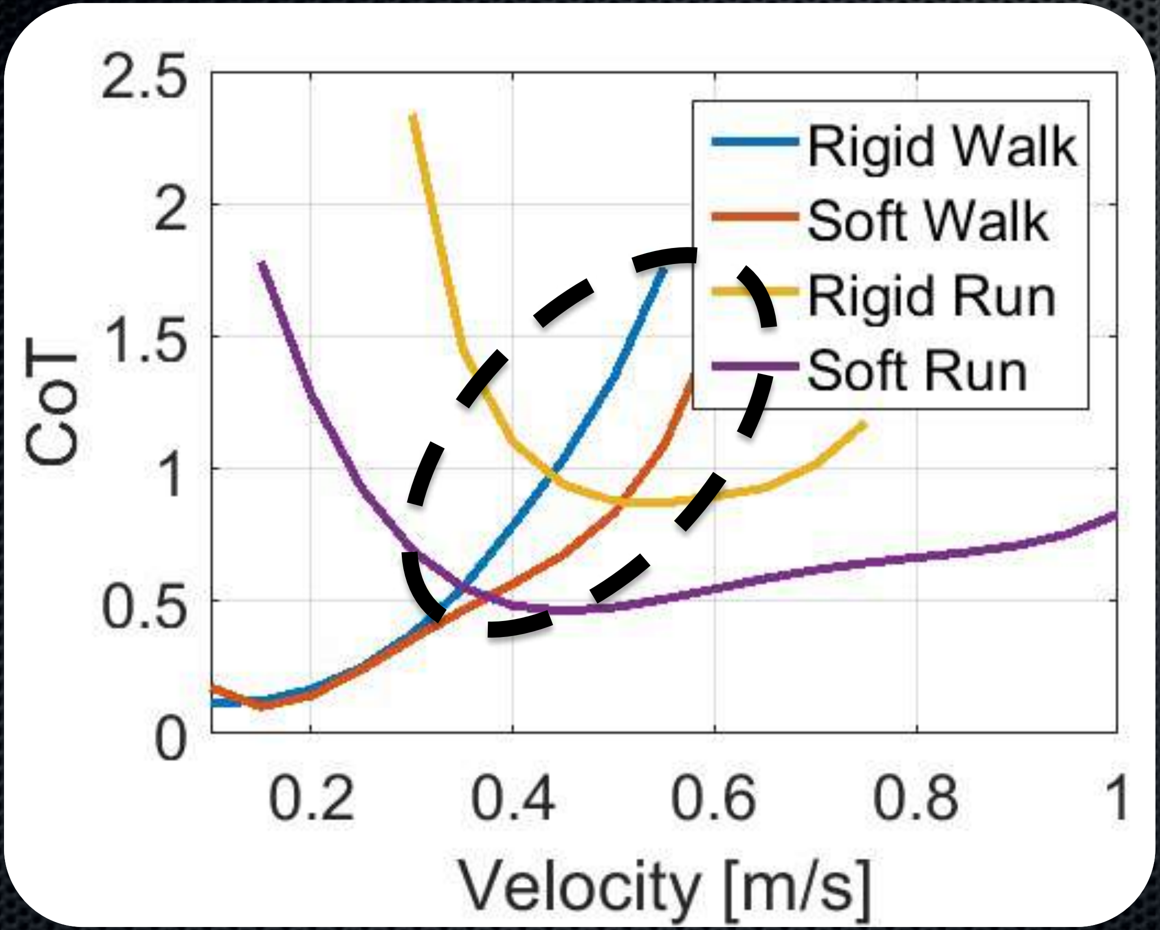
# Opt. Rigid Locomotion Vs Opt. Soft Locomotion




If compared to the rigid case, joint elasticity:

has a minor effect on slow walking

# Opt. Rigid Locomotion Vs Opt. Soft Locomotion

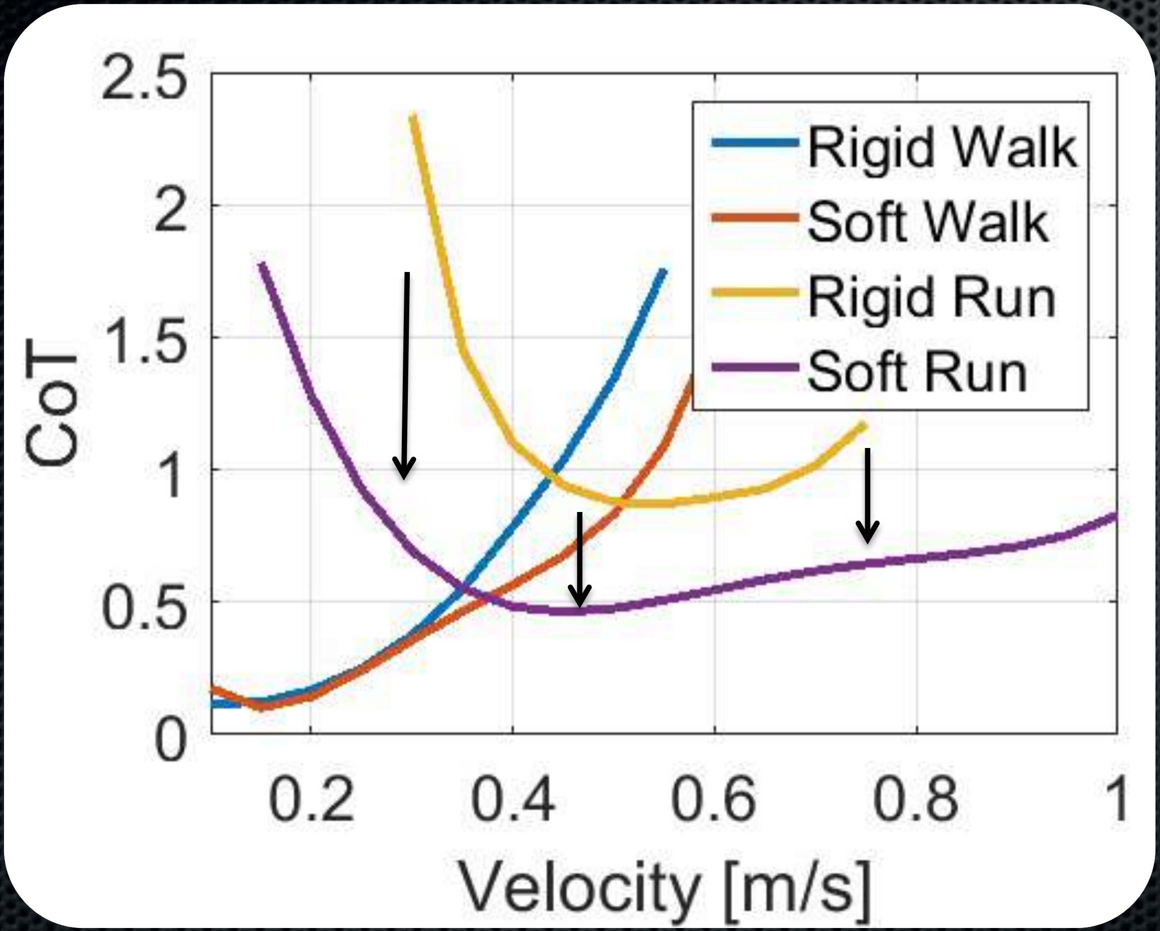



If compared to the rigid case, joint elasticity:

has a minor effect on slow walking

substantially reduces the CoT at fast walking

# Opt. Rigid Locomotion Vs Opt. Soft Locomotion



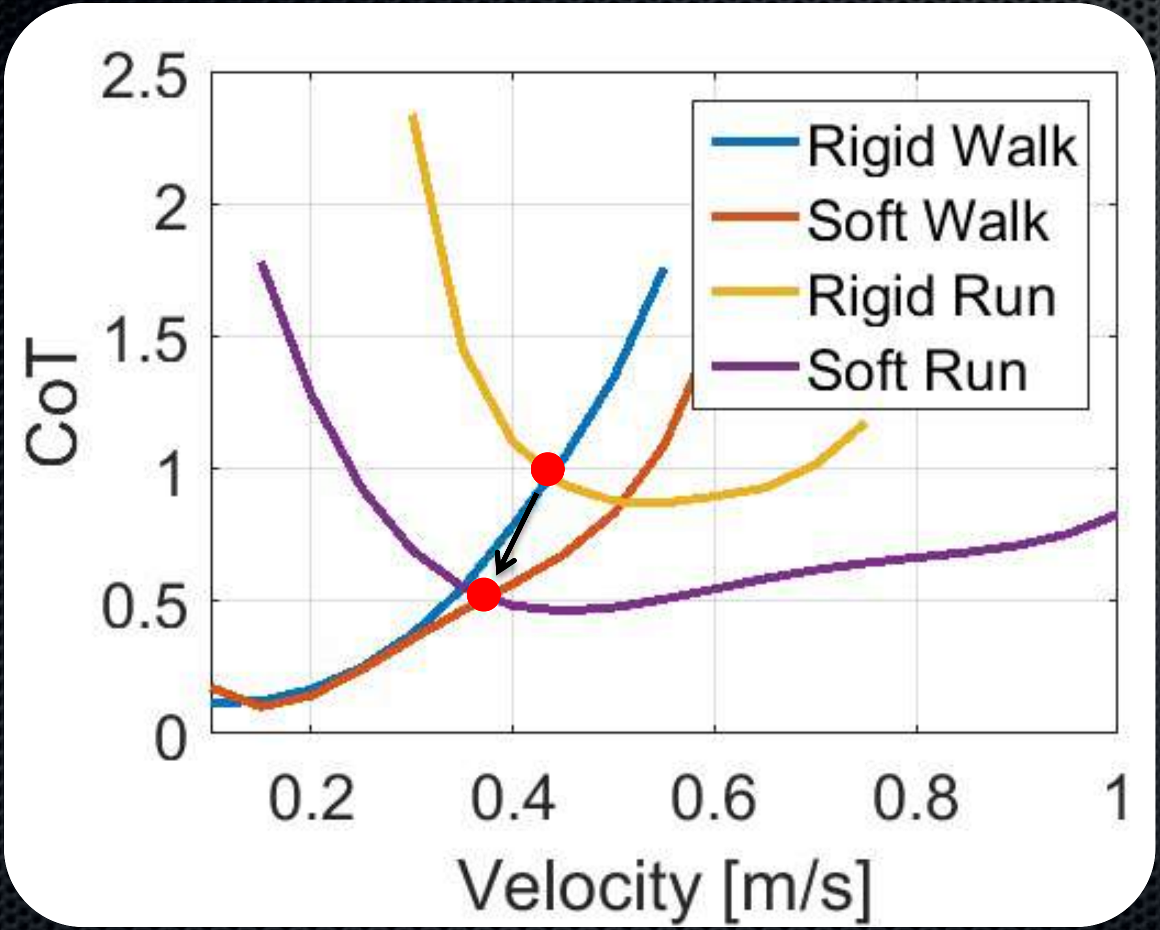

If compared to the rigid case, joint elasticity:

has a minor effect on slow walking

substantially reduces the CoT at fast walking

allows for remarkable energy savings in running

# Opt. Rigid Locomotion Vs Opt. Soft Locomotion




If compared to the rigid case, joint elasticity:

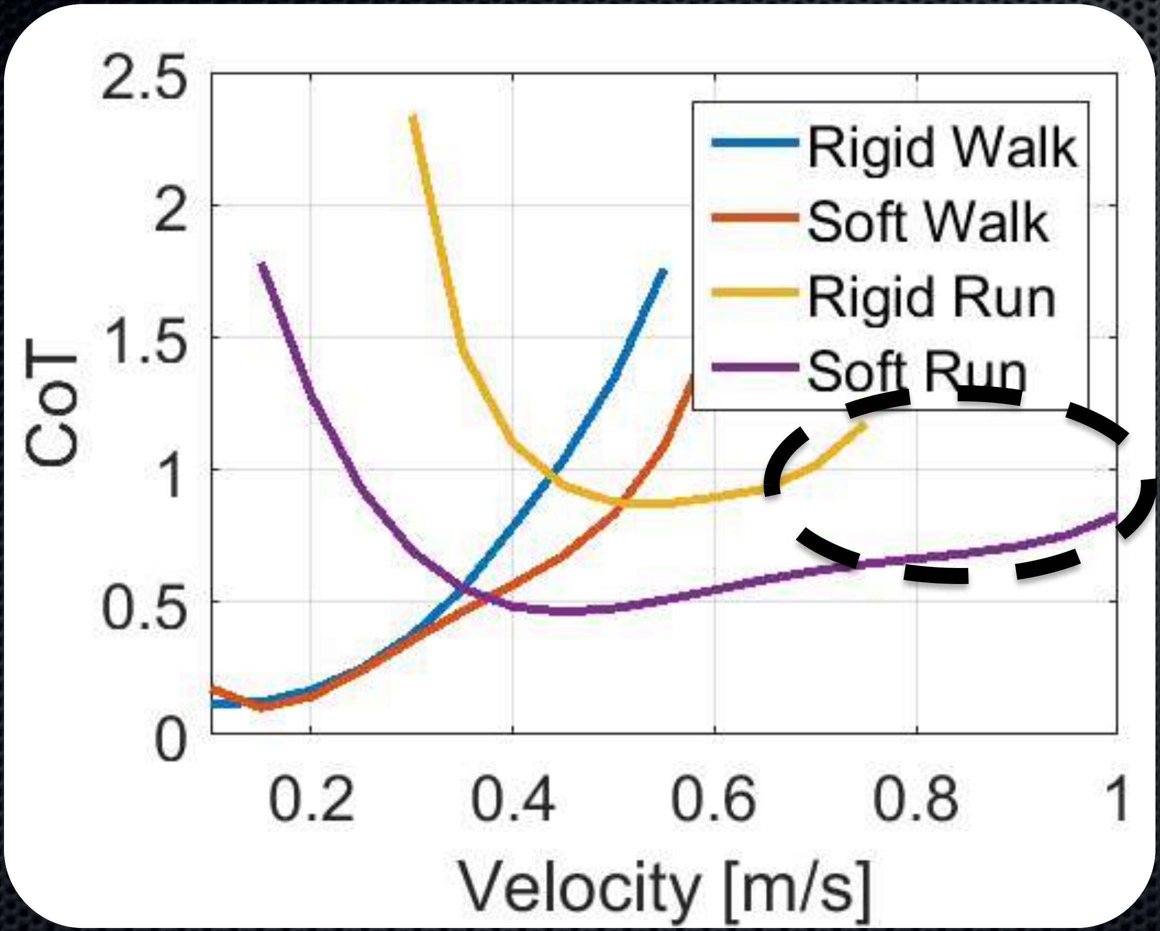
has a minor effect on slow walking

has a substantial effect on fast walking

allows for remarkable energy savings in running

reduces the walk-to-run transition speed

# Opt. Rigid Locomotion Vs Opt. Soft Locomotion




If compared to the rigid case, joint elasticity:

has a minor effect on slow walking

has a substantial effect on fast walking

allows for remarkable energy savings in running

reduces the walk2run transition speed

Increases the forward speed values for which running is feasible

# Experimental Setup: a planar 6 DoF Soft Biped

Powered by VSA
motor position controlled

Stiffness 60 Nm/rad

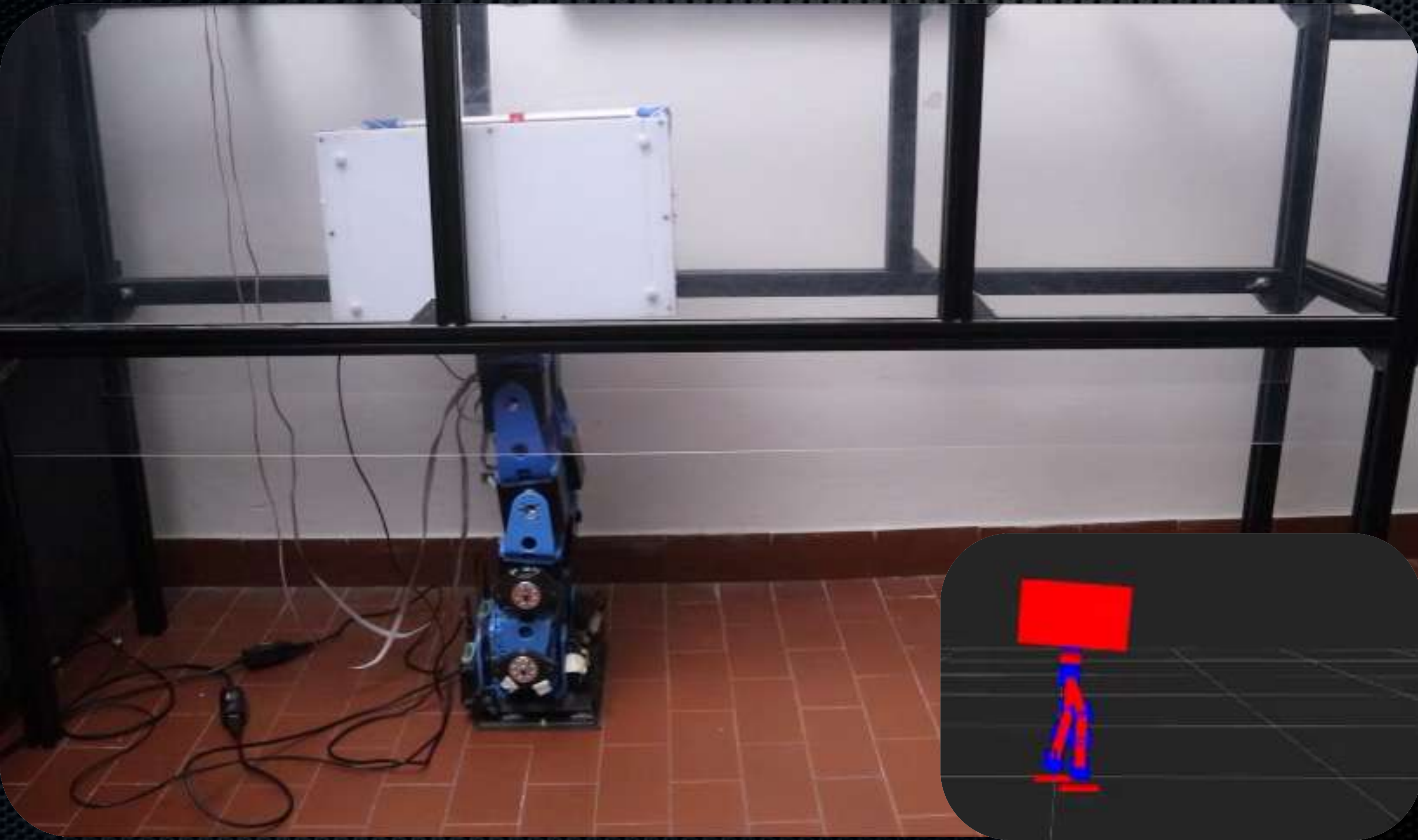
Remotized
ankle pitch

4 3-A Force sensors

Structure to constraint
the robot to evolve in
the sagittal plane

Total Mass 7.2 kg

# from simulations to experiments @ 0.1 m/s



0.1 m/s

0.2 m/s

0.3 m/s

Speeding Up

from 0.04 to 0.1 m/s

Slowing Down

from 0.1 to 0.04 m/s

# Is Numerical Optimization Viable?

Problem

The optimizations can not be performed at run-time

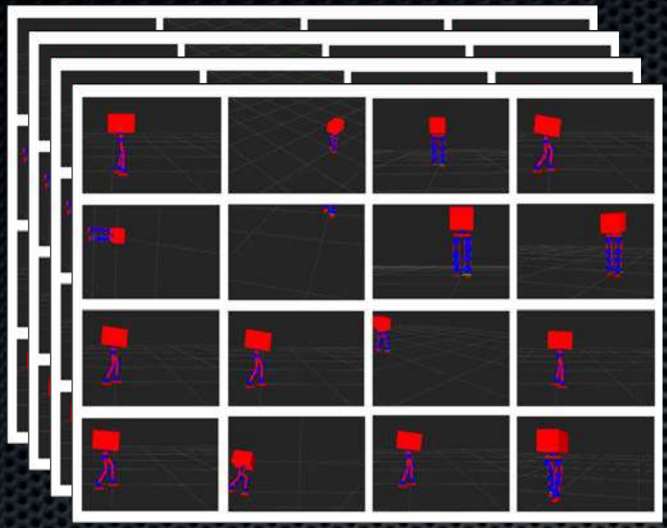
A possible solution: the off-line/on-line approach

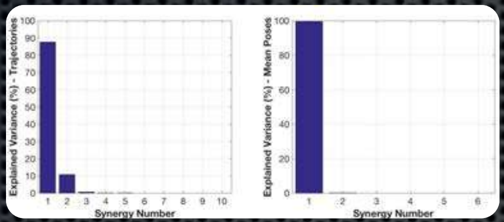
1) off-line phase to evaluate and store a Library of Optimized Trajectories
2) on-line phase to search the Library for a Trajectory and execute it

Example: full humanoid robot

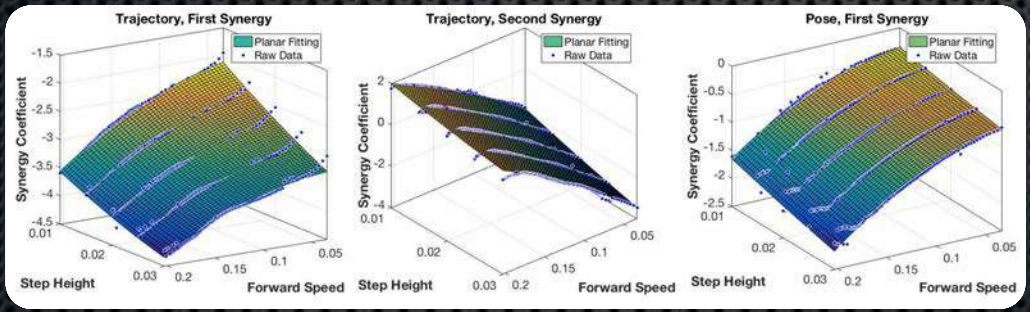
30 joints, 100 sample to describe the trajectories
10 task parameters (speeds, swing foot pose, ground params, …)
10 samples to span each parameter range

**The Optimized Trajectory Library would need 10^2 Tb**











**3 Second Order Polynomial functions encode 500 optimized trajectories**

## Problem

The Trajectory Library size for a realistic example is not manageable

## A possible Solution

Trajectory Library Encoding

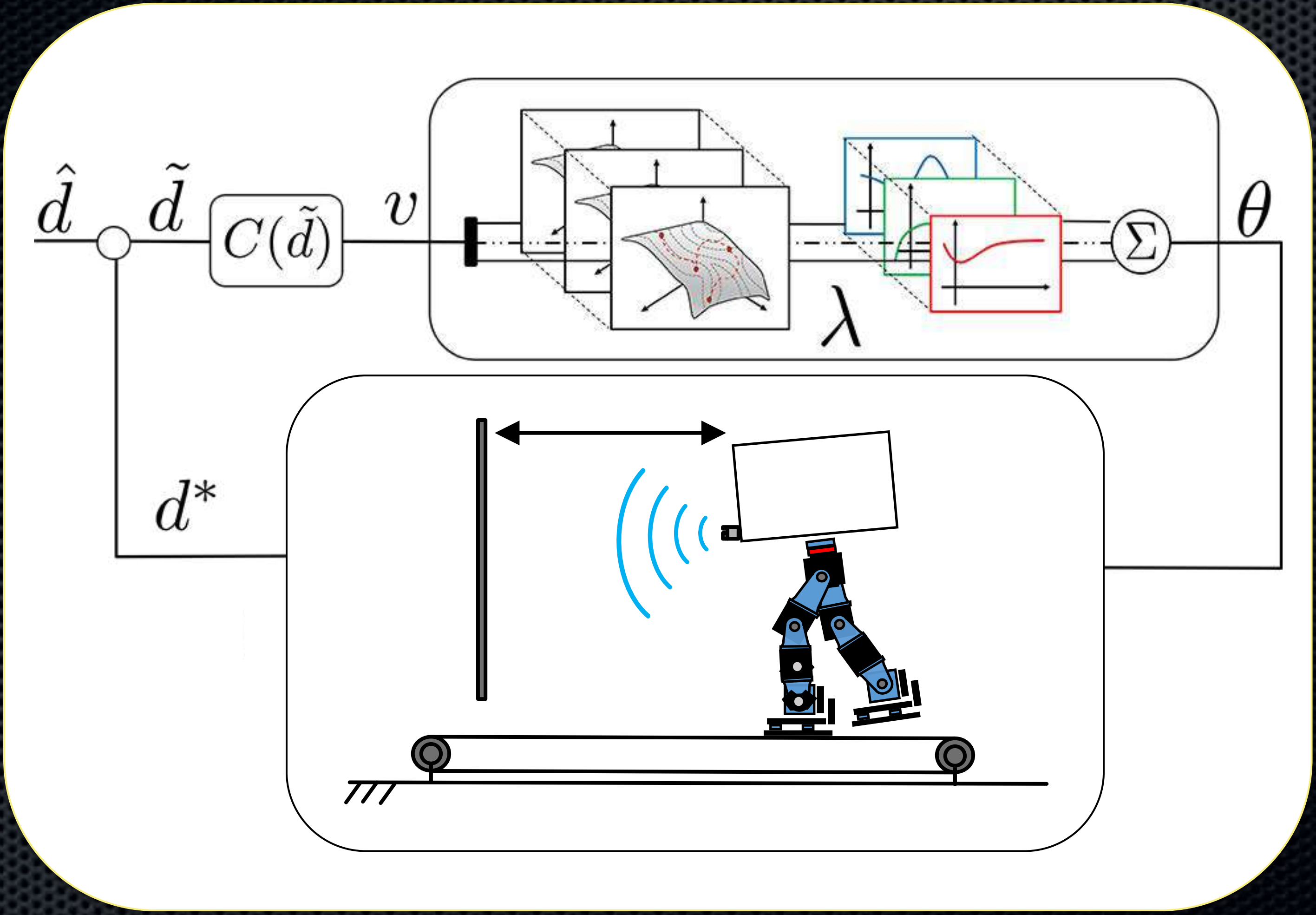
1) PCA

3 principal components allow to explain the 99% of the variance of 500 tasks

2) Mapping Functions

second order polynomials to express the dependence of the principal component weights on the task params

# Close the Loop

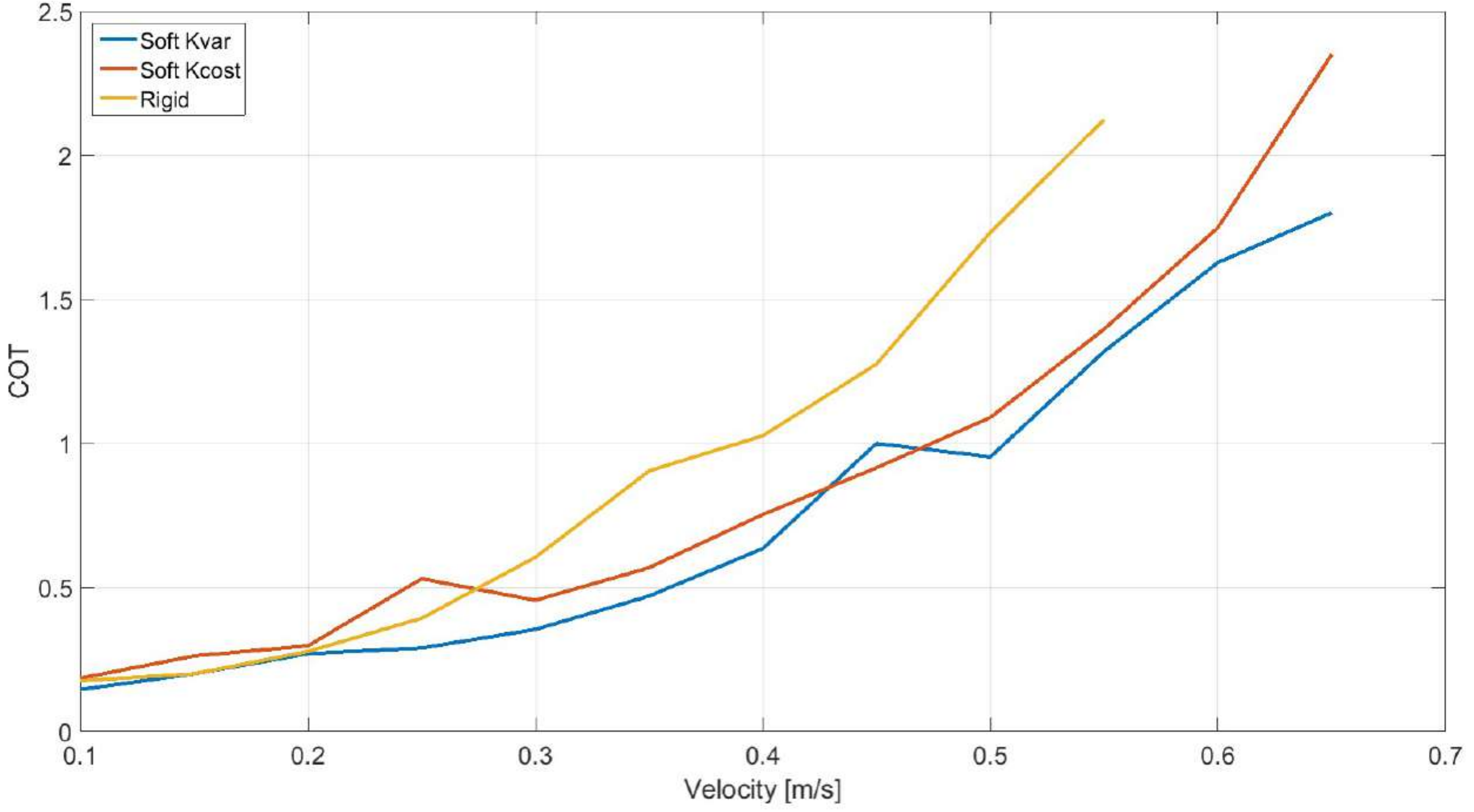


# Distance Regulation on Treadmill
# Start & Stop

# Distance Regulation on Treadmill

In 3 minutes treadmill speed goes from 0.3 km/h to 0.7 km/h and back

# Opt. Rigid Locomotion Vs Opt. Soft Locomotion



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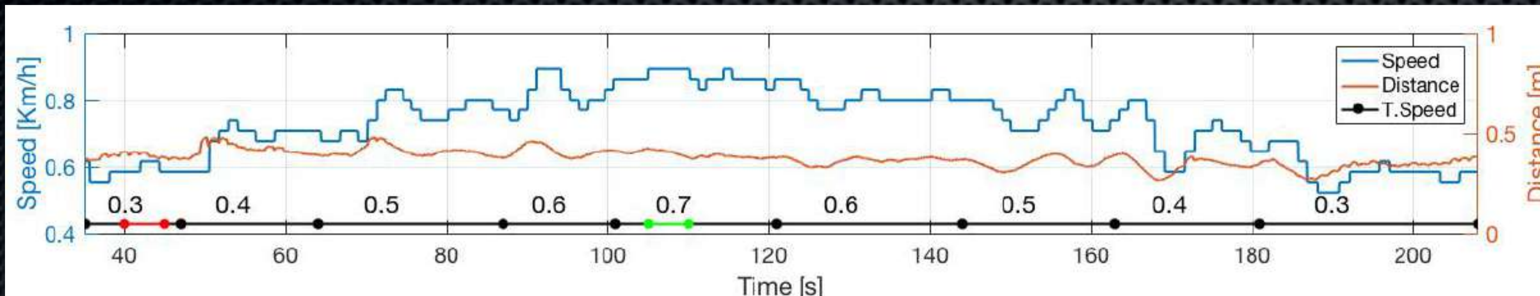
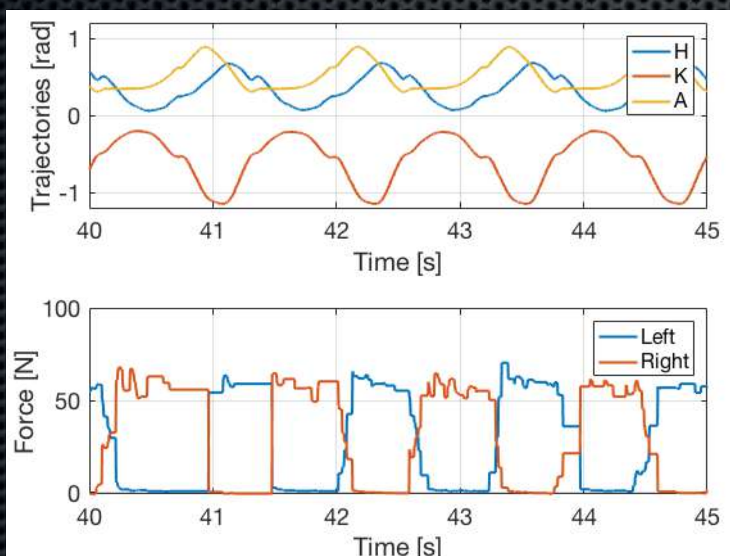
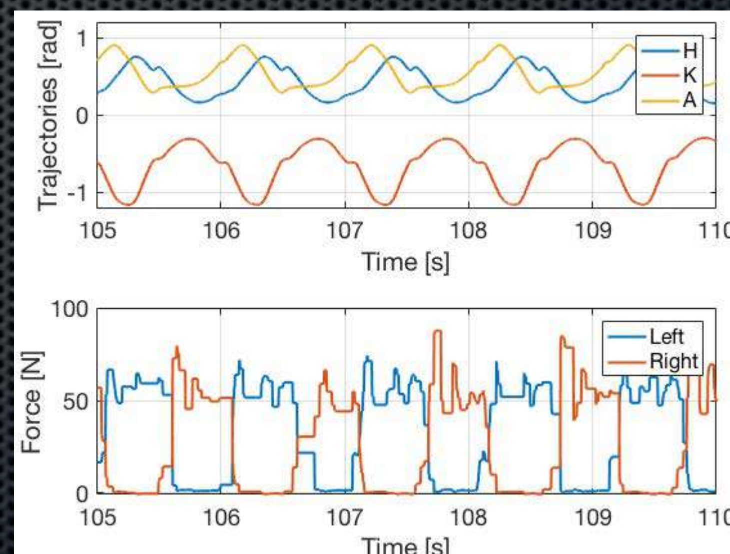



(a)

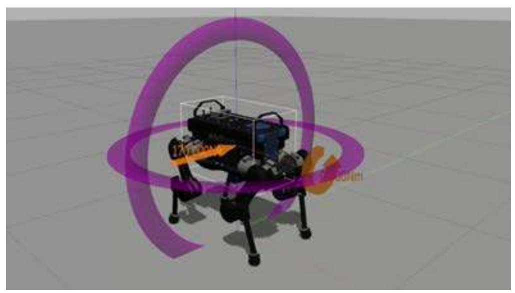
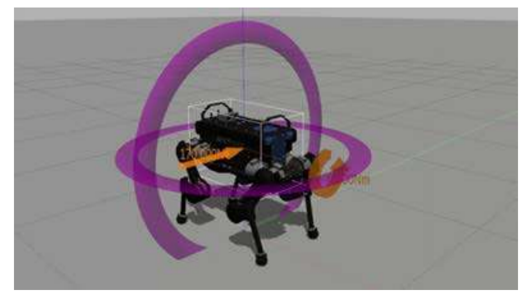
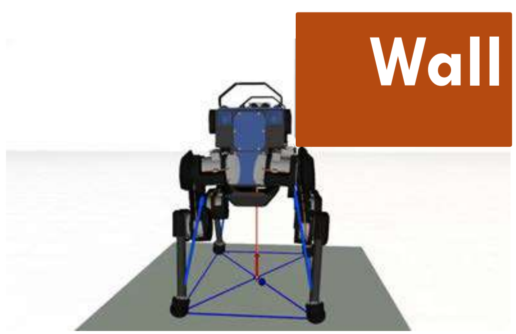
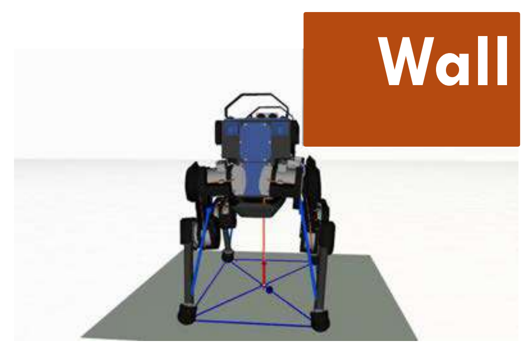


# ONLINE OPTIMAL IMPEDANCE PLANNING FOR LEGGED ROBOTS

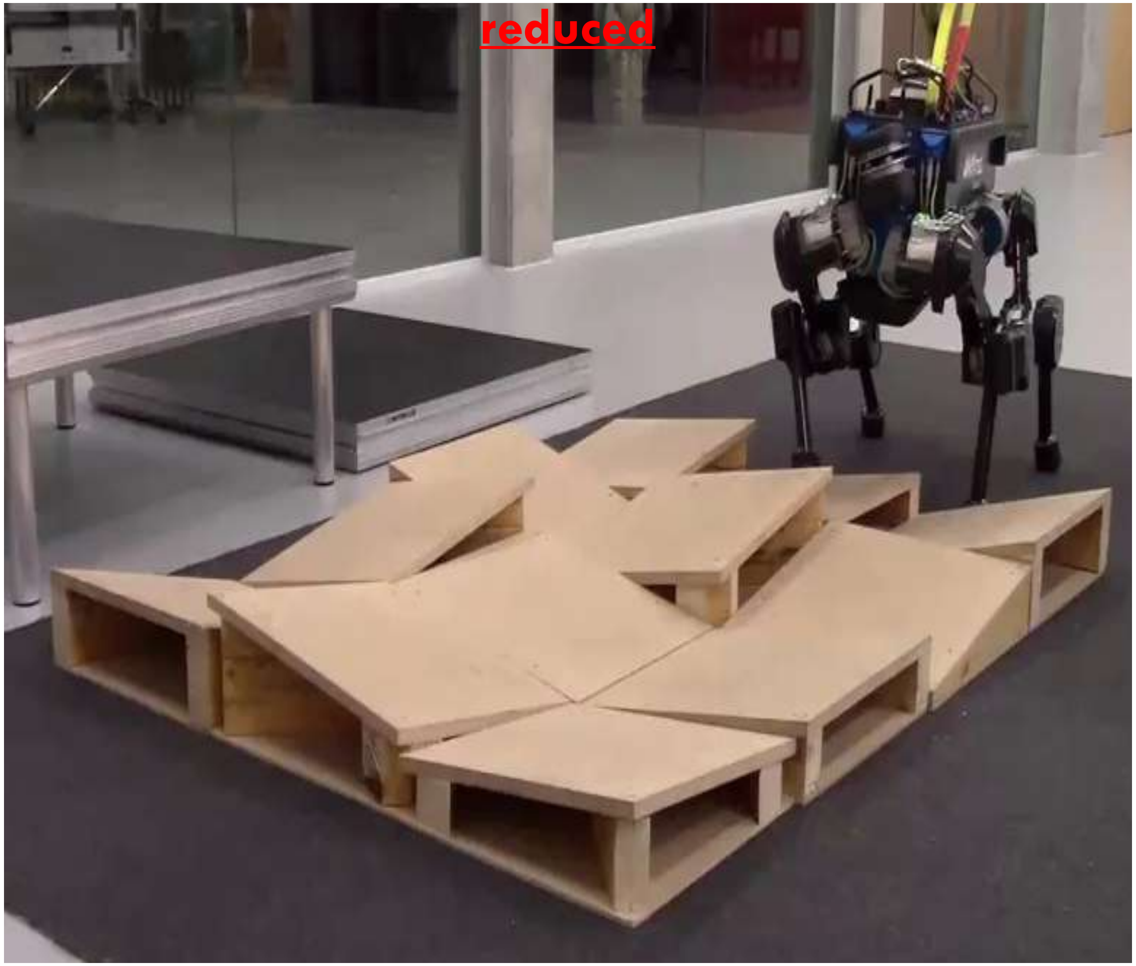
Franco Angelini, Guiyang Xin, Wouter J. Wolfslag, Carlo Tiseo, Michael Mistry, Manolo Garabini, Antonio Bicchi, and Sethu Vijayakumar

# Gains Trade-Off: Issue

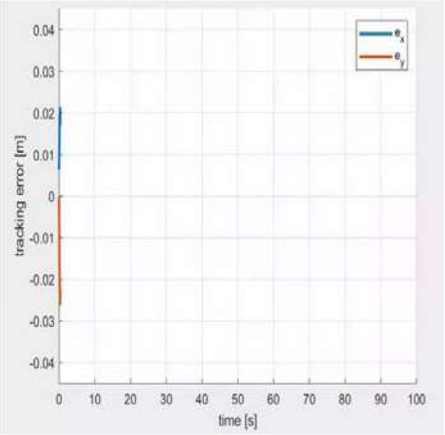
| | **<u>Low impedance</u>** | **<u>High impedance</u>** |
|---|---|---|
| Force disturbance |  |  |
| Wall interaction |   |   |

# Results – Walking on Rough Terrain

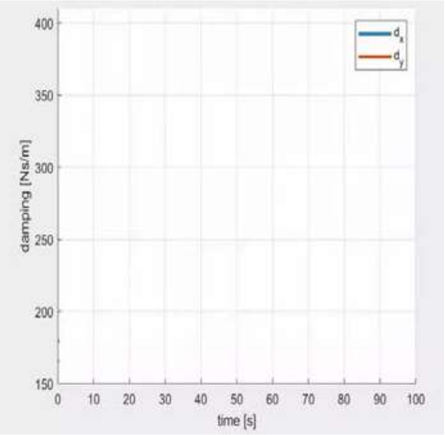
**Increasing task requirements: tracking error reduced**



Tracking error




Damping



# What's next?

- the model is not perfect!
  - exploit data as much as possible
    - Iterative Learning Control
  - combine model based and data driven techniques
- From articulated to continuous soft robots
  - modeling (fidelity/simplicity trade-off)
  - planning and control
    - the relative degree problem
      - Pierallini et al.

# What’s next?

2050

## The Dream

We proposed that the ultimate goal of the RoboCup Initiative to be stated as follows:

> *By the middle of the 21st century, a team of fully autonomous humanoid robot soccer players shall win a soccer game, complying with the official rules of FIFA, against the winner of the most recent World Cup.*

# What’s next?

RoboCup

2050

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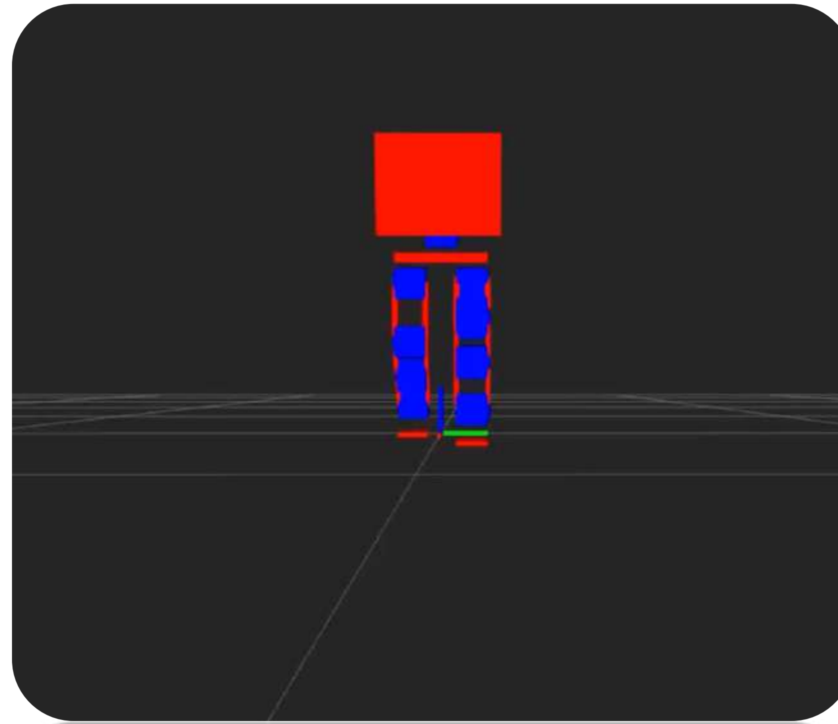
To make the Robocup dream come true robots will need soft bodies!

SIDRA Summer School 2021

# THANK YOU!



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# Optimal Planning and Control of Articulated Soft Robots

Manolo Garabini