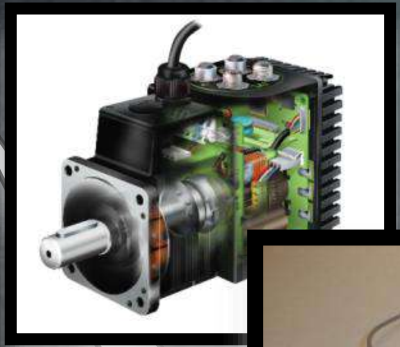
Characterization and Estimation of the Stiffness (or Impedance) of a Robot

by Giorgio Grioli


Part 0
a recap on Soft Robots and their actuation
by Giorgio Grioli

Evolution of actuation

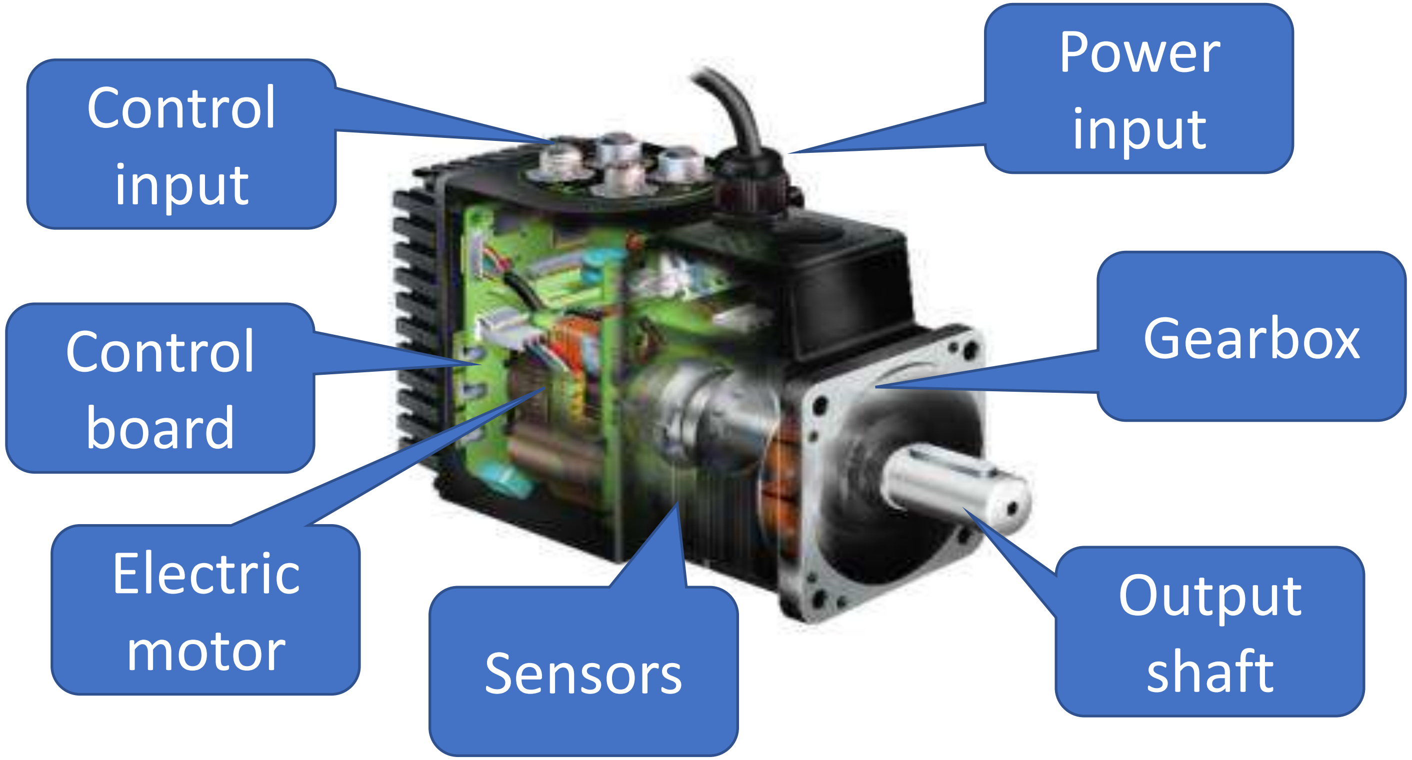




- Rigid actuation (e.g.: servomotors)
- Series Elastic Actuation
- Variable Stiffness and Variable Impedance Actuation

Evolution of robotic actuators

- Rigid actuators: servomotors
  - Standard of the robotic and automation industry



Evolution of robotic actuators

- Rigid actuators: servomotors
  - Standard of the robotic and automation industry
  - Designed to be fast, precise, repeatable
    → **rigid** by design
  - Most of them close an internal position loop
    →designed to behave as ideal position actuators

    given a desired position q_d their output shaft moves to that desired position

$$q \rightarrow q_d$$

Evolution of robotic actuators

- Rigid actuators: servomotors
  - Pro's
    - Very easy to use
    - Very accurate
  - Con's
    - Position sources do not handle constraints very well

When external constraints limit the output position

$$q \leq q_{limit}$$

but a position outside the limit is commanded $q_d > q_{limit}$

→ BAD things happen:

very high interaction torques can arise

which can be very dangerous for the robot & the environment

Evolution of robotic actuators

- Rigid actuators: servomotors
- Rigid robots II: torque-controlled servomotors

One more sensor:
A torque sensor

They measure the torque they apply on the load τ, and close a feedback on that, to make it follow a desired torque τ_d

$$\tau \to \tau_d$$

Evolution of robotic actuators

- Rigid actuators: servomotors
- Rigid robots II (or Soft robots 0?): torque-controlled servomotors
  - Pros's
    - Can be used to implement more advanced control strategies

    e.g.: impedance control
    - Nicer behavior on slow interaction tasks

  - Con's
    - The system is still rigid

    → fast interaction are still problematic

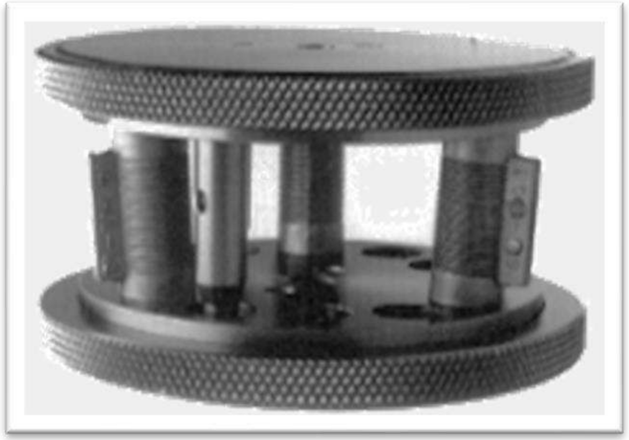
    How fast is "fast" is determined by the controller speed

Evolution of robotic actuators

- Rigid actuators
- Rigid robots III (Soft robots 0.5): soft add-ons

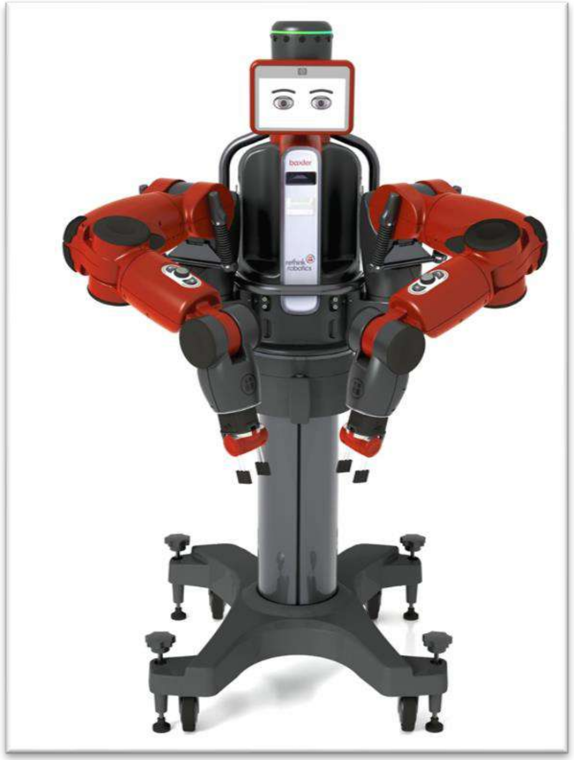
  A traditional rigid robot can be made soft using soft covers and/or end-effectors

  - Remote Center of Compliance are the most famous and used examples



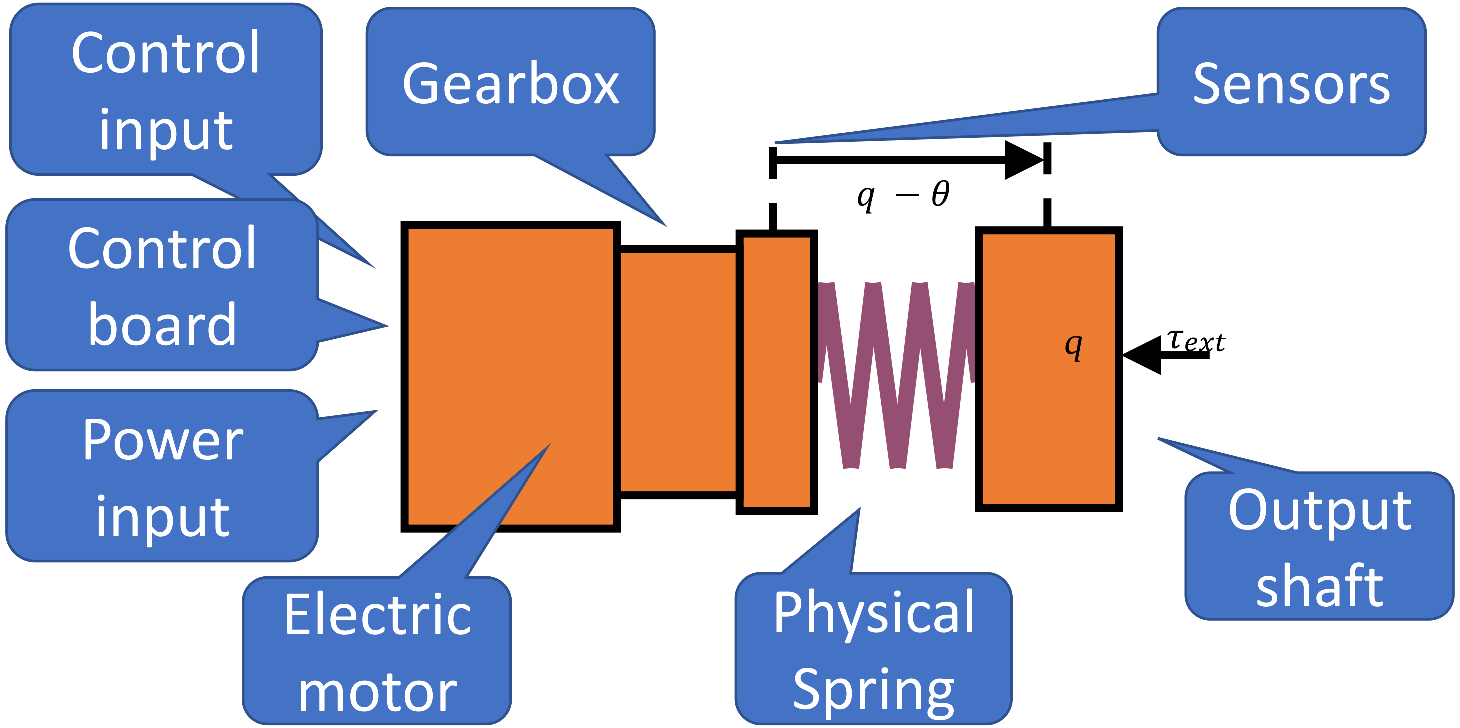
History and evolution

- Rigid actuators
- Soft actuators 1: Series Elastic Actuators (SEAs)
  - Standard in research
  - Early industrial products
    - KUKA LWR
    - Rethink robotics - Baxter



History and evolution

- Rigid actuators
- Soft actuators 1: Series Elastic Actuators (SEAs)



History and evolution

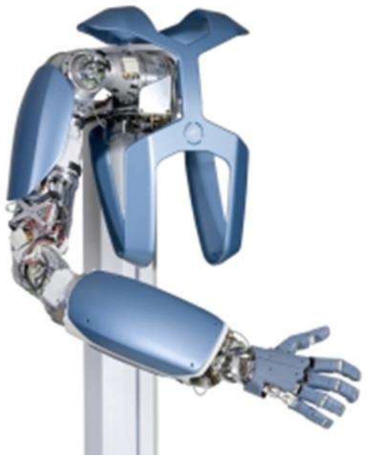
- Rigid actuators
- Soft actuators 1: Series Elastic Actuators (SEAs)
  - A physical spring with elastic constant k is put in series with the motor to the external load
  - The position of the motor θ and the position of the output shaft q are no longer the same
  - A feedback loop can be closed on the motor position θ to let the torque on the link τ_{ext} follow a desired torque τ_d
  - The physical spring in series with the motor yields that even when something happens at speeds that are faster than the controller (e.g. impacts), the system still behaves as the spring.

History and evolution

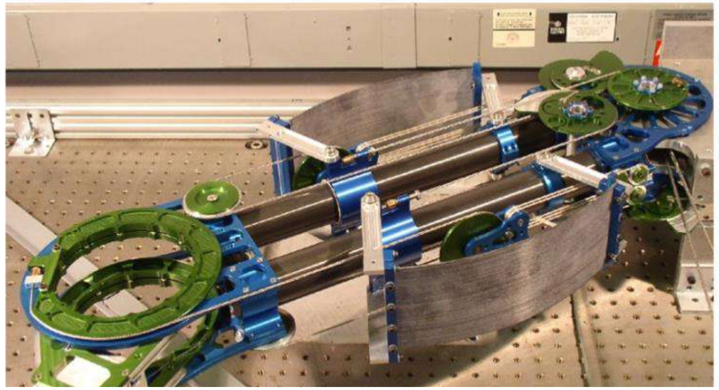
- Rigid actuators
- Soft actuators 1: Series Elastic Actuators (SEAs)
  - Pro’s
    - At low frequencies the system can be torque controlled
    - At high frequencies the system behavior is elastic
  - Con’s
    - The spring (along with the link inertia) yields a natural oscillating frequency of the system
    - This limits, in practice, the bandwidth of the torque which can be transmitted from the motor side to the link side
    - Ultimately limits the performance of the control

History and evolution

- Rigid actuators
- Soft actuators 1: Series Elastic Actuators (SEAs)
- Soft actuators 1I: Variable Stiffness Actuators (VSAs)
  - Advanced research stage
  - Lots of solutions studied and developed
  - Starting to exit outside creators' laboratories



DLR Hand-Arm system, 2012



AMASC – Hurst – Migliore, 2004

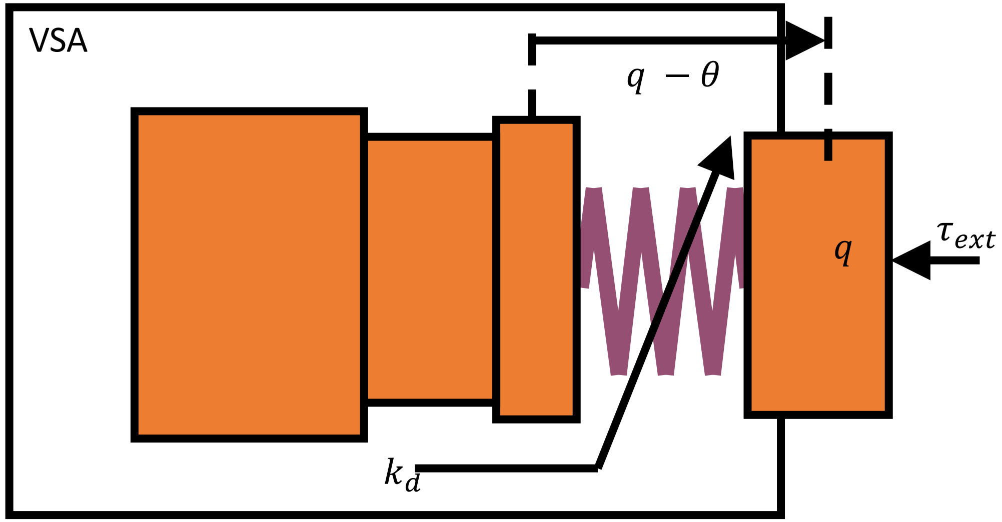
History and evolution

- Rigid actuators
- Soft actuators 1: Series Elastic Actuators (SEAs)
- Soft actuators 1I: Variable Stiffness Actuators (VSAs)
  - Since the stiffness k of the SEA limits its maximum bandwidth
  - Since my bandwidth requirements are not the same every time
    - E.g.: soft in case of hits, or while pushing against a surface, rigid while doing fast accelerations/decelerations or when precision is needed

→ The perfect solution would be to be able to change the stiffness

History and evolution

- Rigid actuators
- Soft actuators 1: Series Elastic Actuators (SEAs)
- Soft actuators 1I: Variable Stiffness Actuators (VSAs)
  - First definition of a VSA:
    an SEA-like system, where the stiffness can be changed online



History and evolution

- Rigid actuators
- Soft actuators 1: Series Elastic Actuators (SEAs)
- Soft actuators 1I: Variable Stiffness Actuators (VSAs)

We will be talking about VSA later.
…but since we are looking at the landscape,
let’s keep sightseeing for a little while…

History and evolution

- Rigid actuators
- Soft actuators 1: Series Elastic Actuators (SEAs)
- Soft actuators 1I: Variable Stiffness Actuators (VSAs)
- Soft actuators III: Variable Impedance Actuators (VIAs)
  - Early prototypes
  - Hot research topic

History and evolution

- Rigid actuators
- Soft actuators 1: Series Elastic Actuators (SEAs)
- Soft actuators 1I: Variable Stiffness Actuators (VSAs)
- Soft actuators III: Variable Impedance Actuators (VIAs)
  - Stiffness is not the only physical element which ca be put in series with a motor
    - Damping is the very next thing that comes in someone's mind
      → variable damping actuators
    - Variable Inertia could be another possibility

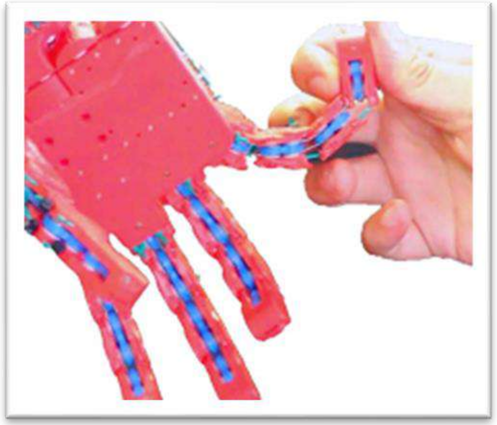
    Related work can be found in the field of energy harvesting: KERS and high efficiency flywheels
    - Also non-linear SEA are classically included in this category
    - Multiple parallel SEA or SPEA

History and evolution

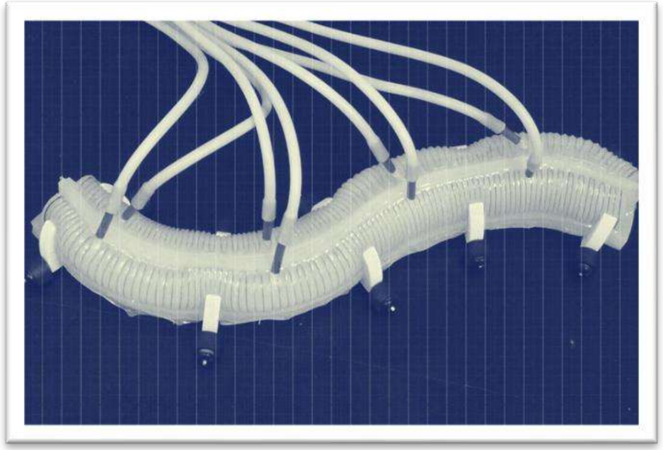
- Rigid actuators
- Soft actuators 1: Series Elastic Actuators (SEAs)
- Soft actuators 1I: Variable Stiffness Actuators (VSAs)
- Soft actuators III: Variable Impedance Actuators (VIAs)
- Soft structure:
  - Another hot topic

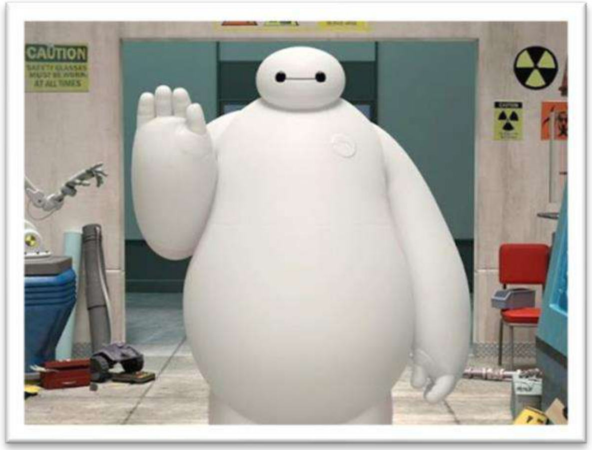
History and evolution

- Rigid actuators
- Soft actuators 1: Series Elastic Actuators (SEAs)
- Soft actuators 1I: Variable Stiffness Actuators (VSAs)
- Soft actuators III: Variable Impedance Actuators (VIAs)



- Soft structure: distributed softness robot
  - Soft joints
  - Soft links
  - Soft bodies





History and evolution

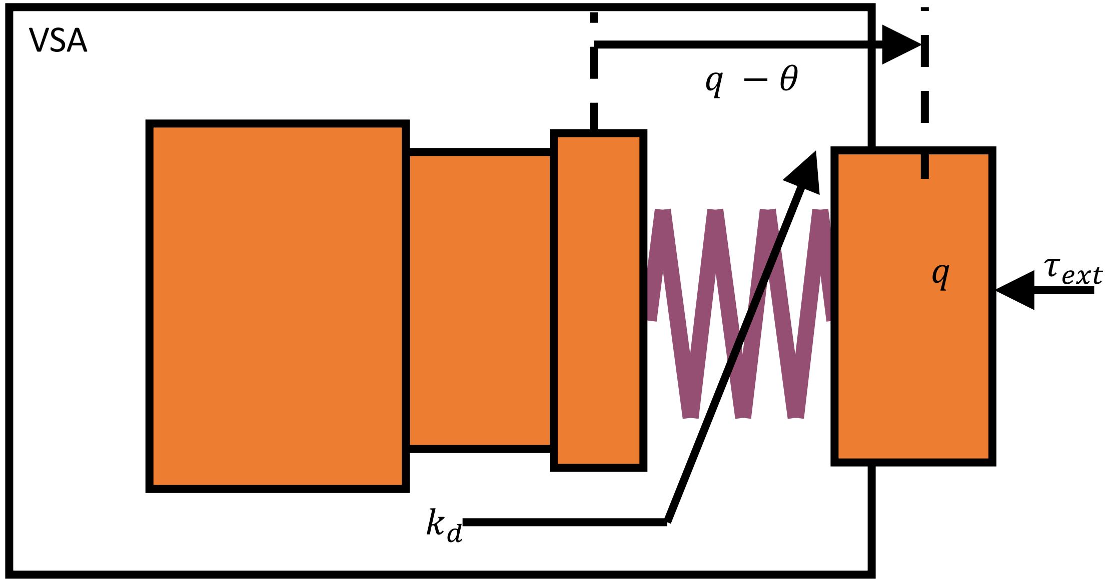
- Rigid actuators
- Soft actuators 1: Series Elastic Actuators (SEAs)
- Soft actuators 1I: Variable Stiffness Actuators (VSAs)
- Soft actuators III: Variable Impedance Actuators (VIAs)
- Soft structure: distributed softness robot

A focus on VSAs

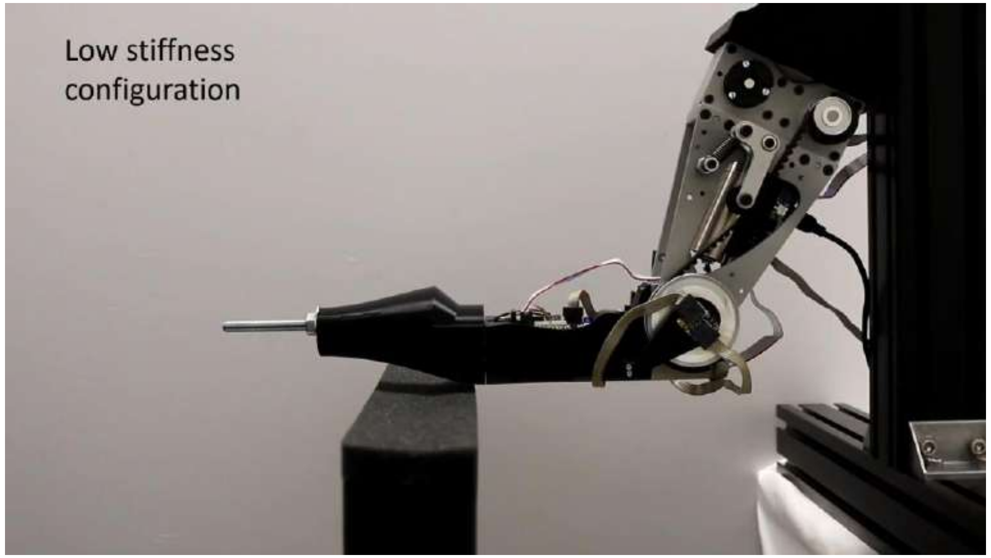
Models of VSAs

Models of VSAs

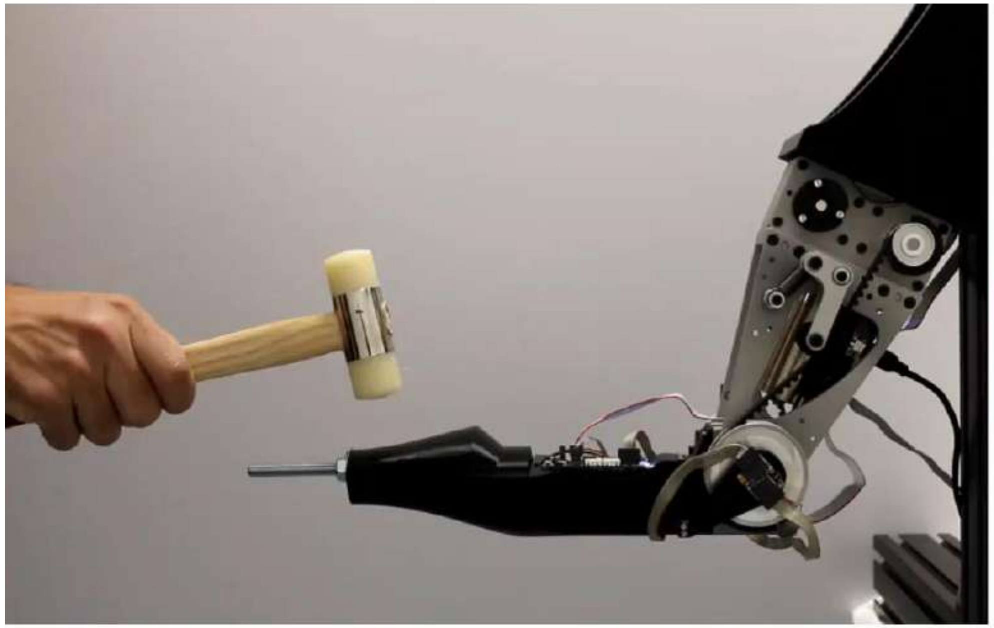
- Previous definition of a VSA:
  an SEA-like system, where the stiffness can be changed online



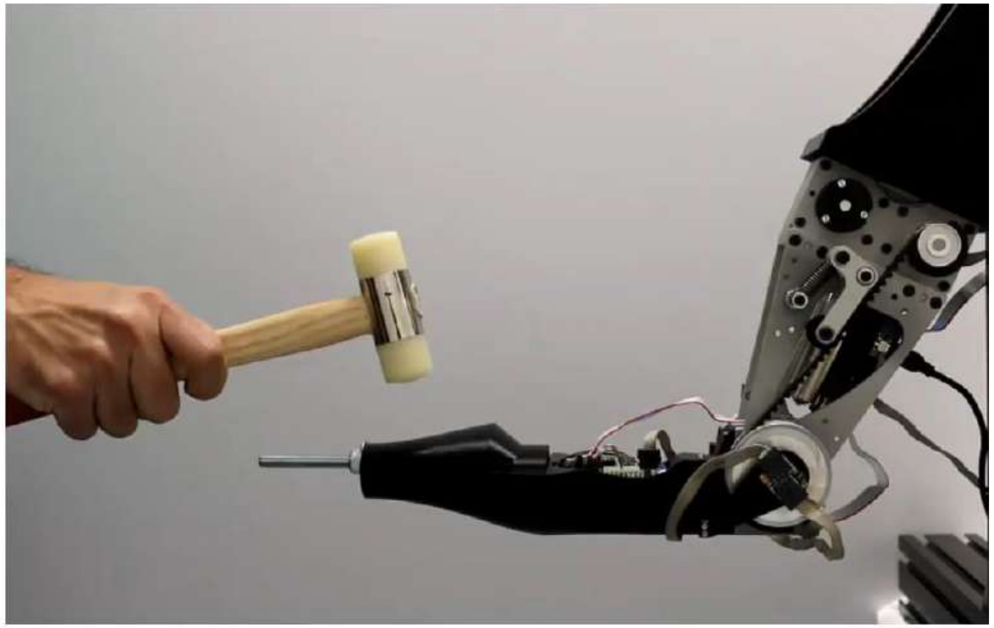
- Can we do something more precise? Sure…



Low stiffness configuration

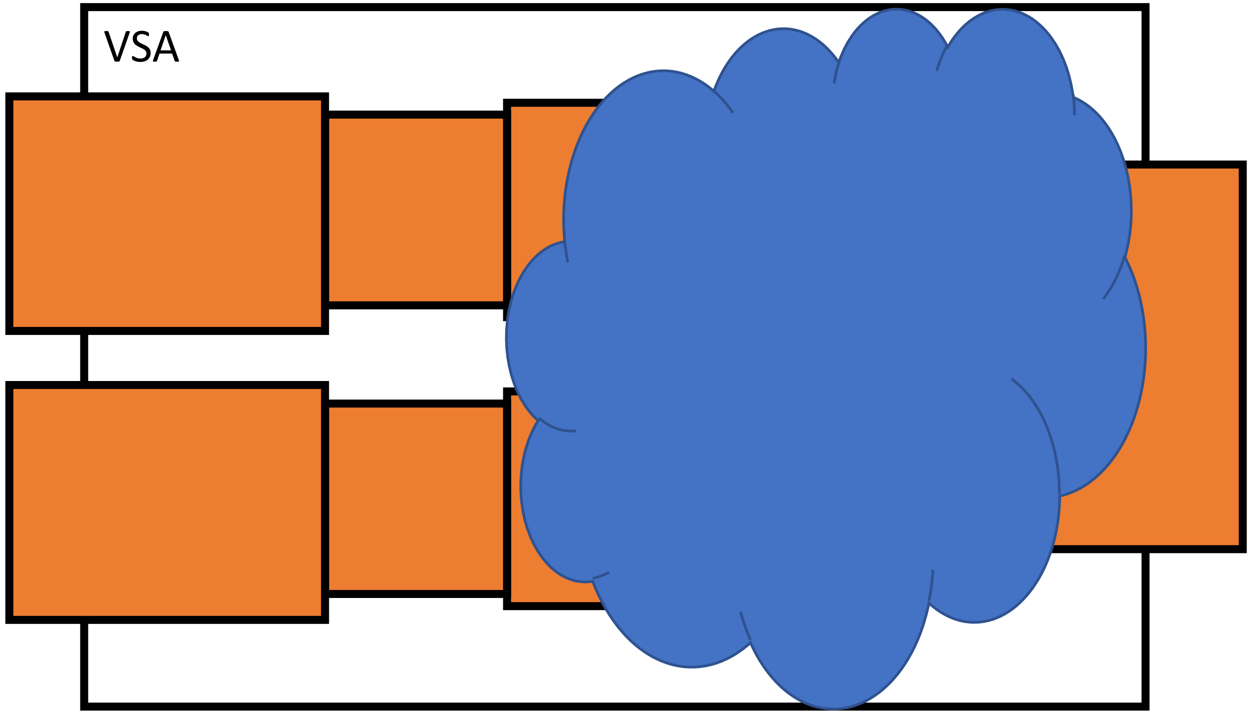


High stiffness configuration



Generic model of a VSA

- A mechanical system which relates the dynamics of three bodies (two motors and the link)



Generic model of a VSA

- A mechanical system which relates the dynamics of three bodies (two motors and the link)

- Dynamics

$$\begin{cases} B(q)\,\ddot{q} + N(q,\dot{q}) = U_q(q,\theta_1,\theta_2) + \tau_{ext} \\ I_1\ddot{\theta}_1 + d_1\dot{\theta}_1 = U_{\theta_1}(q,\theta_1,\theta_2) + \tau_1 \\ I_2\ddot{\theta}_2 + d_2\dot{\theta}_2 = U_{\theta_2}(q,\theta_1,\theta_2) + \tau_2 \\ \dot{\tau}_1 = -p_1\tau_1 - c_1\dot{\theta}_1 + b_1 v_1 \\ \dot{\tau}_2 = -p_2\tau_2 - c_2\dot{\theta}_2 + b_2 v_2 \end{cases}$$

- State $[q,\theta_1,\theta_2,\dot{q},\dot{\theta}_1,\dot{\theta}_2]$

Generic model of a VSA

- A closer look.

Robot dynamics

Motors dynamics

Electrical dynamics

$$
\begin{cases}
B(q)\,\ddot{q} + N(q,\dot{q}) = U_q(q,\theta_1,\theta_2) + \tau_{ext} \\
I_1\ddot{\theta}_1 + d_1\dot{\theta}_1 = U_{\theta_1}(q,\theta_1,\theta_2) + \tau_1 \\
I_2\ddot{\theta}_2 + d_2\dot{\theta}_2 = U_{\theta_2}(q,\theta_1,\theta_2) + \tau_2 \\
\dot{\tau}_1 = -p_1\tau_1 - c_1\theta_1 + b_1 v_1 \\
\dot{\tau}_2 = -p_2\tau_2 - c_2\dot{\theta}_2 + b_2 v_2
\end{cases}
$$

Generic model of a VSA

- A closer look.

$$\begin{cases} B(q)\,\ddot{q} + N(q,\dot{q}) = U_q(q,\theta_1,\theta_2) + \boxed{\tau_{ext}} \\ I_1\ddot{\theta}_1 + d_1\dot{\theta}_1 = U_{\theta_1}(q,\theta_1,\theta_2) + \tau_1 \\ I_2\ddot{\theta}_2 + d_2\dot{\theta}_2 = U_{\theta_2}(q,\theta_1,\theta_2) + \tau_2 \\ \dot{\tau}_1 = -p_1\tau_1 - c_1\dot{\theta}_1 + b_1 v_1 \\ \dot{\tau}_2 = -p_2\tau_2 - c_2\dot{\theta}_2 + b_2 v_2 \end{cases}$$

Disturbance

Inputs (2)

Configuration $[q,\theta_1,\theta_2] \in R^3$

Inputs $[\tau_1,\tau_2] \in R^2$

UNDERACTUATED!!!

Generic model of a VSA

- A closer look.

$$\begin{cases} B(q)\,\ddot{q} + N(q,\dot{q}) = U_q(q,\theta_1,\theta_2) + \boxed{\tau_{ext}} \\ I_1\ddot{\theta}_1 + d_1\dot{\theta}_1 = U_{\theta_1}(q,\theta_1,\theta_2) + \boxed{\tau_1} \\ I_2\ddot{\theta}_2 + d_2\dot{\theta}_2 = U_{\theta_2}(q,\theta_1,\theta_2) + \boxed{\tau_2} \\ \dot{\tau}_1 = -p_1\tau_1 - c_1\dot{\theta}_1 + b_1 v_1 \\ \dot{\tau}_2 = -p_2\tau_2 - c_2\dot{\theta}_2 + b_2 v_2 \end{cases}$$

Disturbance

Inputs (2)

Configuration $[q,\theta_1,\theta_2] \in R^3$

Inputs $[\tau_1,\tau_2] \in R^2$

UNDERACTUATED!!!

Generic model of a VSA

- The Electric part can be neglected (it's usually much faster)

$$\begin{cases} B(q)\,\ddot{q} + N(q,\dot{q}) = \boxed{U_q(q,\theta_1,\theta_2)} + \tau_{ext} \\ I_1\ddot{\theta}_1 + D_1\dot{\theta}_1 = \boxed{U_{\theta_1}(q,\theta_1,\theta_2)} + \tau_1 \\ I_2\ddot{\theta}_2 + D_2\dot{\theta}_2 = \boxed{U_{\theta_2}(q,\theta_1,\theta_2)} + \tau_2 \end{cases}$$

Elastic torques
The partial derivatives of the energy function $U(q,\theta_1,\theta_2)$

The second derivative
$U_{qq}(q,\theta_1,\theta_2) = \sigma$
it is, by definition, the **stiffness**

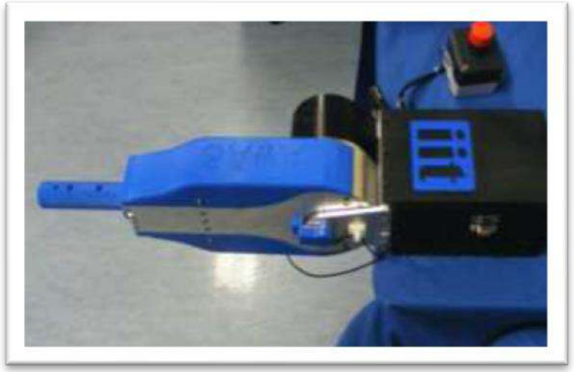
Categories of VSAs

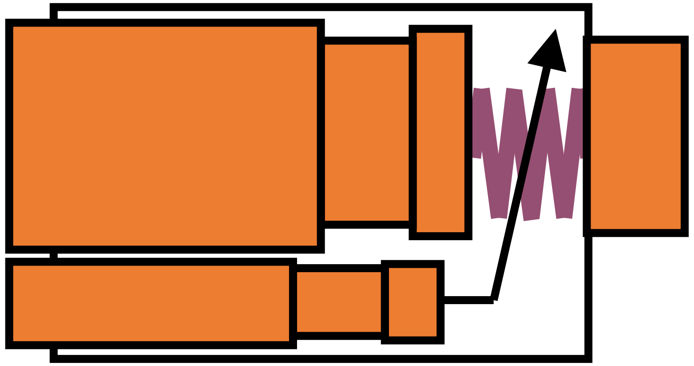
Special models of VSAs

Explicit Stiffness Variation Actuators

$$\begin{cases} B(q)\,\ddot{q} + N(q,\dot{q}) = U_q(\theta_1 - q, \theta_2) + \tau_{ext} \\ I_1\ddot{\theta}_1 + D_1\dot{\theta}_1 = U_{\theta_1}(\theta_1 - q, \theta_2) + \tau_1 \\ I_2\ddot{\theta}_2 + D_2\dot{\theta}_2 = U_{\theta_2}(\theta_1 - q, \theta_2) + \tau_2 \end{cases}$$

- One motor drives the equilibrium position alone (θ_1)
- The other drives the stiffness change
- Very similar to the original idea of a Variable SEA

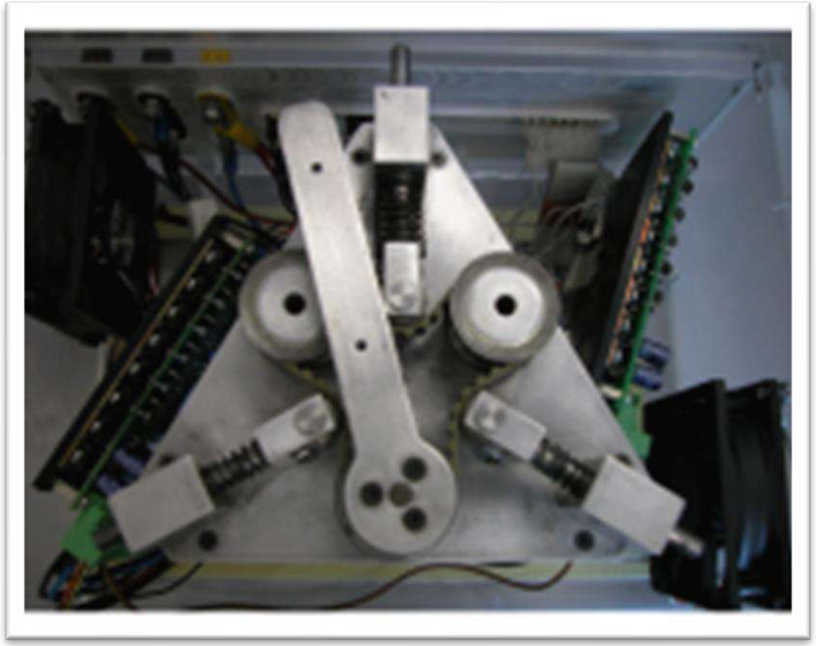




Agonist-Antagonist VSA

$$\begin{cases} B(q)\,\ddot{q} + N(q,\dot{q}) = U_q(\theta_1 - q, \theta_2 - q) + \tau_{ext} \\ I_1\ddot{\theta}_1 + D_1\dot{\theta}_1 = U_{\theta_1}(\theta_1 - q, \theta_2 - q) + \tau_1 \\ I_2\ddot{\theta}_2 + D_2\dot{\theta}_2 = U_{\theta_2}(\theta_1 - q, \theta_2 - q) + \tau_2 \end{cases}$$

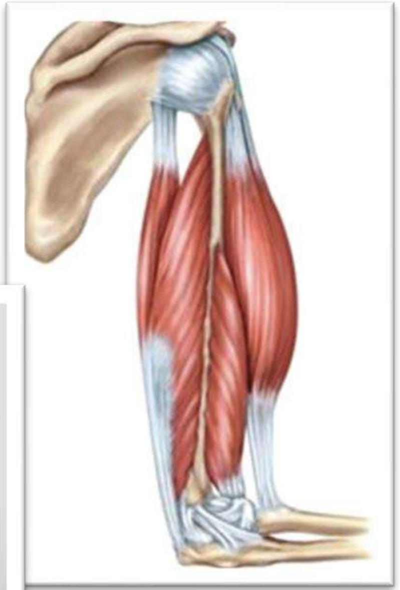
- Both motors act on the output, in parallel
- Both motors contribute simultaneously to change the stiffness and to move the output shaft



Decoupled Agonist-Antagonist VSA

$$\begin{cases} B(q)\,\ddot{q} + N(q,\dot{q}) = -U_{\theta_1}(\theta_1 - q)\; - U_{\theta_2}(\theta_2 - q) + \tau_{ext} \\ I_1\ddot{\theta}_1 + D_1\dot{\theta}_1 = U_{\theta_1}(\theta_1 - q) + \tau_1 \\ I_2\ddot{\theta}_2 + D_2\dot{\theta}_2 = U_{\theta_2}(\theta_2 - q) + \tau_2 \end{cases}$$

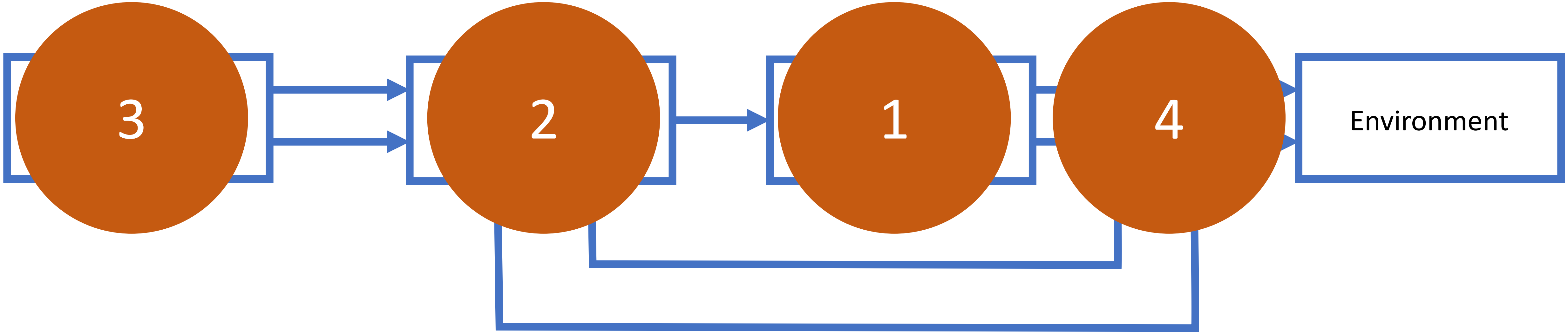
- Both motors act on the output in parallel
- Each through a separate spring
  - The two motors are thus decoupled



You are here!

a
map
of Soft
Robotics

What’s the Matter with VSA/VIA Robots



Three major challenges

1. Design
2. Control
3. Planning
4. Sensing

VSA: the 4 challenges

1. Design: how to make a VSA
   - Variable Spring pre-loading
   - Variable Spring geometry
   - Variable Transmission geometry
   - …

see: Vanderborght, B., Albu-Schäffer, A., Bicchi, A., Burdet, E., Caldwell, D. G., Carloni, R., ... & Wolf, S. (2013). Variable impedance actuators: A review. *Robotics and autonomous systems*, *61*(12), 1601-1614.

VSA: the 4 challenges

1. Design: how to make a VSA
2. Control: how to get desired position and impedance
    - PD control
    - […]
    - Feedback linearization

VSA: the 4 challenges

1. Design: how to make a VSA
2. Control: how to get desired position and impedance
3. Planning: what to do with impedance
    - Safety → safe brachistochrone
    - Efficiency → energy optimization
    - Adaptability → explicit impedance control
    - Bio-mimesis → …
    - Learning → …
    - Robustness → …
    - …

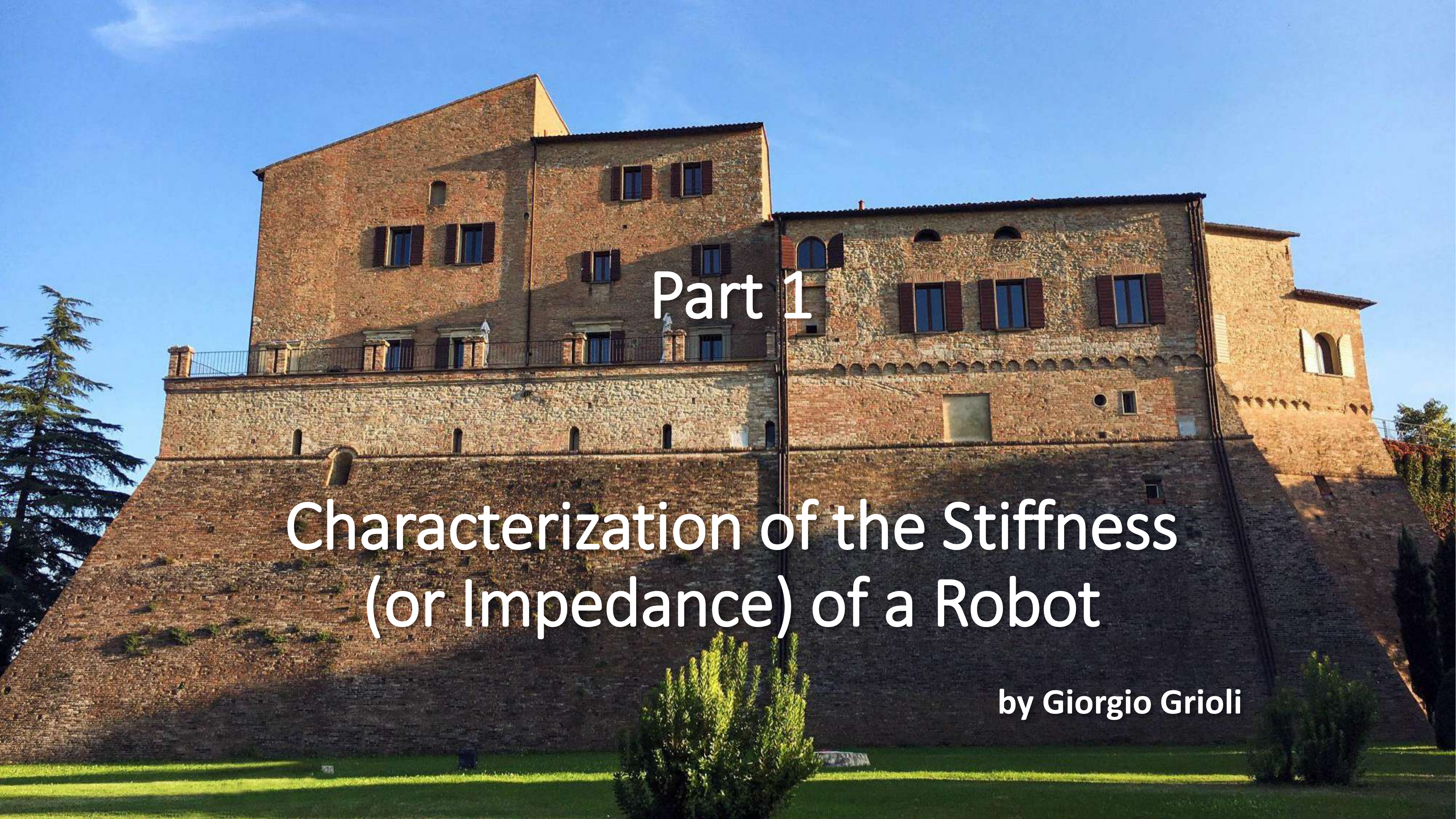
VSA: the 4 challenges

1. Design: how to make a VSA
2. Control: how to get desired position and impedance
3. Planning: what to do with impedance
4. Sensing: measuring impedance to feedback…

There are no stiff-o-meters

(stiffness sensors)!

Are we really closing a loop?


Part 1
Characterization of the Stiffness
(or Impedance) of a Robot
by Giorgio Grioli

Why is difficult to measure stiffness/impedance

- There are no "Stiffness Sensors”

- Stiffness is the relationship between two quantities

$$\sigma(y) = \frac{\partial f(y)}{\partial y}$$

  → need to measure both force and deformation

- Stiffness is a form of “reaction”

  → needs to excited to be observed

Impedance Measurement

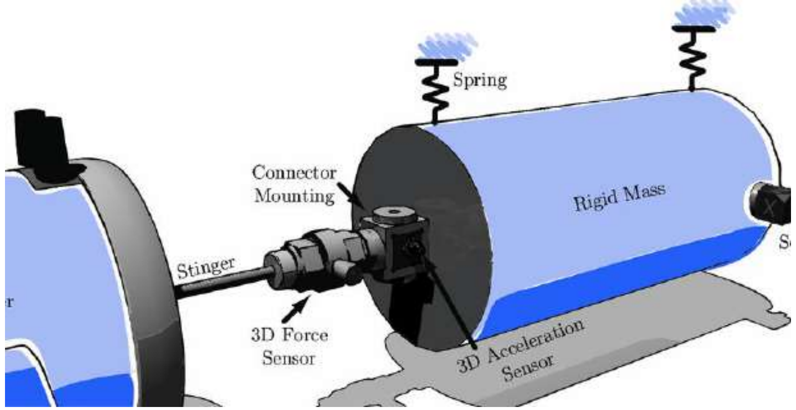
- Not only feedback needs to measure impedance…
- Measurements are one of the basis of scientific approach

> *“Misura ciò che e misurabile e rendi misurabile ciò che non lo è”*
> *(Measure what is measurable and make measurable what is not so.)*
>
> Galileo de’ Galilei

- State of Art
  - In Mechanical Engineering
  - In Biomechanics
  - In Robotics, etc.

Solutions in engineering (offline)

- Impedance heads



- Universal testing machines (Instron)

Solutions in motor sciences

It is not an easy problem

Why is difficult to measure stiffness/impedance in humans

- Reflexes change completely the game

- Not all the variables are easy to access

  - we measure what happens “outside” more easily than what happens “inside”

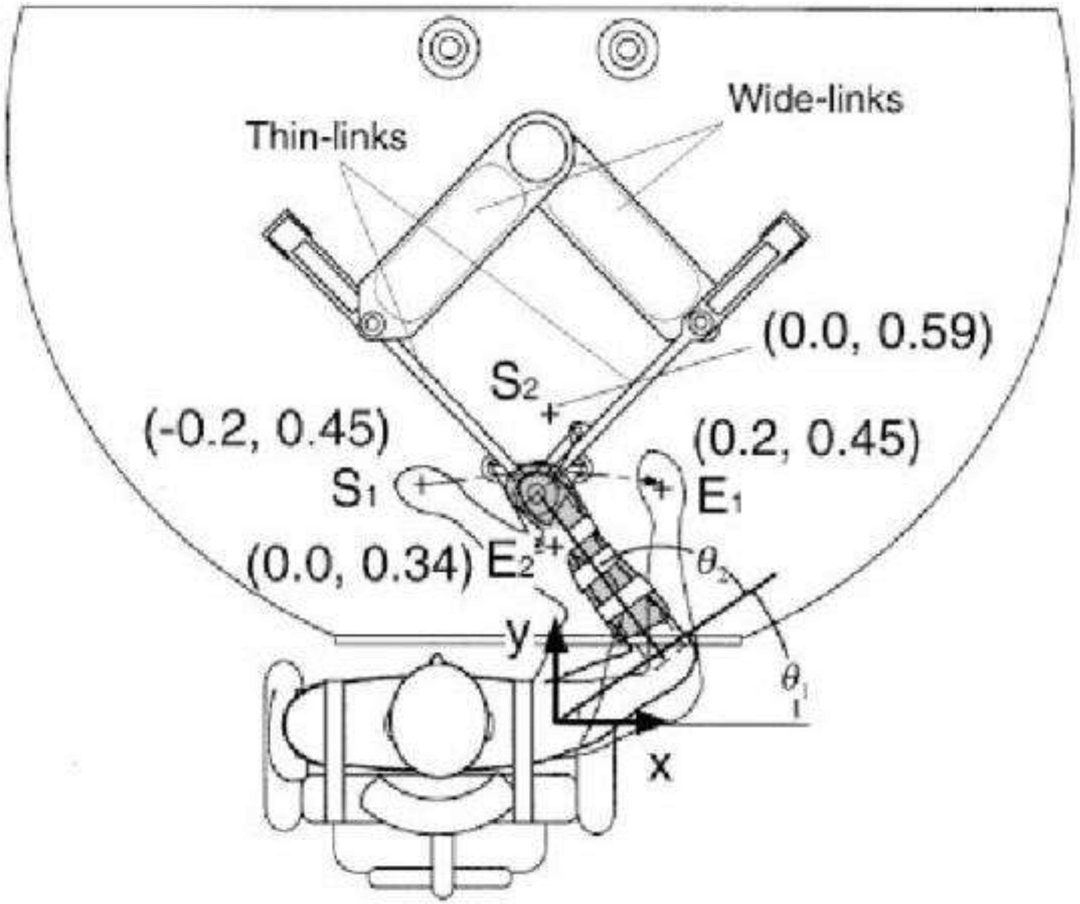
  - examples of variables that are difficult to access are, e.g.:
    - rest length of muscles under some level of activation
    - force on single muscles and tendons
    - muscle activation (EMG is related but not the same)

Solutions in the field of motor sciences

- Manipulandum-based experiments
- Lots of repetitions
- A robot introduces very fast perturbations

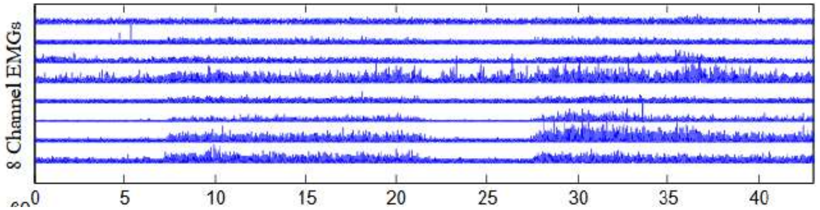
Gomi, Hiroaki, Yasuharu Koike, and Mitsuo Kawato. "Human hand stiffness during discrete point-to-point multi-joint movement." *1992 14th Annual International Conference of the IEEE Engineering in Medicine and Biology Society*. Vol. 4. IEEE, 1992.

Burdet, E., et al. "A method for measuring endpoint stiffness during multi-joint arm movements." Journal of biomechanics 33.12 (2000): 1705-1709.



Solutions in the field of motor sciences

- EMGs + models



EMG1

EMG2

Activation Model

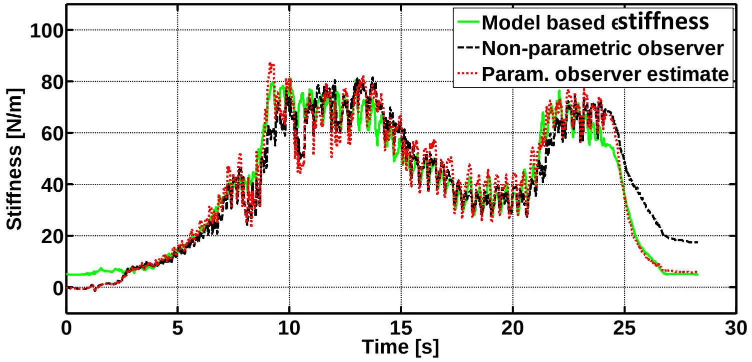
A1, A2

Muscle Model (e.g. Gribble's)

$\boldsymbol{Flexor\ force}$: f_1= ρ($e^{\delta A_1}$ −1)

$\boldsymbol{Extensor\ force}$: f_2= -ρ($e^{\delta A_2}$ −1)

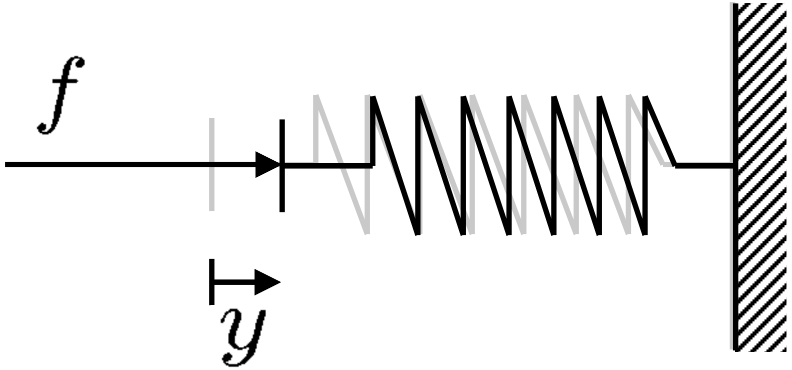
stiffness



Impedance Measurement

- Common Characteristics of S.o.A.
  - Typically: repeated experiments with probing perturbations
  - Mostly: not applicable in real time
  - Almost always: linear, time invariant impedance
- Difficulty arises because **Impedance** is a **differential operator**

Definition of Stiffness and Impedance

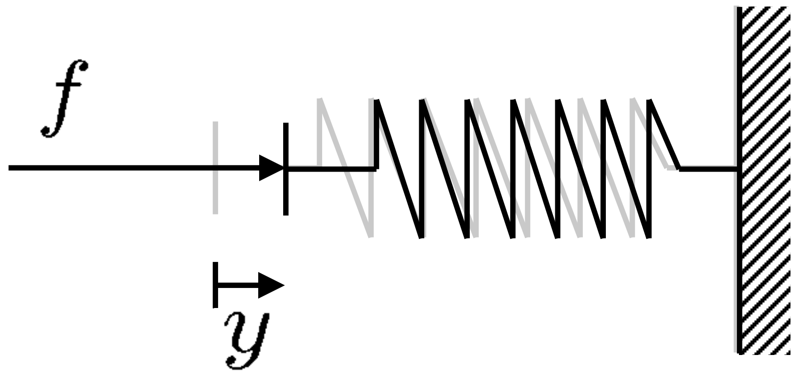


- Compliance is the ability of a mechanical system to respond to an external stimulus by adapting.
- The simplest example is that of a spring, that when subject to a force f , deforms of some amount y.
- In linear spring $f = Ky$ wo quantities are proportional and follow the Hooke's law

- Where the constant K is called the Stiffness

Definition of Stiffness and Impedance

- Linear Springs $f = Ky$

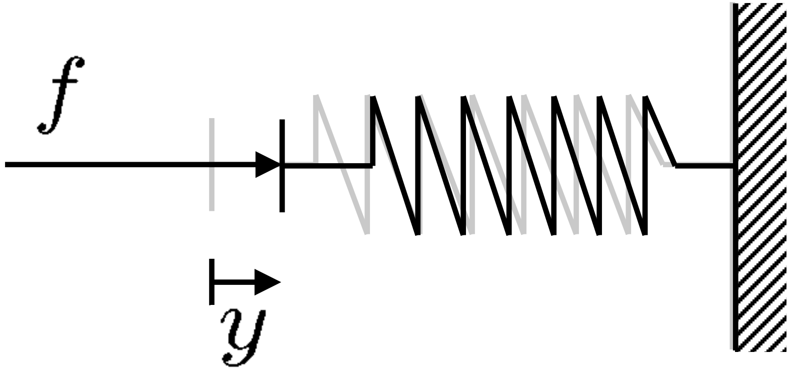


- In general, the rate at which the force and the deformation change does not need to be a constant, so it is possible to generalize stiffness for non-linear elastic systems using partial derivatives

$$y = f(y) \Longrightarrow \sigma(y) = \frac{\partial f(y)}{\partial y}$$

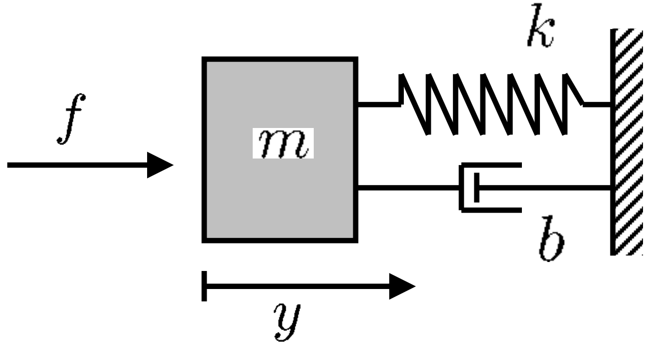
Definition of Stiffness and Impedance

- Linear Springs $f = Ky$



- Non-linear Stiffness $\sigma(y) = \dfrac{\partial f(y)}{\partial y}$

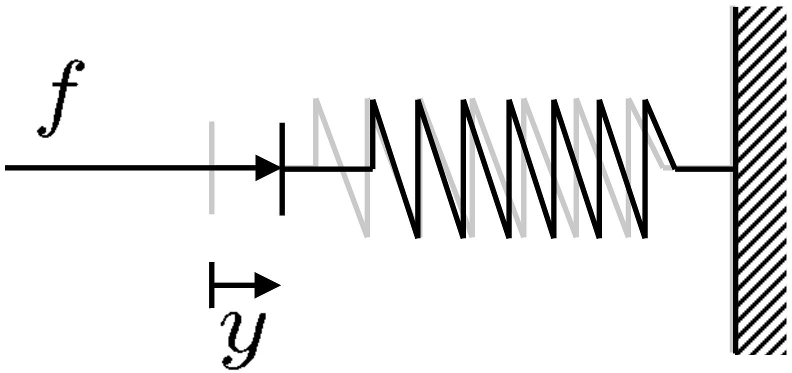
- More in general, in mechanical systems, the force does not depend on deformation only, but also on the speed and acceleration at which the deformation changes, and other parameters, leading to the concept of mechanical impedance



$$f = m\ddot{y} + b\dot{y}\ ky \quad \Longrightarrow \quad F(s) = (ms^2y + bs + k)Y(s) = Z(s)Y(s)$$

Definition of Stiffness and Impedance

- Linear Springs $f = Ky$



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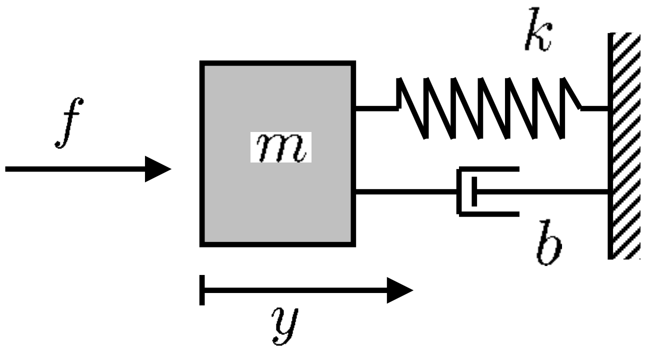
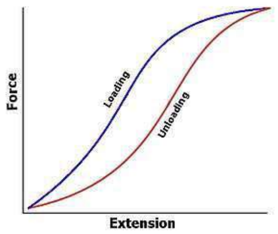
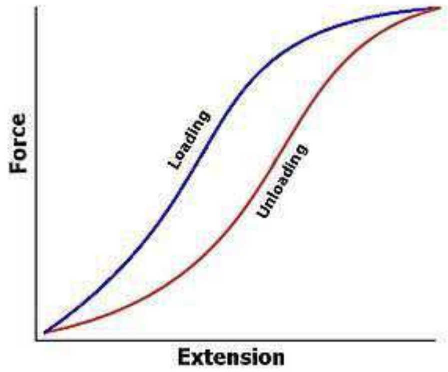


$$G(f, y, \dot{y}, \ddot{y}, u) = 0 \quad \Rightarrow \quad \delta f = m(d)\,\delta\ddot{y} + b(d)\,\delta\dot{y} + k(d)\,\delta y + \nu(d)\,\delta u$$



Definition of Stiffness and Impedance

- Generalizing Impedance:
  - Graph $G \subset F \times Y \times DY \times D^2Y \times U$



  - Analytical Description: $G(f, y, \dot{y}, \ddot{y}, u) = 0$
  - Regular point: d_0

    s.t. exists locally $f(y, \dot{y}, \ddot{y}, u)$

  - Fréchet differential

$$\delta f = m(d)\,\delta\ddot{y} + b(d)\,\delta\dot{y} + k(d)\,\delta y + \nu(d)\,\delta u$$

$$k(d) = -\left(\frac{\partial G(d)}{\partial f}\right)^{-1} \frac{\partial G(d)}{\partial y}$$

$$b(d) = -\left(\frac{\partial G(d)}{\partial f}\right)^{-1} \frac{\partial G(d)}{\partial \dot{y}}$$

$$m(d) = -\left(\frac{\partial G(d)}{\partial f}\right)^{-1} \frac{\partial G(d)}{\partial \ddot{y}}$$

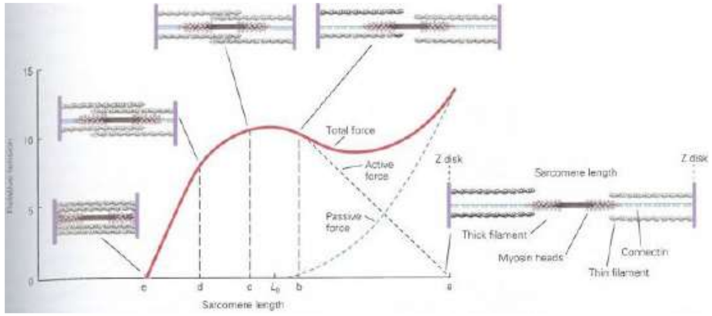
$$\nu(d) = -\left(\frac{\partial G(d)}{\partial f}\right)^{-1} \frac{\partial G(d)}{\partial u}$$

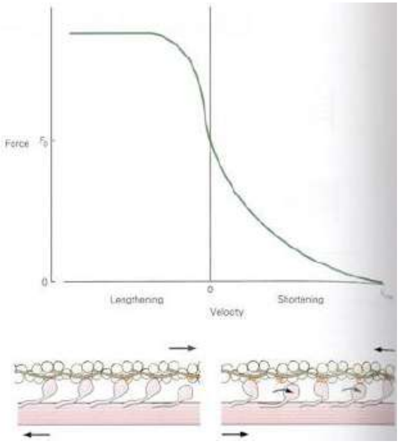
Examples of different impedance behaviors

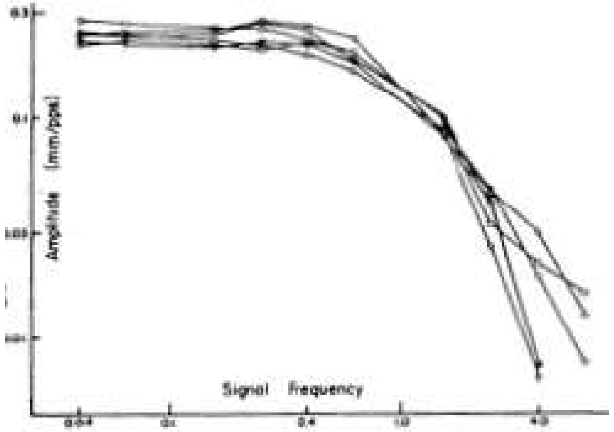
$$\delta f = m(d)\,\delta\ddot{y} + b(d)\,\delta\dot{y} + k(d)\,\delta y + \nu(d)\,\delta u$$

- Force / deformation
- Force / velocity
- Force / activation

(moto-neuron firing rate)



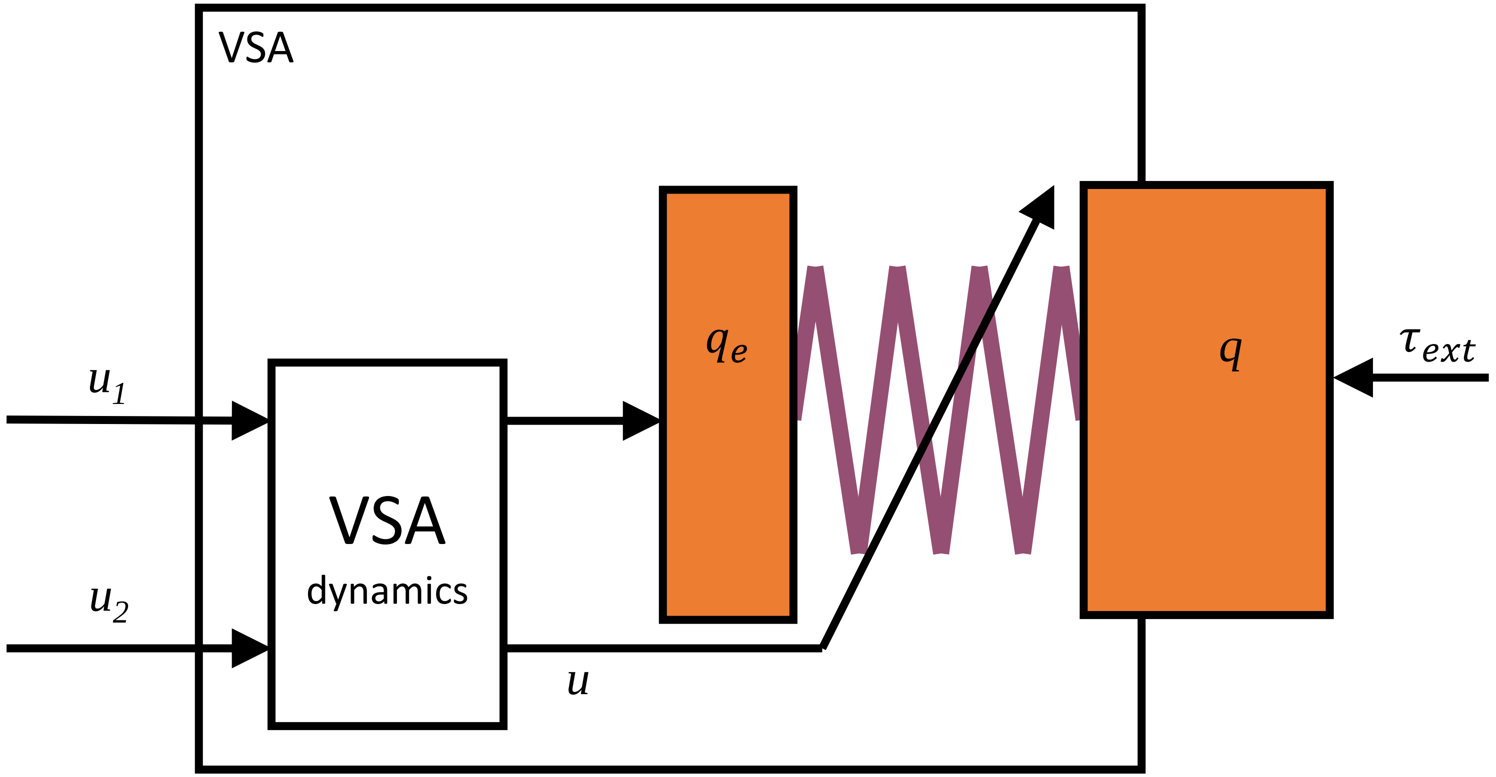




How to characterize a VSA?

The golden rule: abstraction

- Black box model

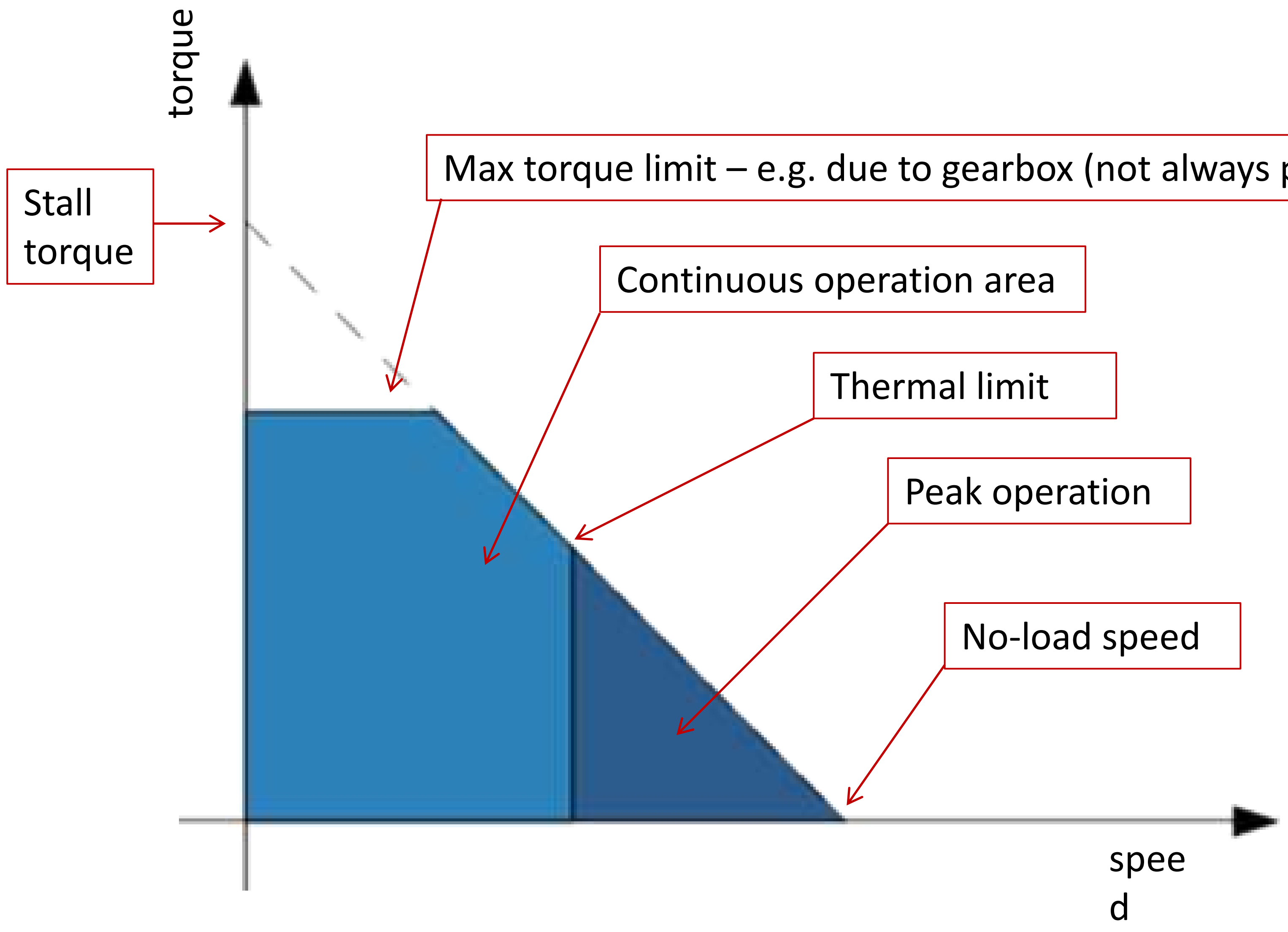


Black box model

- Physical behavior
  - Performance parameters
  - What are the performance parameters of a VSA?

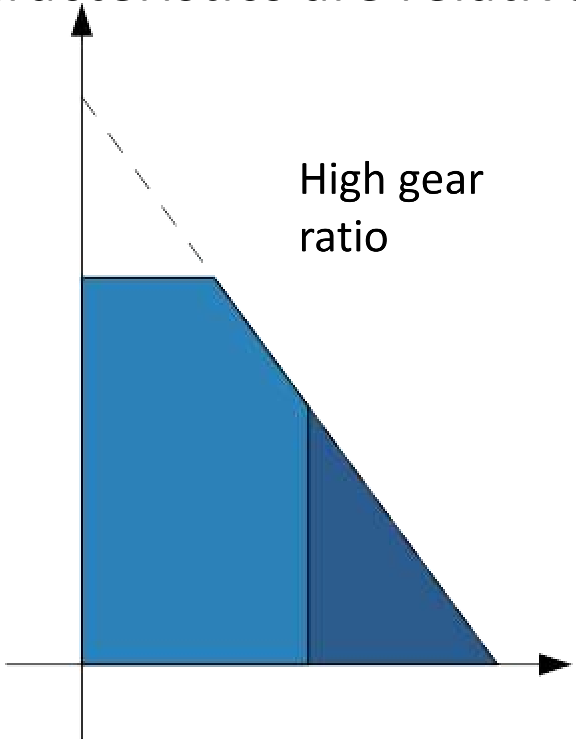
- Let’s look at the bases:
  - How do you choose a “normal” motor?

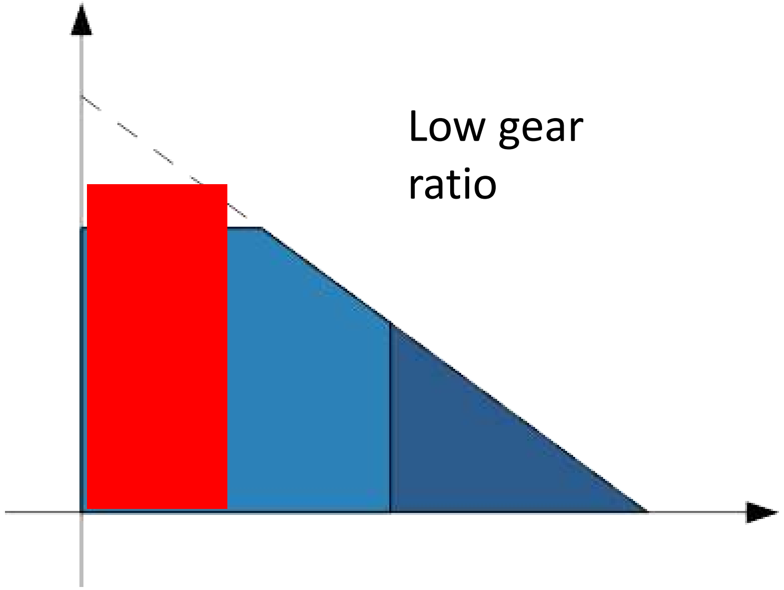
Torque – speed characteristic



Torque – speed characteristic

- Remember there are gearboxes!
- This two characteristics are relative to the same motor with different gear ratios!





Back to VSAs

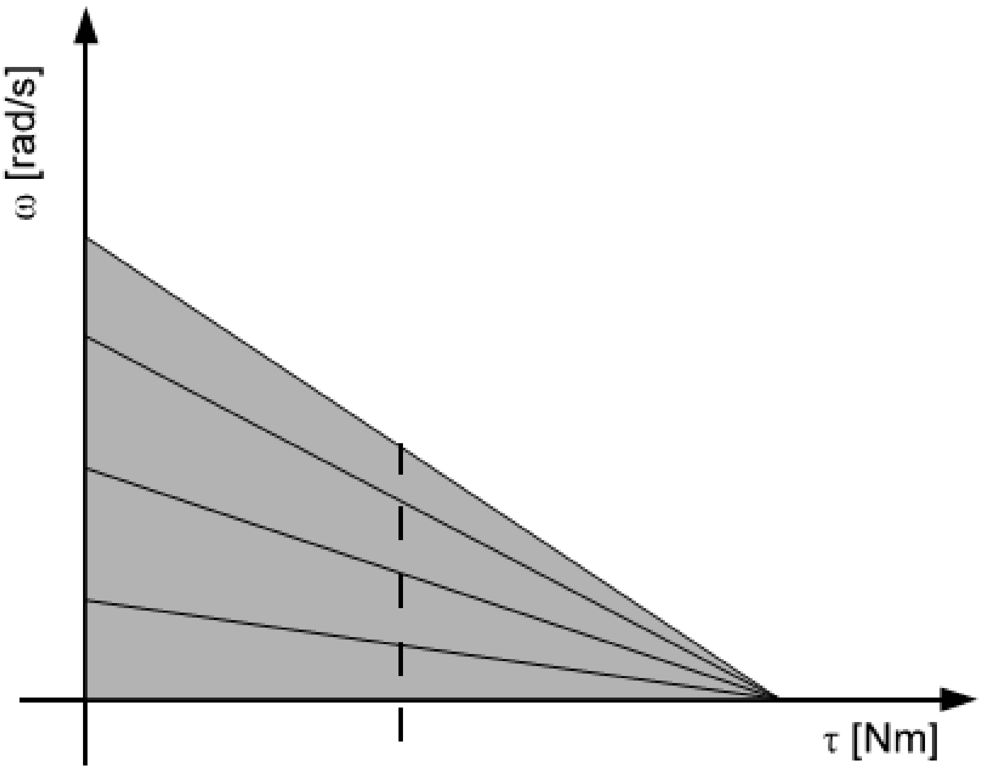
What are the main parameters of a VSA?

Main parameters of a VSA

- A VSA is primarily a motor, so we still have
  - Torque
  - Speed
- The core characteristic of a VSA is the variable stiffness
  - Stiffness range

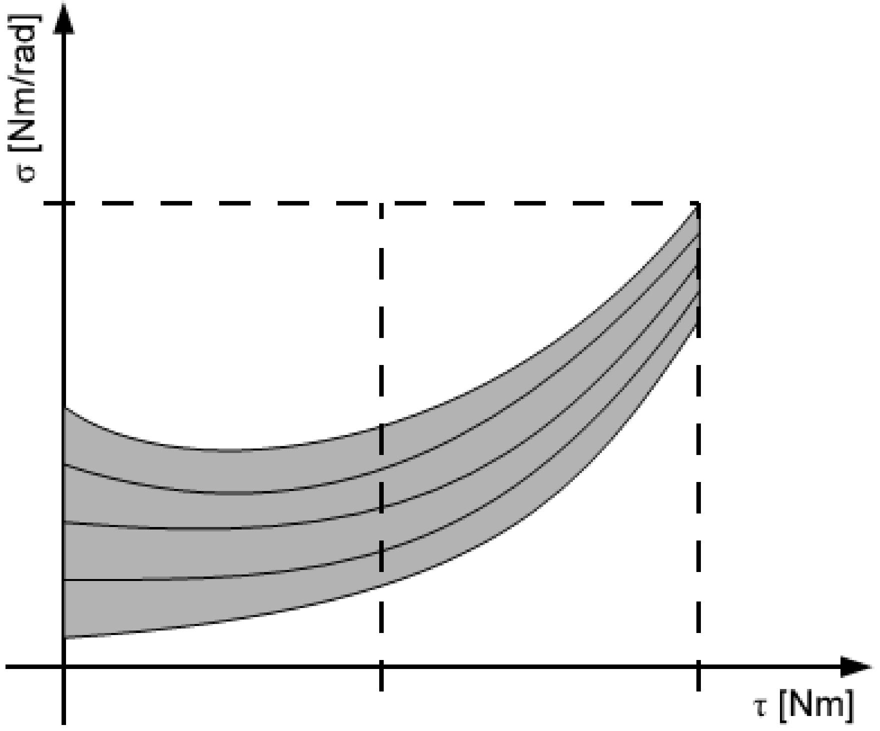
Main parameters of a VSA

- Torque vs Speed
  - Same old story
  - Limited by
    - Motors power
    - Frictions
    - Gearboxes
    - Heat dissipation
    - …



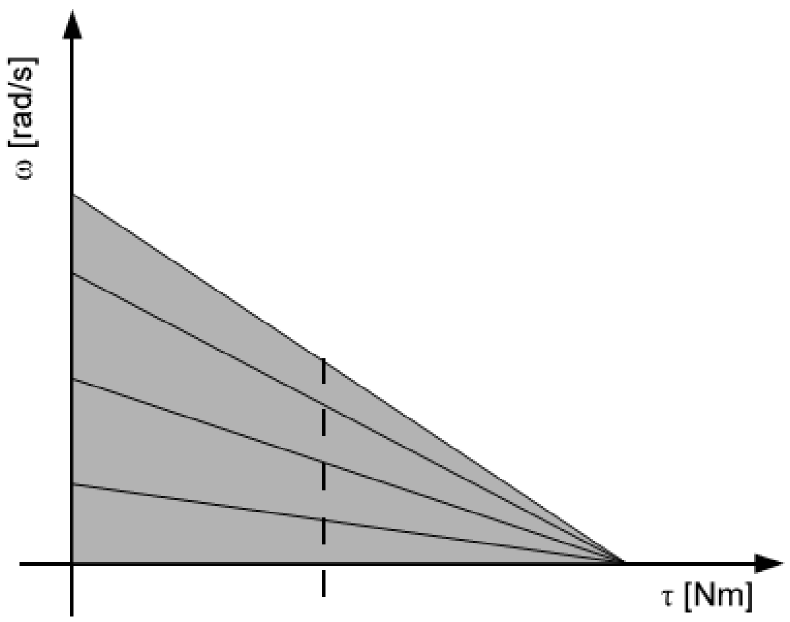
Main parameters of a VSA

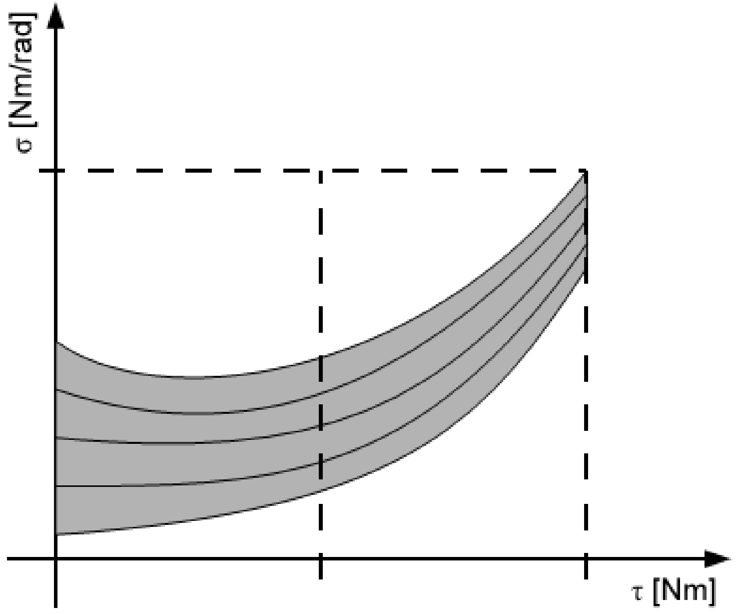
- Torque vs Stiffness range
  - The stiffness range can be, in general, affected by the load
  - Almost always the case
    If the system has
    non-linear springs
  - Has to be accounted
    when controlling



Main parameters of a VSA

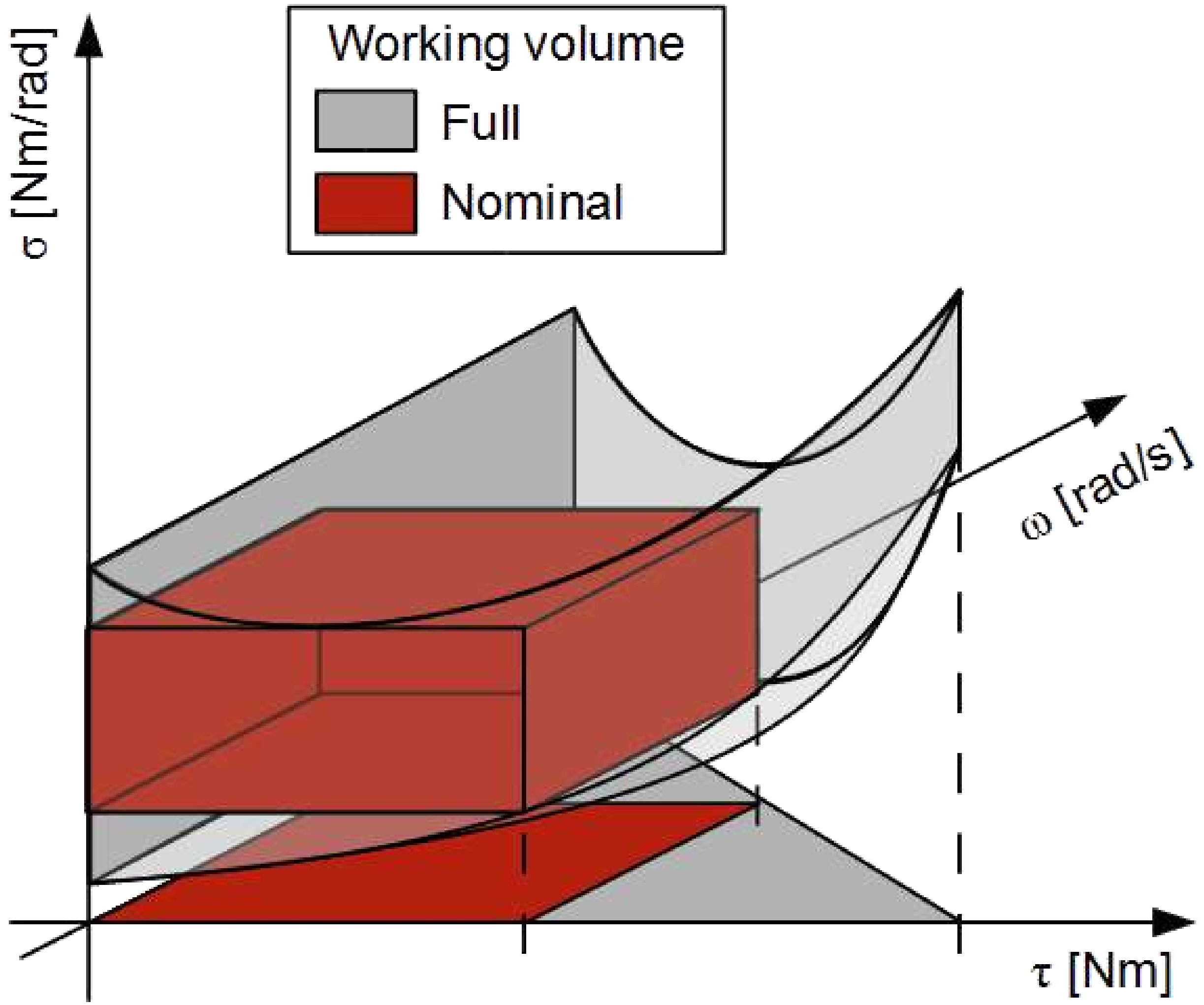
- Speed vs Stiffness range
  - The two previous plots, combined, yield a relationship also between rotation speed and stiffness





Main parameters of a VSA

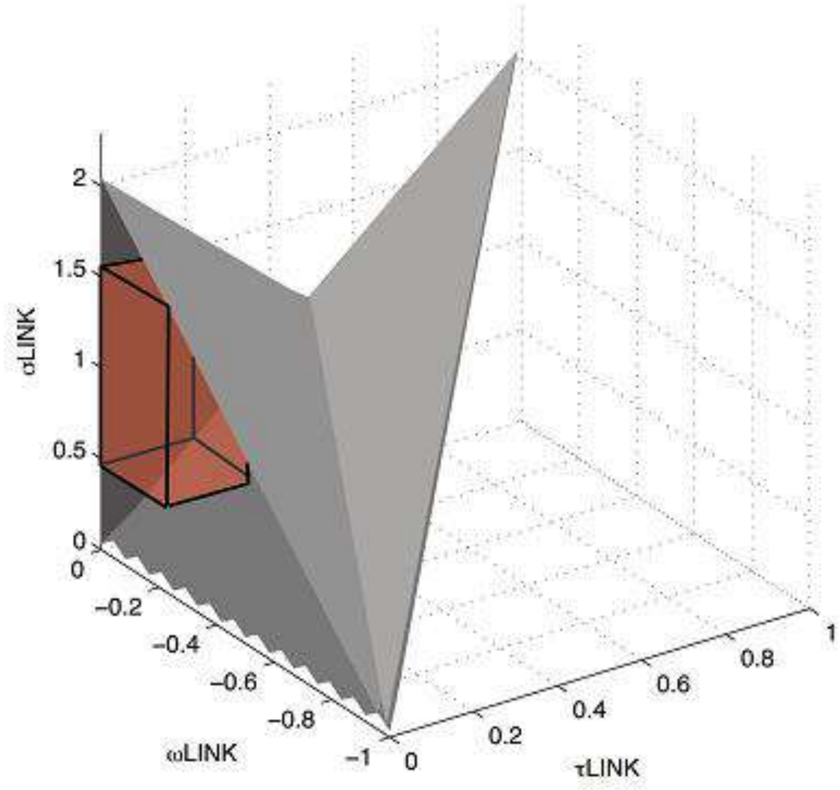
- 3d working volume
  - Torque
  - Speed
  - Stiffness range



Main parameters of a VSA

- Working volume
  - Actuator vs Application

The application volume must fit within the volume of the actuator



Main parameters of a VSA

- Stiffness has a speed too
  - The real figure should be 4D
    - Difficult to visualize and to work with
    - What we really care about is the time to change stiffness

  - Given the previous points, this time could, in general, be different when there is a load applied and when there is not, thus the parameters to read are:
    - Nominal stiffness variation time with no load
    - Nominal stiffness variation time with nominal load

  - Important for rapid tasks
    - e.g.: bang-bang optimal control
      (see lesson of Manolo Garabini Monday)

Main parameters of a VSA

- Elasticity means deflection
  - Variable stiffness → variable deflection
  - Maximum deflection with minimum stiffness
  - Maximum deflection with maximum stiffness

- Elasticity means storable energy
  - Maximum energy storable in the springs

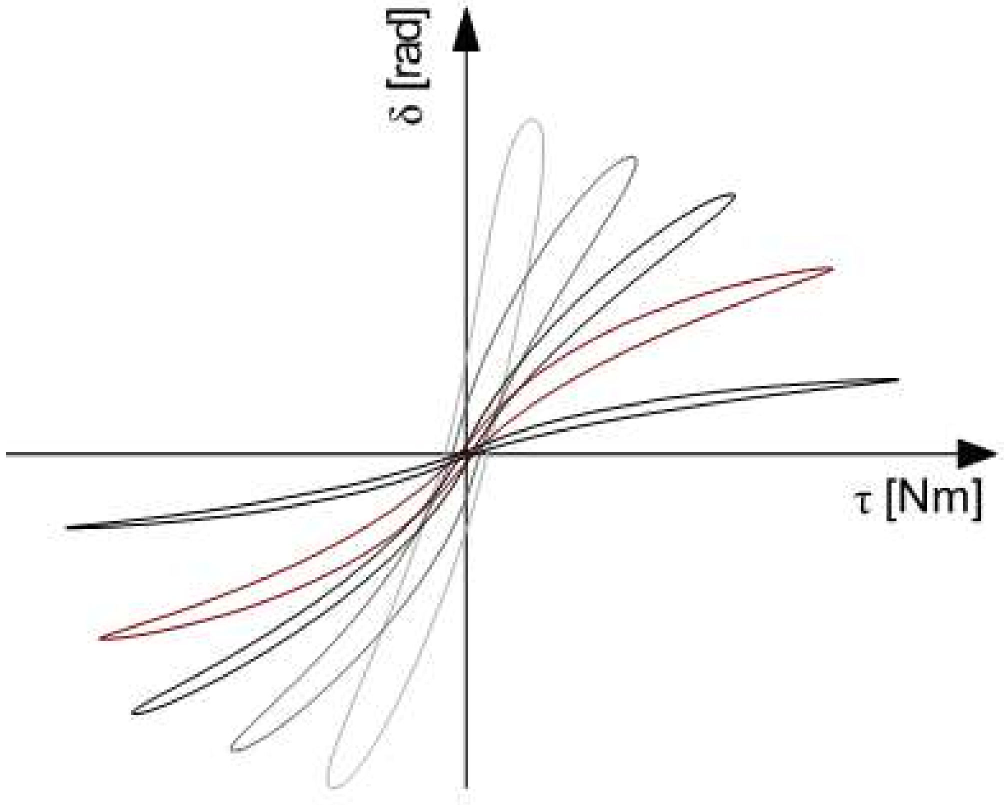
  - Important for
    - Shock absorption
    - Exploiting natural oscillations

Other aspects of a VSA

- Active rotation angle
  - Not all actuators can rotate continuously
    - Limits can derive from
      - Sensors
      - Type of transmission
      - Shape of system
  - Since the main application is robot joints this is usually not a problem
  - For some application a limited rotation angle could be problematic
  - In general, you should check that the range of the active rotation angle is enough do perform your intended task

Other aspects of a VSA

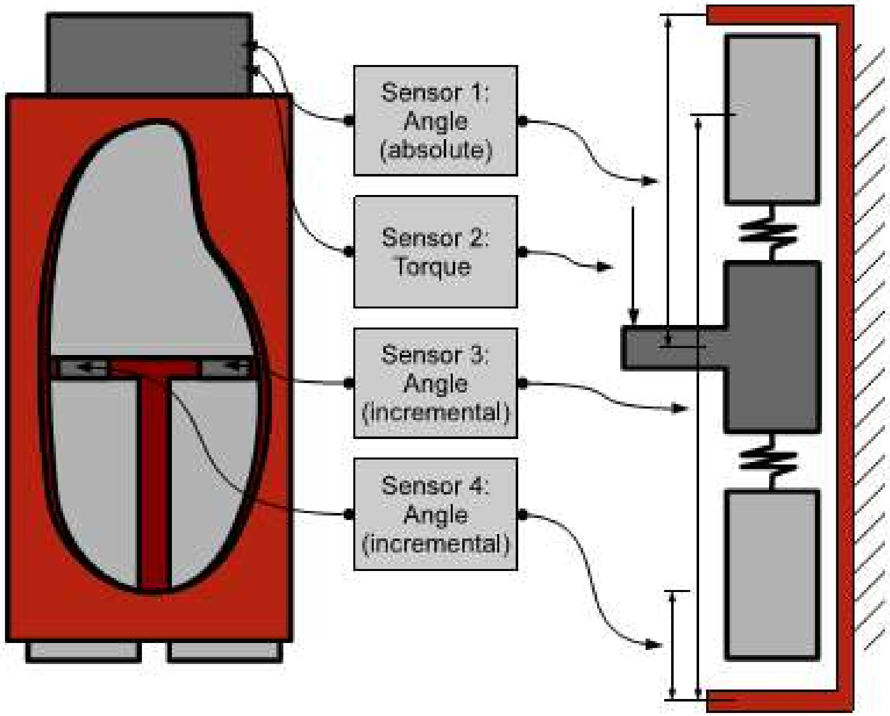
- Hysteresis
  - Real systems have friction
  - This implies some hysteresis on the torque-deformation characteristic
  - It can be quantified in terms of deflection or in term of torque



- Very important if you want precise torque and/or position actuation!

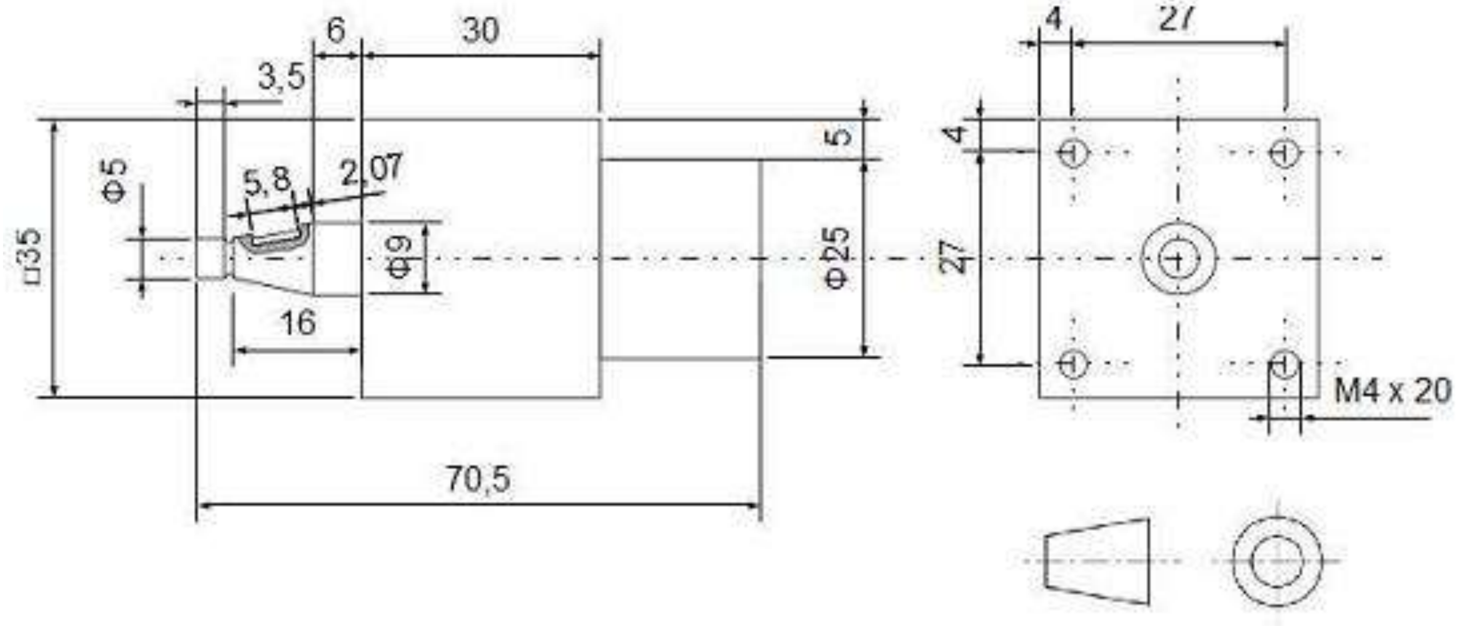
Other aspects of a VSA

- Transducers
    - Output shaft sensors
        - Position sensor: type, resolution, precision…
    - Internal sensors
        - Usually at least other two position sensors (on the motor shafts)
        - Position, type, etc…
        - Torque sensor(?): type, range, resolution…



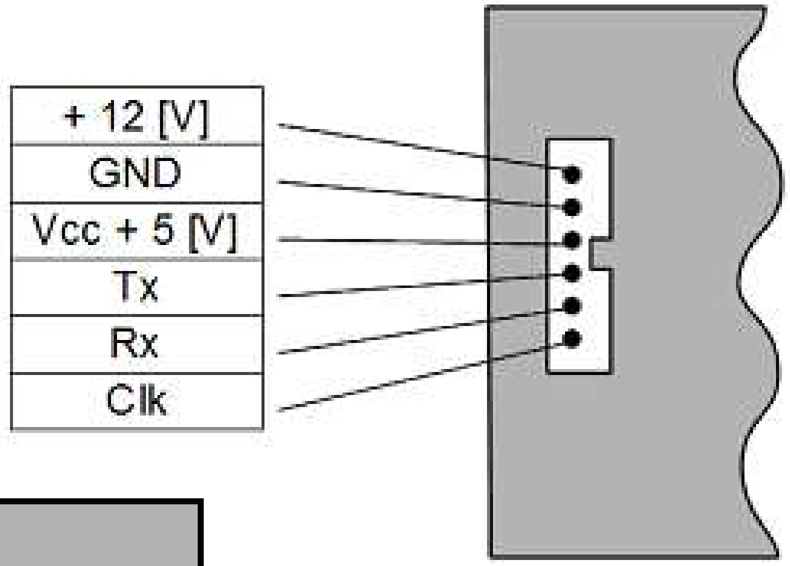
Other aspects of a VSA

- An architecture made of components relies on **interfaces**
    - Mechanical interfaces:
        - Drawings



Other aspects of a VSA

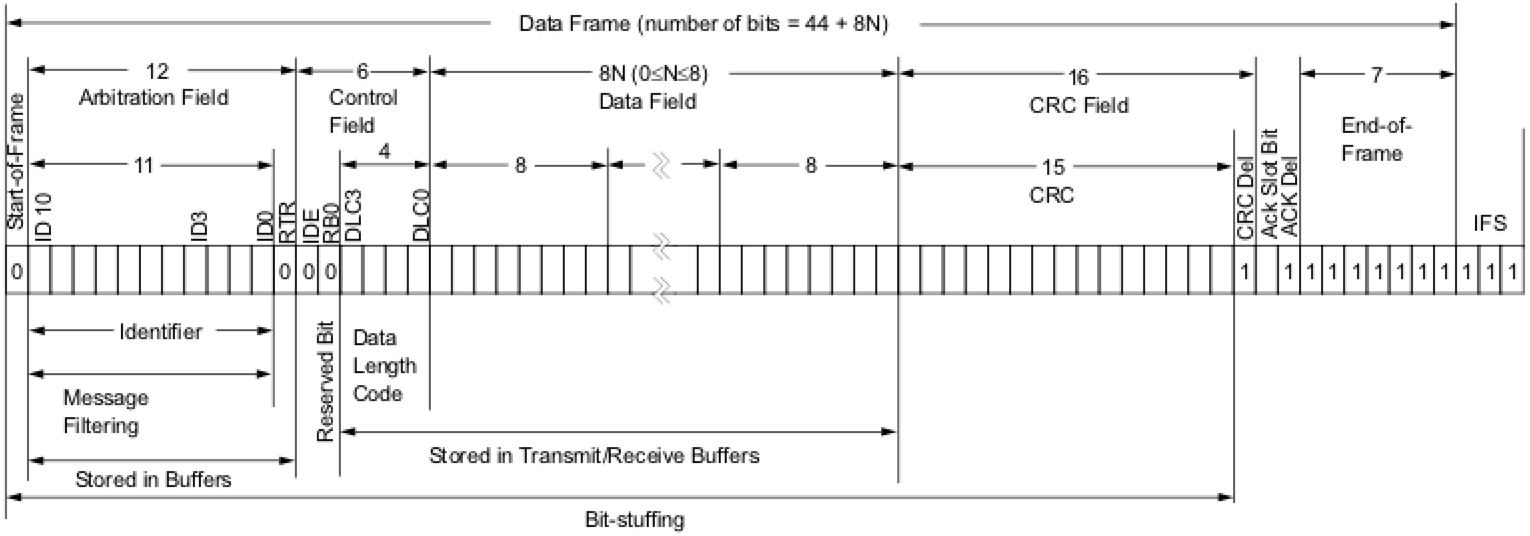
- An architecture made of components relies on **interfaces**
  - Electrical interfaces:
    - Connectors
    - Voltage
    - …

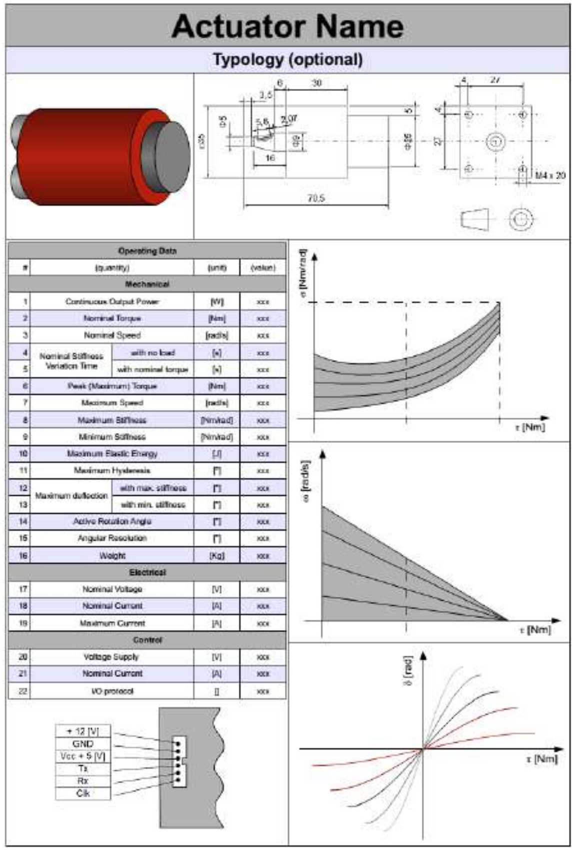


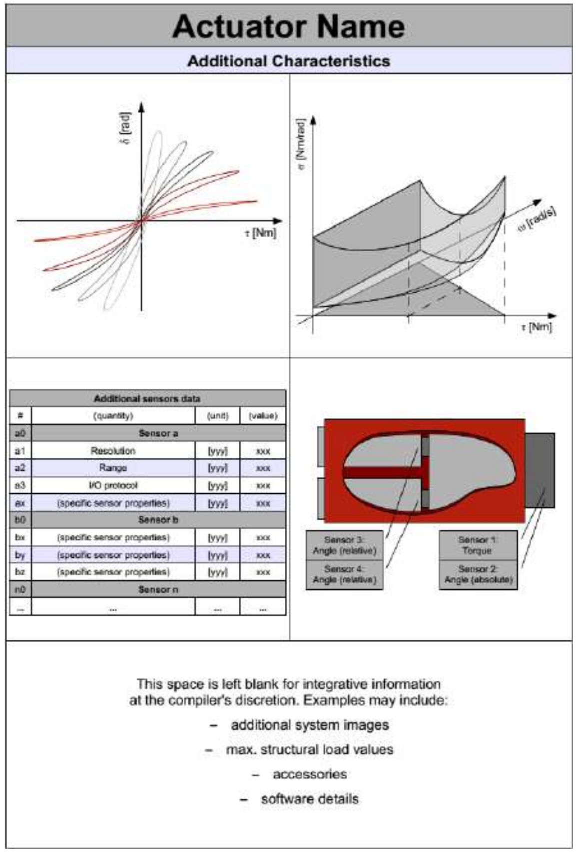
Electrical		
Nominal Voltage	[V]	xxx
Nominal Current	[A]	xxx
Maximum Current	[A]	xxx

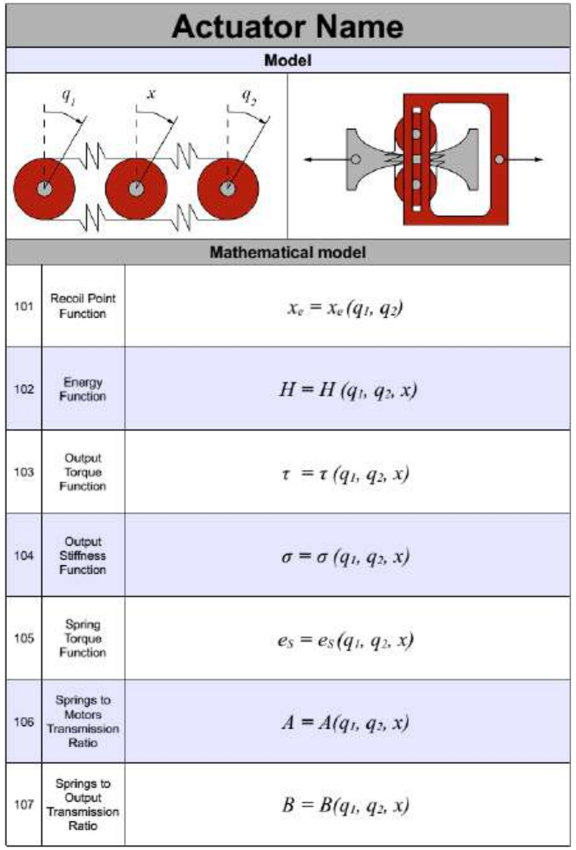
Other aspects of a VSA

- An architecture made of components relies on **interfaces**
  - Control interfaces:
    - Protocol



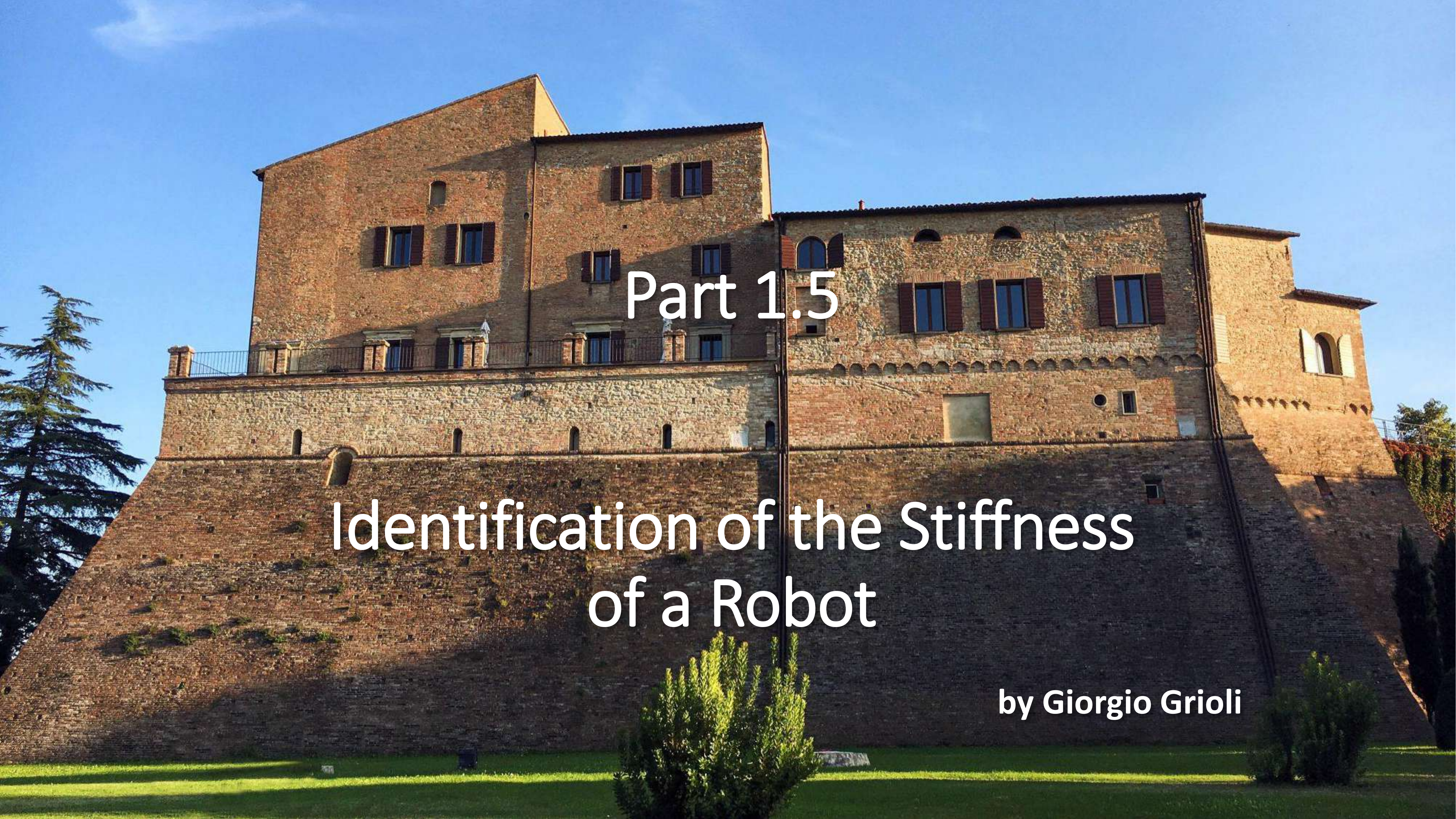






VSA datasheet

Grioli, G., Wolf, S., Garabini, M., Catalano, M., Burdet, E., Caldwell, D., ... & Bicchi, A. (2015). Variable stiffness actuators: The user's point of view. *The International Journal of Robotics Research*, *34*(6), 727-743.



Part 1.5

Identification of the Stiffness of a Robot

by Giorgio Grioli

Mathematical model of the mechanical characteristic

- Energy function

$$U(q, \theta_1, \theta_2)$$

- Output torque function

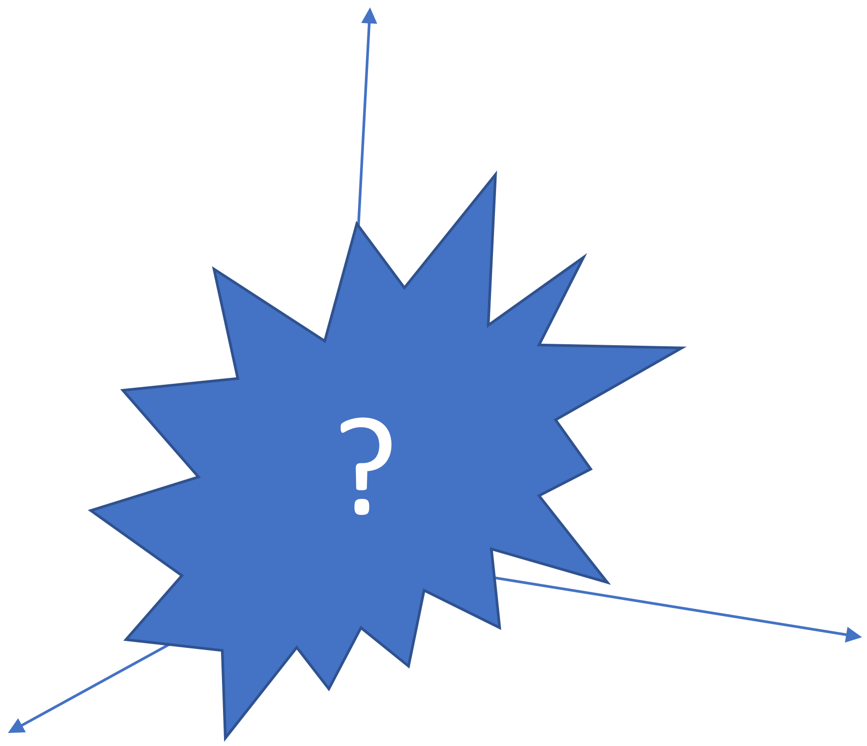
$$\tau = U_q(q, \theta_1, \theta_2) = \frac{\partial U}{\partial q}$$

- Output stiffness function

$$\sigma = U_{qq}(q, \theta_1, \theta_2) = \frac{\partial U_q}{\partial q} = \frac{\partial^2 U}{\partial q^2}$$

- Recoil point function

$$q_e = q_e(\theta_1, \theta_2)$$



Simplification for some VSA (decoupled AA)

- Energy function

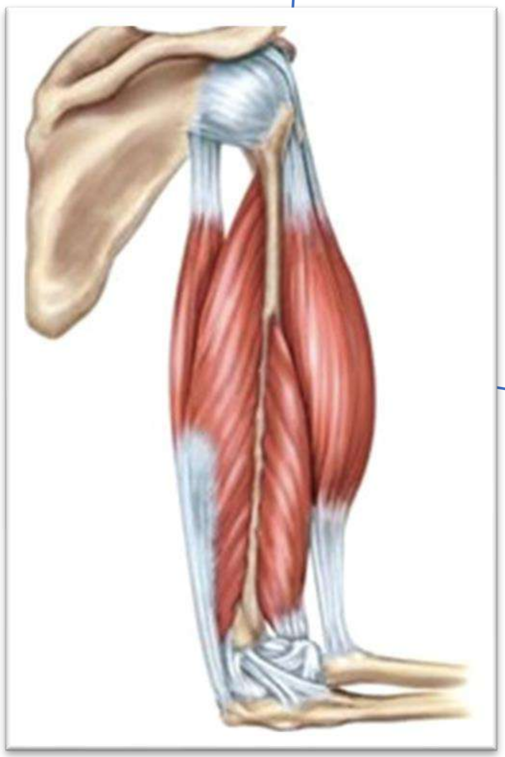
$$U(q,\theta_1,\theta_2) = U(q - \theta_1, q - \theta_2) \triangleq U(\delta_1,\delta_2)$$

- Output torque function

$$\tau = U_q(\delta_1,\delta_2) = \frac{\partial U}{\partial q} = \frac{\partial U}{\partial \delta_1} + \frac{\partial U}{\partial \delta_2}$$

- Output stiffness function

$$\sigma = U_{qq}(\delta_1,\delta_2) = \frac{\partial^2 U}{\partial {\delta_1}^2} + \frac{\partial^2 U}{\partial {\delta_2}^2}$$



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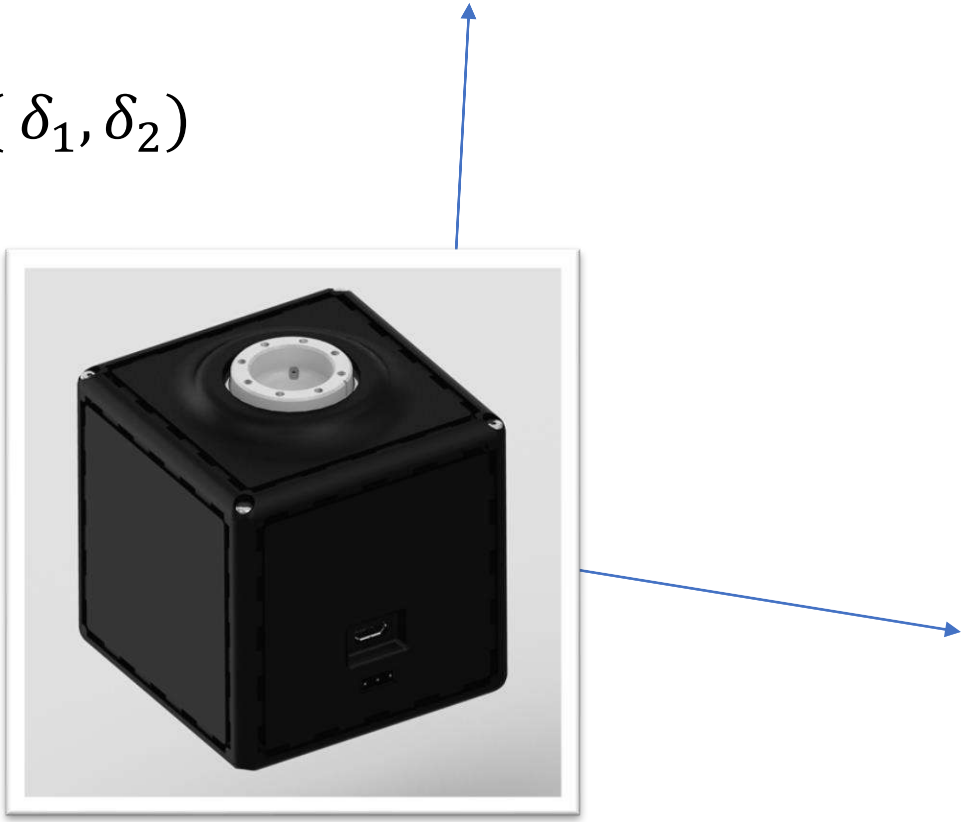


- Recoil point function

$q_e = q_e(\theta_1,\theta_2) = (\theta_1 + \theta_2)/2$ if the mechanism is symmetric (very ideal)

Simplification for other VSA (ESV)

- Energy function

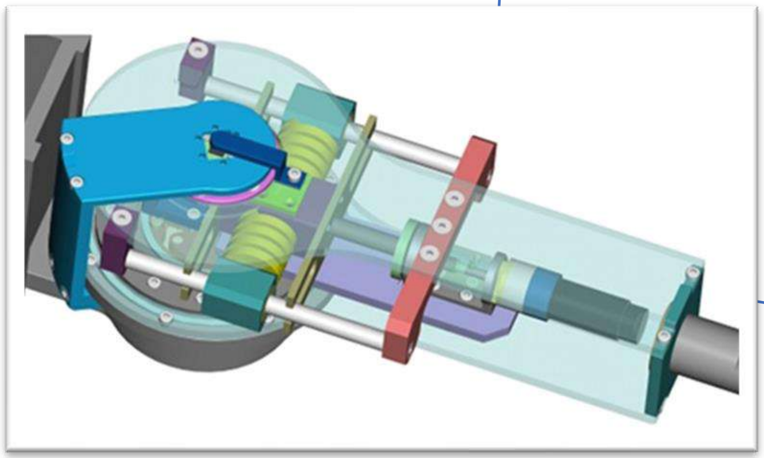
$$U(q,\theta_1,\theta_2) = U(q - \theta_1,\theta_2) \triangleq U(\delta,\theta_2)$$

- Output torque function

$$\tau = U_q(\delta,\theta_2) = \frac{\partial U}{\partial q} = \frac{\partial U}{\partial \delta}$$

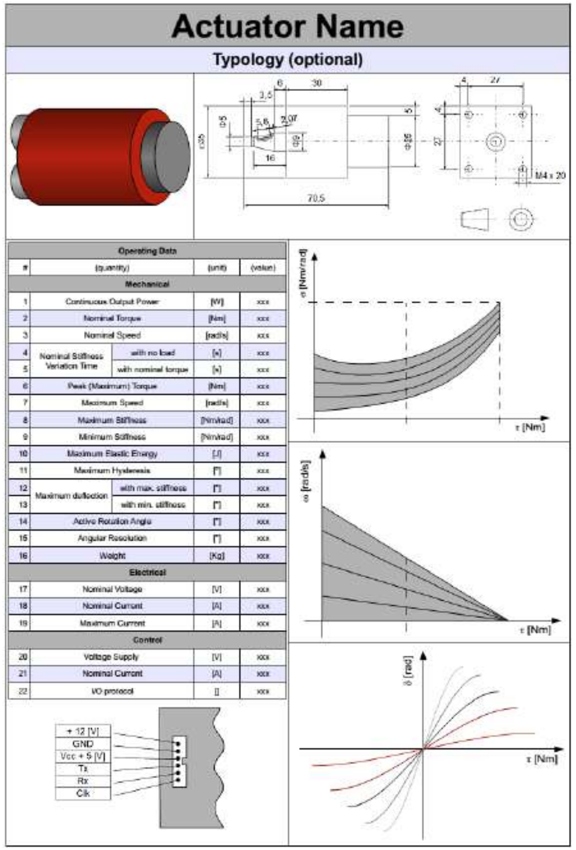
- Output stiffness function

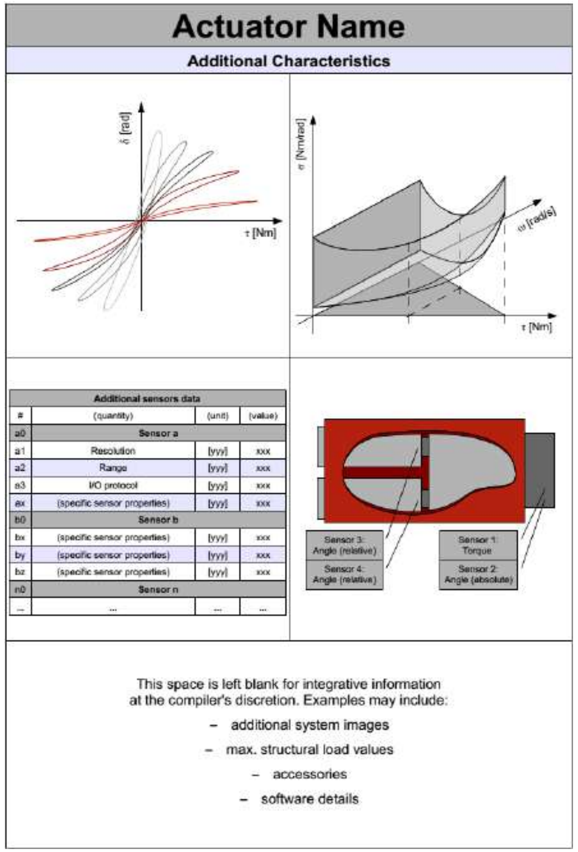
$$\sigma = U_{qq}(\delta_1,\delta_2) = \frac{\partial^2 U}{\partial \delta^2}$$

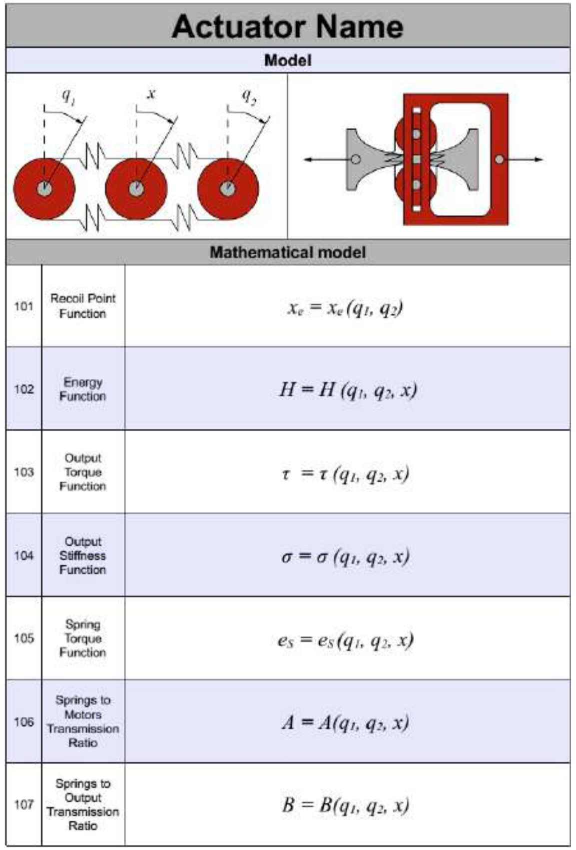


- Recoil point function

$$q_e = q_e(\theta_1,\theta_2) = (\theta_1 + \theta_2)/2$$ if the mechanism is symmetric (very ideal)







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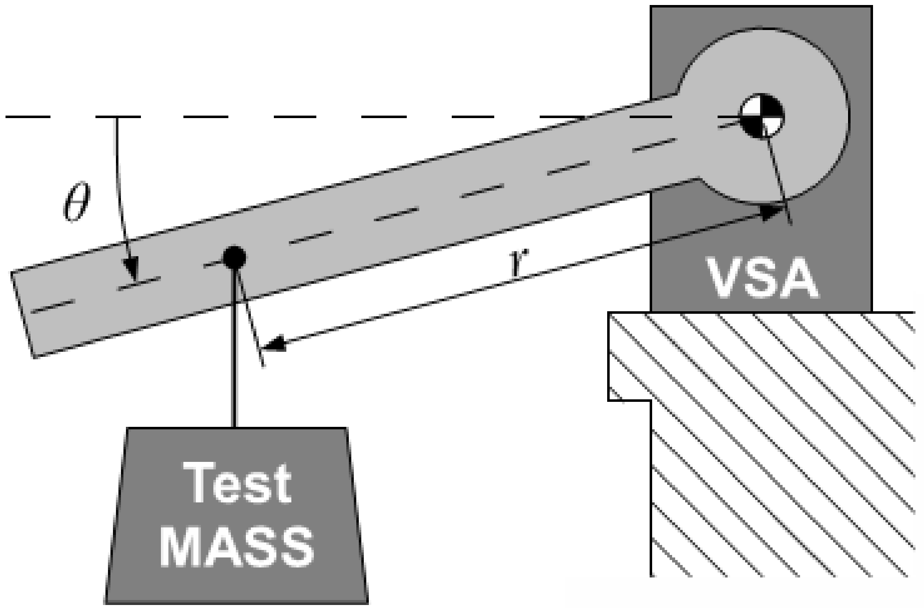
Where do parameters come from?

Experimental characterization of a VSA

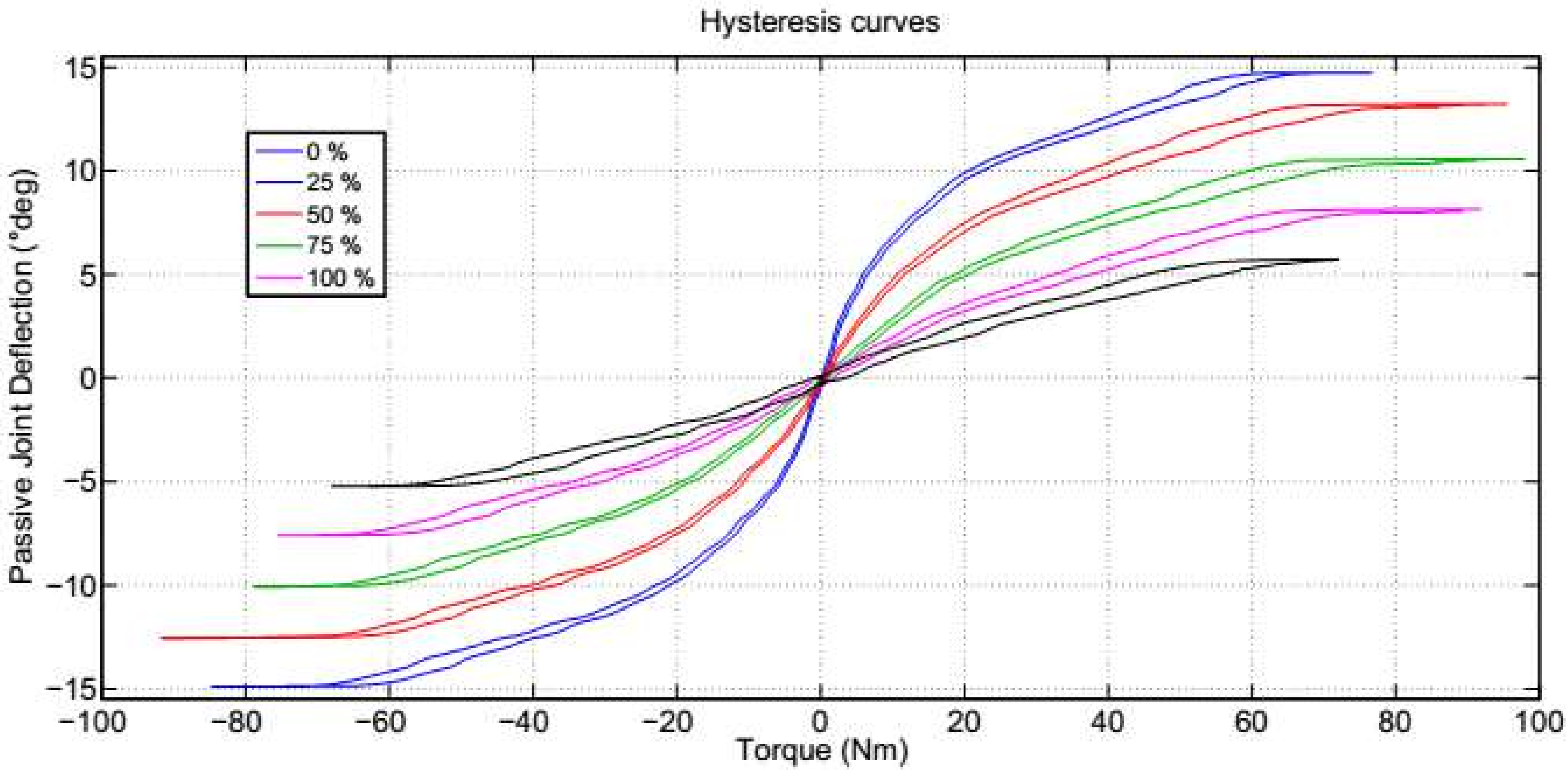
Characterization of a VSA

- Quasi-static load-unload cycles with fixed stiffness preset
  - Experimental setup



Characterization of a VSA

- Quasi-static load-unload cycles with fixed stiffness preset
  - raw data



Characterization of a VSA

- Quasi-static load-unload cycles with fixed stiffness preset
  - Distilled parameters and data
    - Torque/deflection characteristic
    - Torque/stiffness characteristic
    - Maximum deflection with max & min stiffness
    - Max and Min stiffness
    - Hysteresis

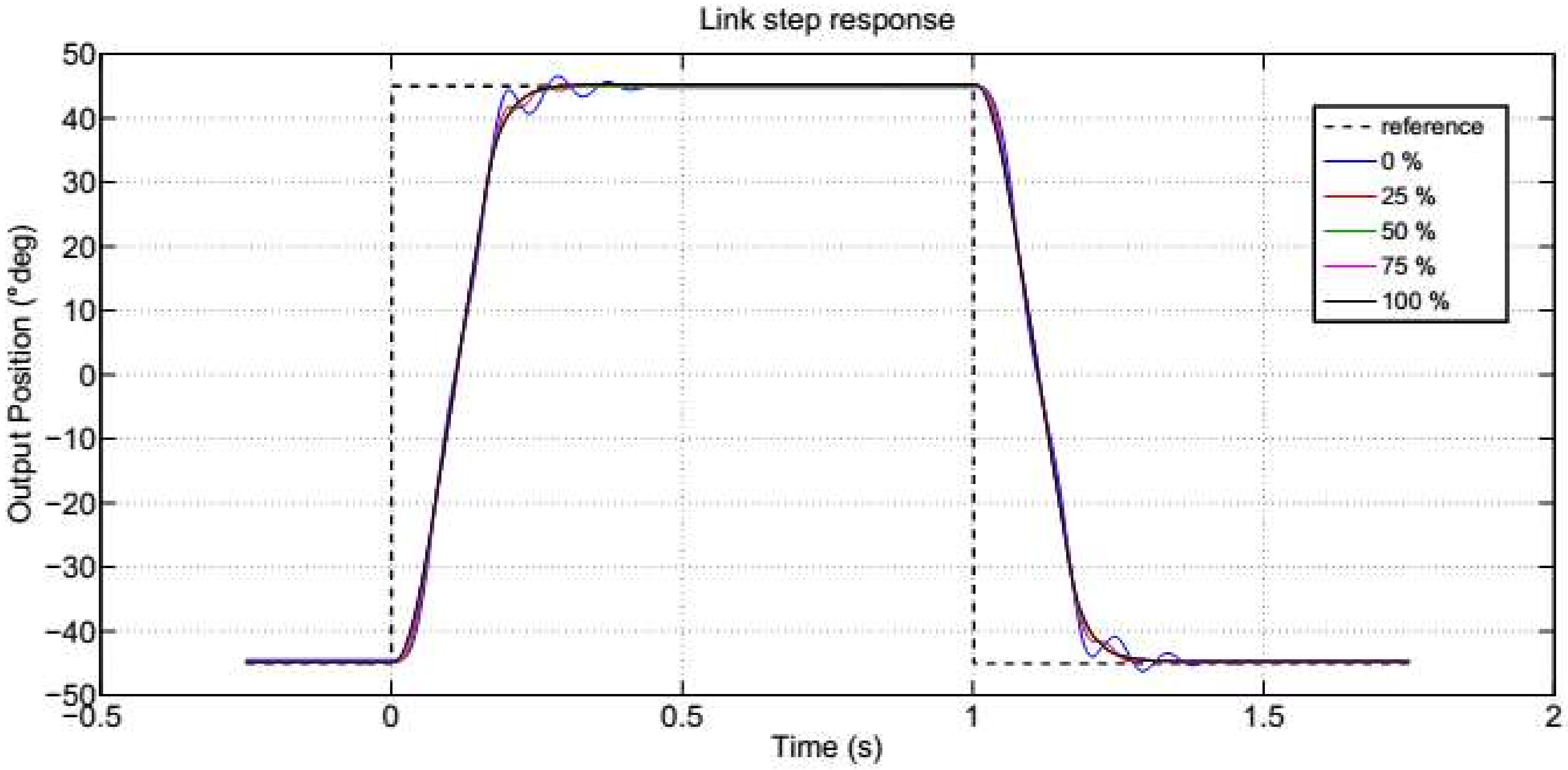
Characterization of a VSA

- Step command(s)
  - Experimental setup
  - Actuator in horizontal, with no added load
    - Step in output position
    - Step in stiffness reference

  - Actuator with nominal load (e.g. with same setup as before)
    - Step in stiffness reference

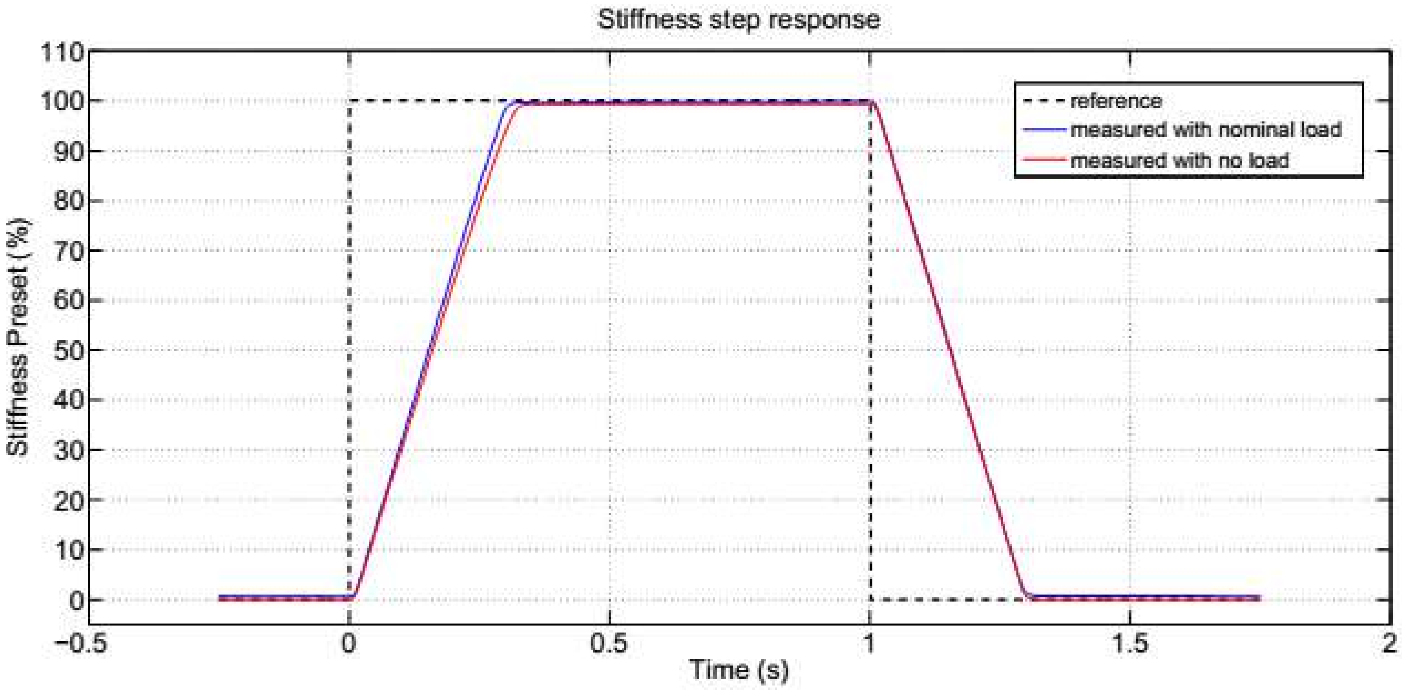
Characterization of a VSA

- Step command(s)
  - Max speed



Characterization of a VSA

- Step command(s)
  - Stiffness variation time with no load
  - Stiffness variation time with nominal load

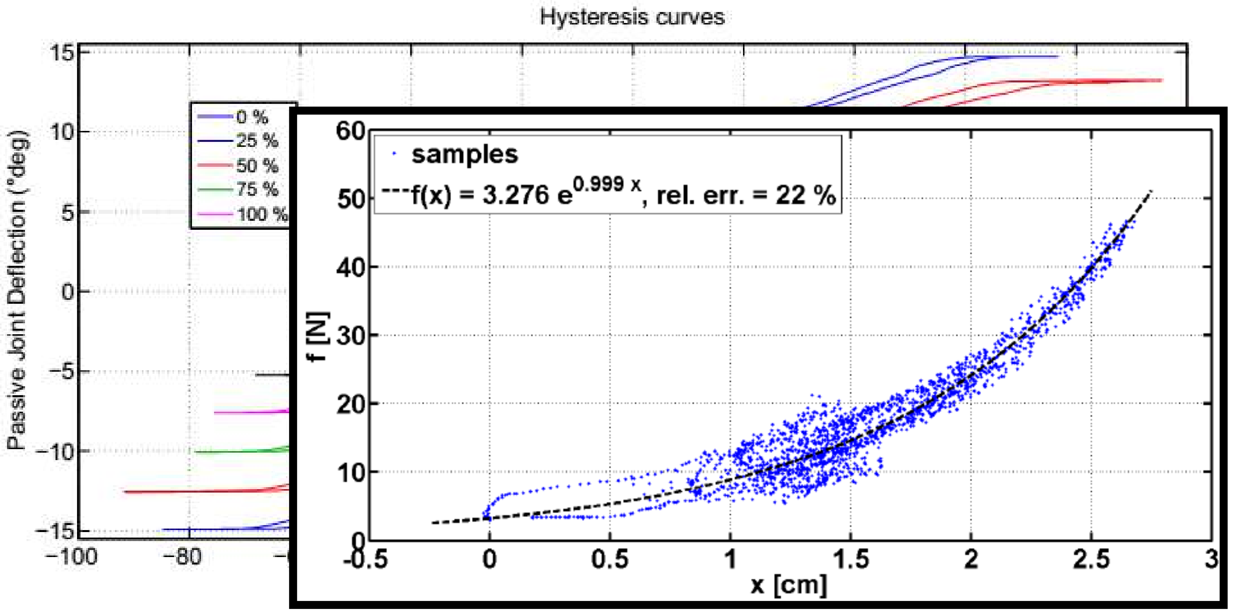


From data to a model

Data can be very noisy and hard to do derivatives on

Solutions:

1. Purely numeric
    1. Filtering a lot
    2. Removing hysteresis
    3. Derivate numerically
2. Fitting data to a model
    1. Define an error functional
    2. Minimize it
    3. Make clean derivatives with on the fitted model



More subject to errors due to noise

More subject to errors due to bad modelling

From Identification to Estimation

Identification has other limits:

- Models can be hard to define in explicit form

  $\cancel{U(q, \theta_1, \theta_2)}$

  → $U(\varepsilon_1, \varepsilon_2), \theta_1 = \theta_1(\varepsilon_1)$ … or $G(f, y, \dot{y}, \ddot{y}, u) = 0$

- Parameters can change over time
  - with use, due to wear
  - with temperature!!

Part II

Estimation of the Stiffness (or Impedance) of a Robot

by Giorgio Grioli

Measuring stiffness

- Linear case

$$f = m\ddot{y} + b\dot{y} + ky \qquad m,\, b,\, k > 0$$

  - Build a non-linear equivalent system

$$\begin{cases} \dot{z} = \begin{bmatrix} z_2 \\ z_1 z_3 + z_2 z_4 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ z_5 \\ 0 \\ 0 \\ 0 \end{bmatrix} f \quad , \; z = \begin{bmatrix} y \\ \dot{y} \\ k \\ b \\ m \end{bmatrix} \\ y = h(z) = z_1 \end{cases}$$

  - Observability Co-distribution

$$\Omega(z) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ z_3 & z_4 & z_1 & z_2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ z_3 z_4 & z_3 + z_4^2 & z_2 + z_1 z_4 & z_1 z_3 + 2 z_2 z_4 & 0 \\ 0 & 0 & 0 & z_5 & 4 \end{bmatrix}$$

$$y, \dot{y} \neq 0$$

$$\Downarrow$$

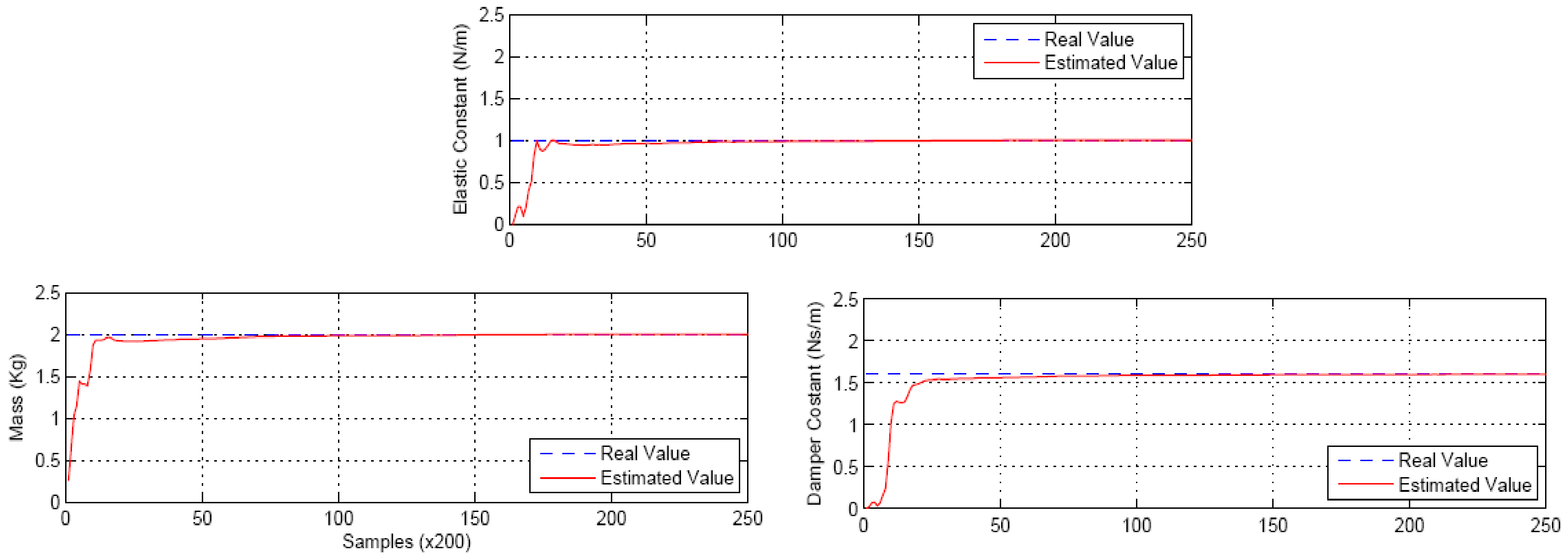
$$\mathrm{rank}(\Omega) = 5$$

$$\Downarrow$$

OBSERVABLE!

Measuring stiffness

- Linear case $f = m\ddot{y} + b\dot{y} + ky$ $m,\ b,\ k > 0$
  - Build a non-linear Observer e.g.: **Extended Kalman Filter**
  - Results can be good



Measuring stiffness

- Non-Linear case

$$f = m(\ddot{y}) + b(\dot{y}) + k(y)$$

  - the former approach is no longer possible (at least, not trivially)

$$\begin{cases} \dot{z} = \begin{bmatrix} z_2 \\ z_1 z_3 + z_2 z_4 \\ ? \\ ? \\ ? \end{bmatrix} + \begin{bmatrix} 0 \\ z_5 \\ ? \\ ? \\ ? \end{bmatrix} f \\ y = h(z) = z_1 \end{cases}$$

We want to try observing it!

Observation of Stiffness: possible approaches

- Model-based
  - A.k.a. White box or Parametric (with few parameters)
  - Pros:
    - \+ Can be based on classic approaches: EKF, particle filters, and the like
    - \+ Few, **meaningful** parameters facilitate
      - Understanding what is doing what
      - Defining bounds
  - Cons:
    - \- Not easy to do adaptive control on (non-linearity on parameters)
    - \- Specific
      - Many VSA = many different equations = many different observers
    - \- Can suffer modeling errors

Observation of Stiffness: possible approaches

- Model-based
- Model-free
  - A.k.a. black box
  - Pros:
    + no model → robust to modeling errors
  - Cons:
    - Use only local information → Have no memory
    - Not so easy to define the interfaces
    - Can use more sensors

Observation of Stiffness: possible approaches

- Model-based
- Model-free
- Gray-box
  - A.k.a. Parametric with “many” parameters
  - Pros:
    - + No model → robust to modeling errors
    - + Parameters store information → Memory effect can help when observability is lost

Measuring stiffness

- Linear case

$$f = m\ddot{y} + b\dot{y} + ky \qquad m,\, b,\, k > 0$$

  - Build a non-linear equivalent system

$$\begin{cases} \dot{z} = \begin{bmatrix} z_2 \\ z_1 z_3 + z_2 z_4 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ z_5 \\ 0 \\ 0 \\ 0 \end{bmatrix} f \\ y = h(z) = z_1 \end{cases}, \quad z = \begin{bmatrix} y \\ \dot{y} \\ k \\ b \\ m \end{bmatrix}$$

  - Observability Co-distribution

$$\Omega(z) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ z_3 & z_4 & z_1 & z_2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ z_3 z_4 & z_3 + z_4^2 & z_2 + z_1 z_4 & z_1 z_3 + 2 z_2 z_4 & 0 \\ 0 & 0 & 0 & z_5 & 4 \end{bmatrix}$$

Loses rank when

$$y, \dot{y} \neq 0$$

i.e., when the spring is not moving!

Observation of Stiffness: possible approaches

- Model-based
- Model-free
- Gray-box
  - A.k.a. Parametric with “many” parameters
  - Pros:
    + No model → robust to modeling errors
    + Parameters store information → Memory effect can help when observability is lost
  - Cons:
    - many parameters without physical interpretation (e.g., a functional basis) complicates
      - Understanding which parameter is contributing to what
      - Defining meaningful bounds for the parameters
      - Defining bounds for the performance of the filter

Observation of Stiffness: possible approaches

- Model-based
- Model-free
- Gray-box

Model-base and Gray-box go together

Vs

Model-free

Example of a model-based approach

18th IFAC World Congress
August 28 - September 2, 2011, Milano, Italy

Stiffness Estimation and Nonlinear Control of Robots with Variable Stiffness Actuation

Courtesy of

Fabrizio Flacco
Alessandro De Luca

Dipartimento di Informatica e Sistemistica A. Ruberti

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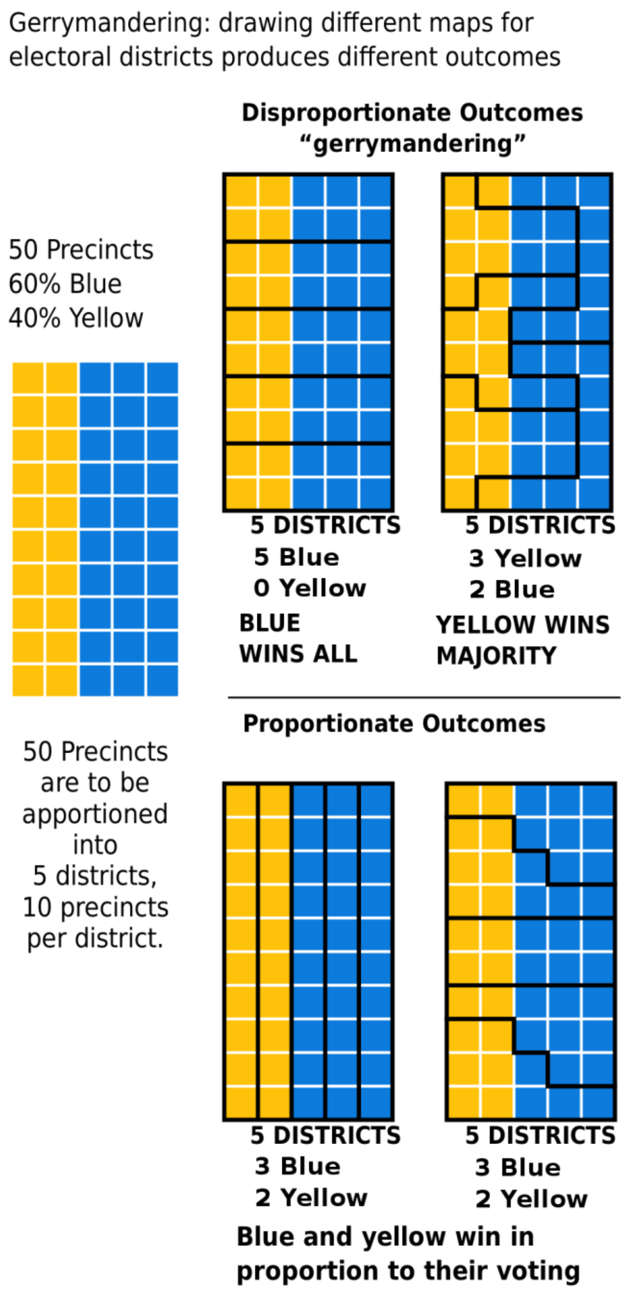
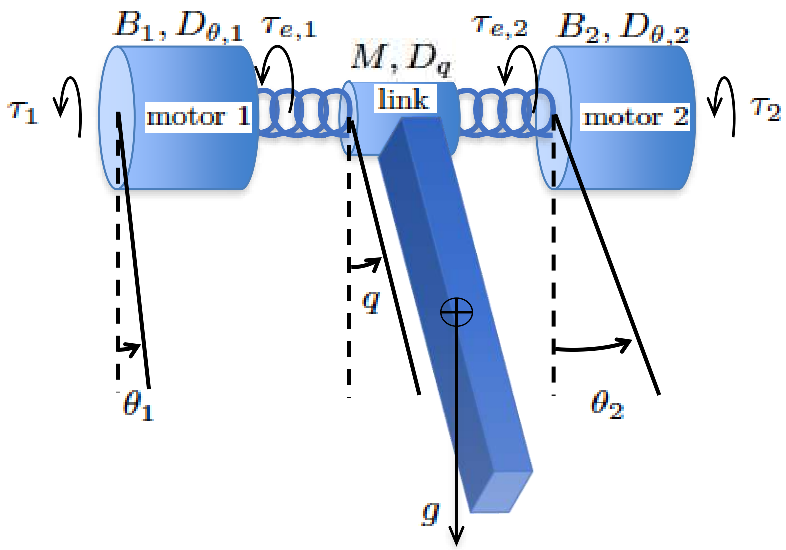
Gerrymandering

- **Gerrymandering** (/ˈdʒɛrimændərɪŋ/ or /ˈɡɛrimændərɪŋ/)[1] is a practice intended to establish an unfair political advantage for a particular party or group by manipulating the boundaries of electoral districts, which is most commonly used in first-past-the-post electoral systems. [source Wikipedia]

Please forgive me for Gerrymandering your presentation



Assume a model is given: e.g. the VSA-II by UniPisa



$$
\begin{aligned}
M\ddot{q} + D_q\dot{q} + \tau_{e,t}(\boldsymbol{\phi}) + g(q) &= \tau_k \\
B_1\ddot{\theta}_1 + D_{\theta,1}\dot{\theta}_1 - \tau_{e,1}(\phi_1) &= \tau_1 \\
B_2\ddot{\theta}_2 + D_{\theta,2}\dot{\theta}_2 - \tau_{e,2}(\phi_2) &= \tau_2
\end{aligned}
$$

typically, with symmetric structure

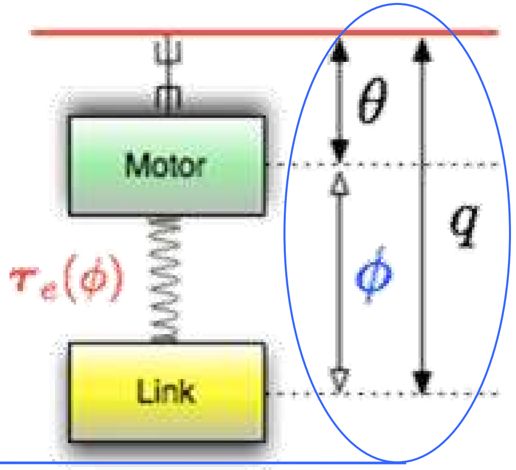
$$
\tau_e(\phi) = 2k\,\beta(\phi)\,\frac{\partial\beta(\phi)}{\partial\phi}
$$

$$
\beta(\phi) = \arcsin\left(C\,\sin\left(\frac{\phi}{2}\right)\right) - \frac{\phi}{2}
$$

two nonlinear flexible transmissions
(four-bar linkages + linear springs)

Two-stage Stiffness Estimator

$$
\begin{aligned}
M\ddot{q} + D_q\dot{q} + \tau_e(\phi) + g(q) &= \tau_k \\
B\ddot{\theta} + D_\theta\dot{\theta} - \tau_e(\phi) &= \tau
\end{aligned}
$$



two-stage approach

residual-based torque flexibility estimator $\xrightarrow{\tau_e(\phi)}$ RLS stiffness estimator $\xrightarrow{\sigma(\phi)}$

first stage

second stage

Residual-based Estimator

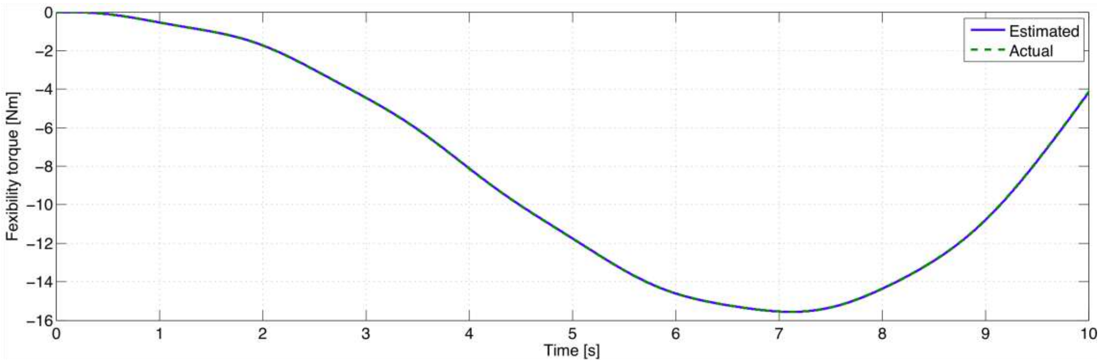
residual generator

$$r_{\tau_e} = K_{\tau_e} \left(B\dot{\theta} + D_\theta \theta - \int_0^t (\tau + r_{\tau_e})\, dt_1 \right)$$

it is easy to check that

$$\dot{r}_{\tau_e} = K_{\tau_e} (\tau_e - r_{\tau_e}) \longrightarrow \boxed{\frac{r_{\tau_e}(s)}{\tau_e(s)} = \frac{K_{\tau_e}}{s + K_{\tau_e}}}$$

first-order stable filter of the flexibility torque



on-line estimated flexibility torque for a flexible joint with constant stiffness

Stiffness Approximation

flexibility torque is approximated by a nonlinear function in the unknown parameter vector $\boldsymbol{\alpha} = (\alpha_1 \ldots \alpha_n)^T$

$$\tau_e(\phi) \simeq f(\phi, \boldsymbol{\alpha})$$

stiffness could then be analytically computed

$$\sigma(\phi) = \frac{\partial f\,(\phi, \boldsymbol{\alpha})}{\partial \phi}$$

linear parameterization (with polynomial basis)

$$f(\phi, \boldsymbol{\alpha}) = \sum_{h=1}^{n} f_h(\phi)\alpha_h = \boldsymbol{F}^T(\phi)\boldsymbol{\alpha}$$

$$f_h(\phi) = \phi^{2h-1}, \quad h = 1, \ldots, n$$

only odd powers

$$\sigma(\phi) = \sum_{h=1}^{n}(2h-1)\phi^{2h-2}\alpha_h$$

Recursive Least Squares (RLS)

the parameter vector $\widehat{\boldsymbol{\alpha}}$ that minimizes the cost function

$$E = \frac{1}{2}\sum_{k=1}^{p}\Big(r_{\tau_e}(k) - f\left(\phi(k), \widehat{\boldsymbol{\alpha}}, n\right)\Big)^2$$

is obtained with a Recursive Least Squares algorithm

$$\widehat{\boldsymbol{\alpha}}(k) = \widehat{\boldsymbol{\alpha}}(k-1) + \Delta\widehat{\boldsymbol{\alpha}}$$

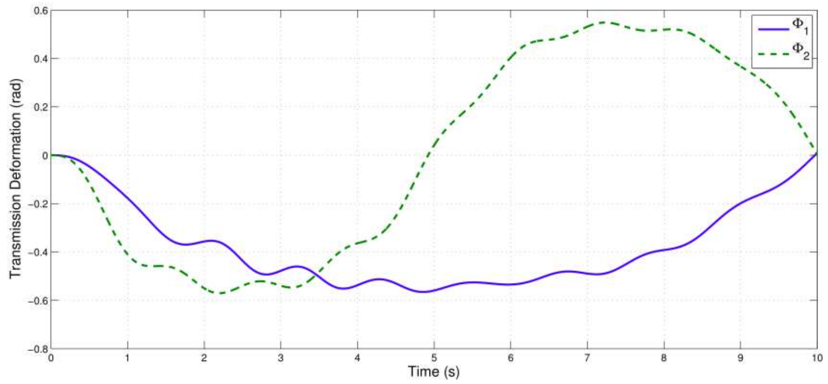
$$\Delta\widehat{\boldsymbol{\alpha}} = \boldsymbol{L}(k)\Big(r_{\tau_e}(k) - \boldsymbol{F}^T(k)\widehat{\boldsymbol{\alpha}}(k-1)\Big)$$

$$\boldsymbol{L}(k) = \frac{\boldsymbol{P}(k-1)\boldsymbol{F}(k)}{1+\boldsymbol{F}^T(k)\boldsymbol{P}(k-1)\boldsymbol{F}(k)}$$

$$\boldsymbol{P}(k) = \Big(\boldsymbol{I} - \boldsymbol{L}(k)\boldsymbol{F}^T(k)\Big)\boldsymbol{P}(k-1)$$

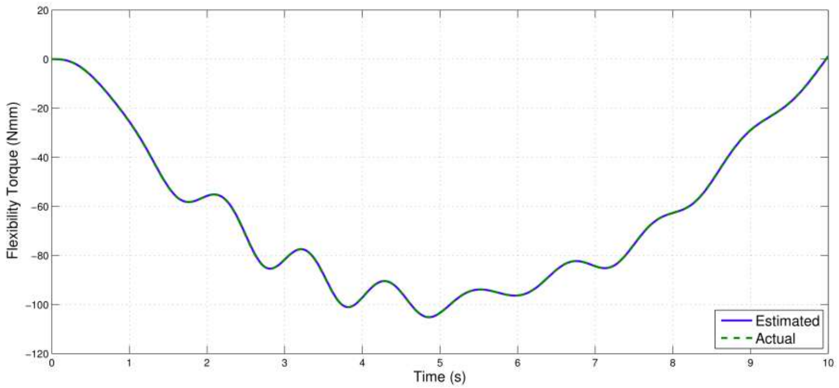
Estimation Results - 1

deformation of the two transmissions during motion

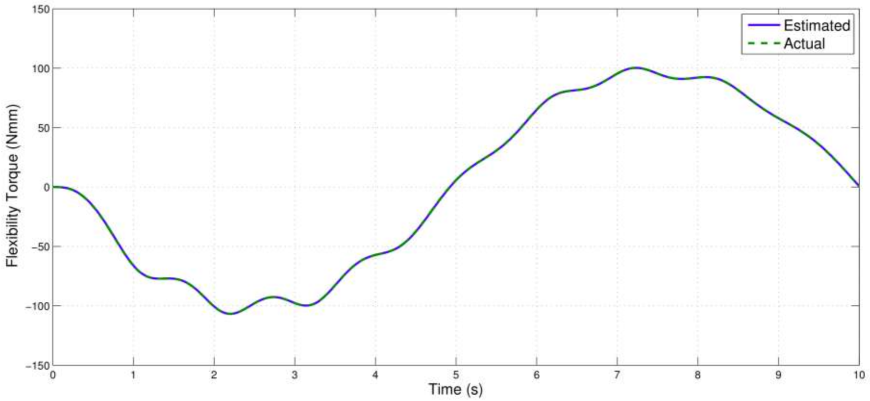


≈ 0.6 rad max

on-line estimated flexibility torques (using residuals)



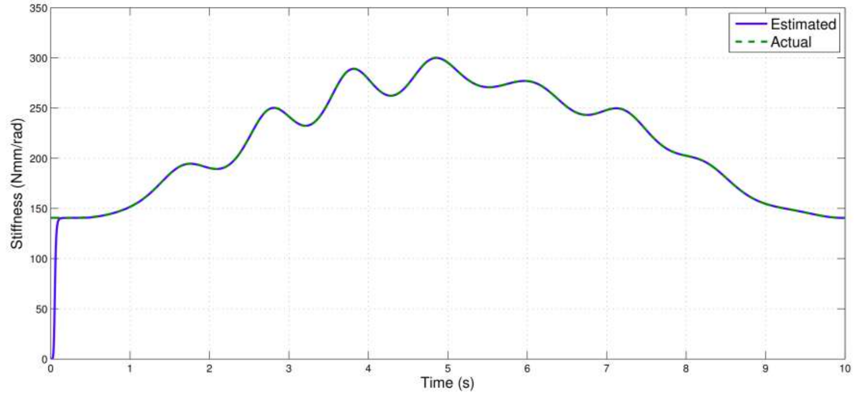
left transmission

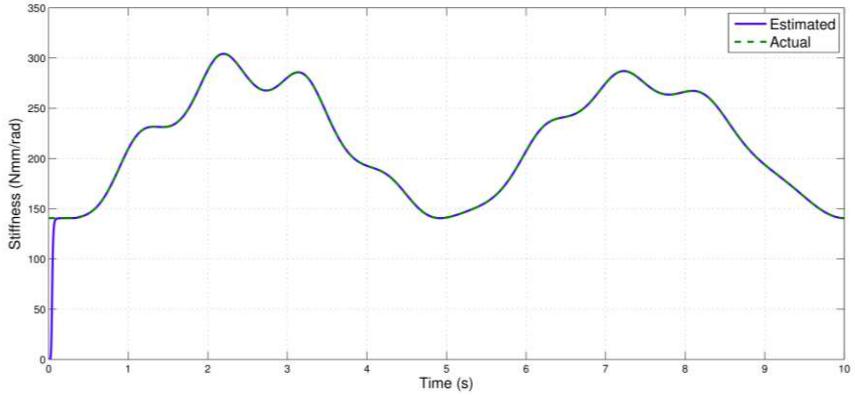


right transmission

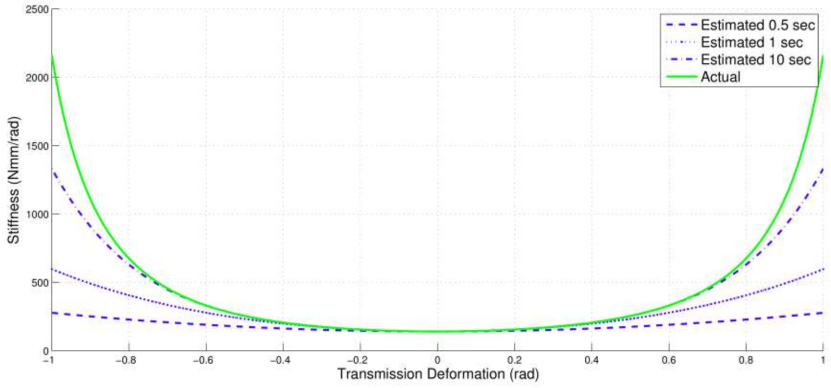
Estimation Results - 2

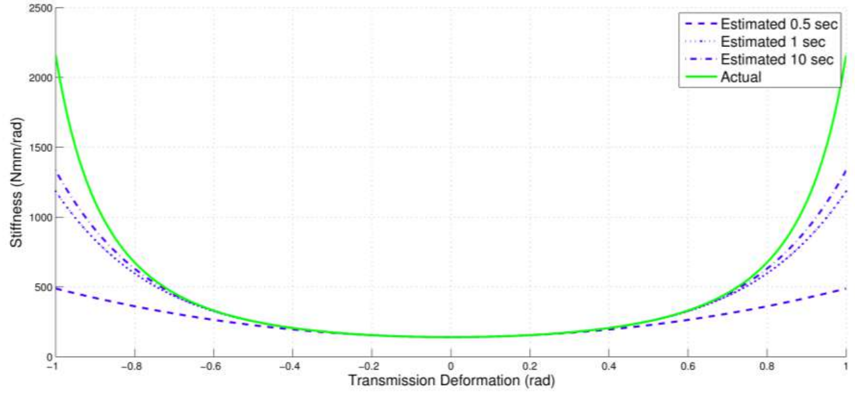
estimated stiffness of the two transmissions





... over time





... stiffness profiles as function of transmission deformations

Examples of model-free approaches

“A Non-Invasive, Real-Time Method for Measuring Variable Stiffness”
G. Grioli, A. Bicchi

Robotic Science and Systems 2010, Zaragoza, Spain.

Model-free Stiffness observer

- **stiffness observer:**
  - given

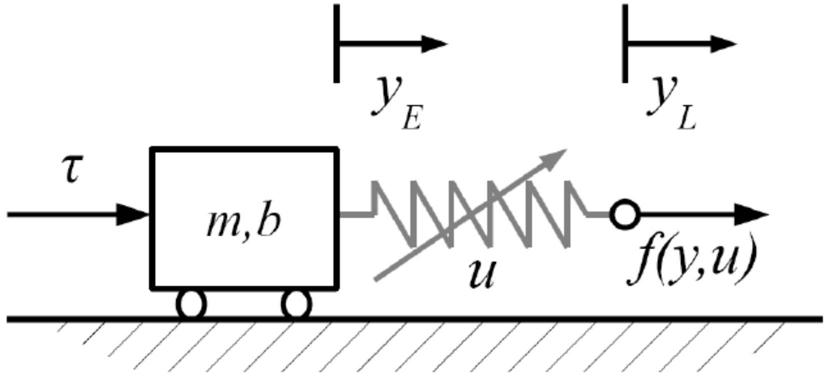
$$\tau = m\ddot{y} + b\dot{y} + f\left(y, u\right)$$

  - differentiation yields

$$\dot{\tau} = m y^{(3)} + b\ddot{y} + \sigma\dot{y}$$

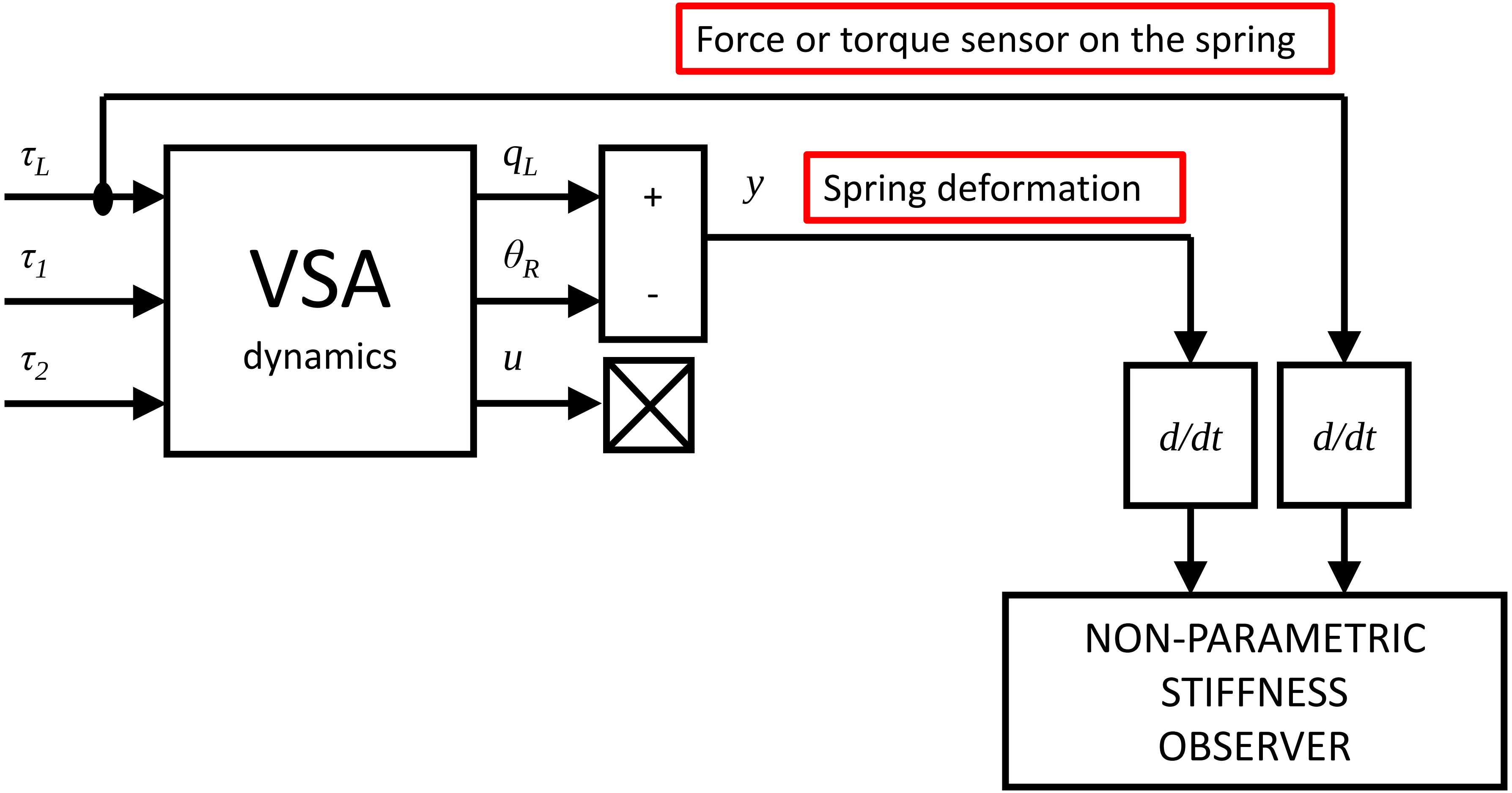
  - build an estimate

$$\dot{\hat{\tau}} = m y^{(3)} + b\ddot{y} + \hat{\sigma}\dot{y}$$



$$\dot{\tilde{\tau}} = \dot{\tau} - \dot{\hat{\tau}} = [\ldots] = \tilde{\sigma}\dot{y}$$

Schematics



Force or torque sensor on the spring
τ_L
τ_1
τ_2
VSA
dynamics
q_L
θ_R
u
+
-
y
Spring deformation
d/dt
d/dt
NON-PARAMETRIC
STIFFNESS
OBSERVER

Model-free Stiffness observer

- **stiffness observer:**
  - Assume an estimate of the stiffness $\hat{\sigma}$
  - Use the update law $\dot{\hat{\sigma}} = \alpha \dot{\hat{\tau}} \operatorname{sgn}(\dot{y})$
  - Define the error function $V_\sigma := \frac{1}{2}\tilde{\sigma}^2$
  - Calculate its time derivative

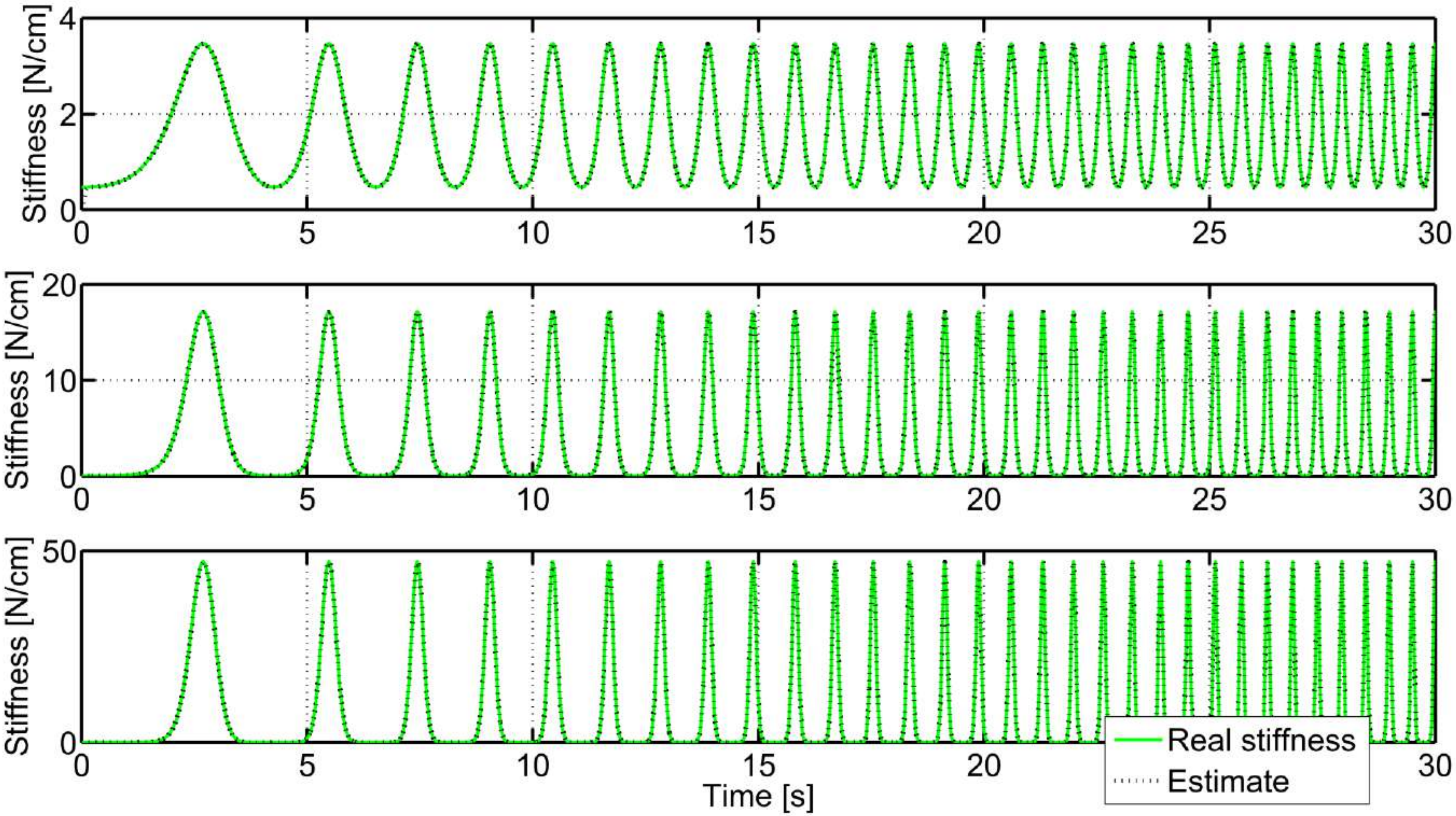
$$\dot{V}_\sigma = \tilde{\sigma}\dot{\tilde{\sigma}} = \tilde{\sigma}\dot{\sigma} - \tilde{\sigma}\dot{\hat{\sigma}} = \tilde{\sigma}\dot{\sigma} - \alpha\tilde{\sigma} s_u \, \dot{u} \, \mathrm{sgn}(\dot{y}) - \alpha\tilde{\sigma}^2 |\dot{y}|$$

where $s_u = \frac{\partial \tau}{\partial u}$ i.e. the derivative of the output torque w.r.t the stiffness input

  - **Converges** to within a **Uniformly Ultimately Bounded** error region near the **real stiffness value**

$$|\tilde{\sigma}| > \frac{|\sigma_y|}{\alpha} + \left(|s_u| + \frac{|\sigma_u|}{\alpha} \right) \frac{\dot{u}}{\dot{y}}$$

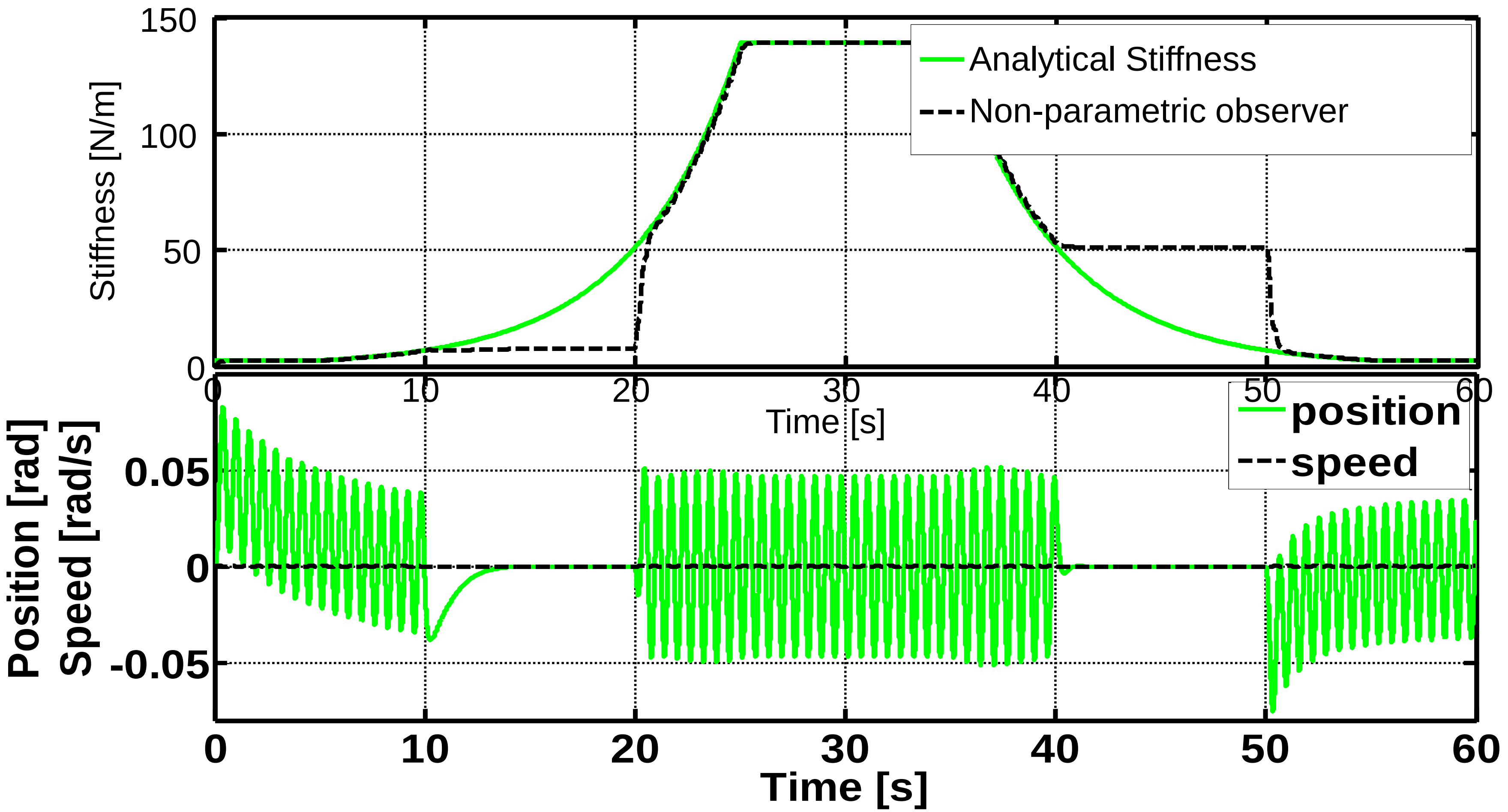
It works



Stiffness [N/cm]
4
2
0
Stiffness [N/cm]
20
10
0
Stiffness [N/cm]
50
0
0
5
10
15
20
25
30
Time [s]
Real stiffness
Estimate

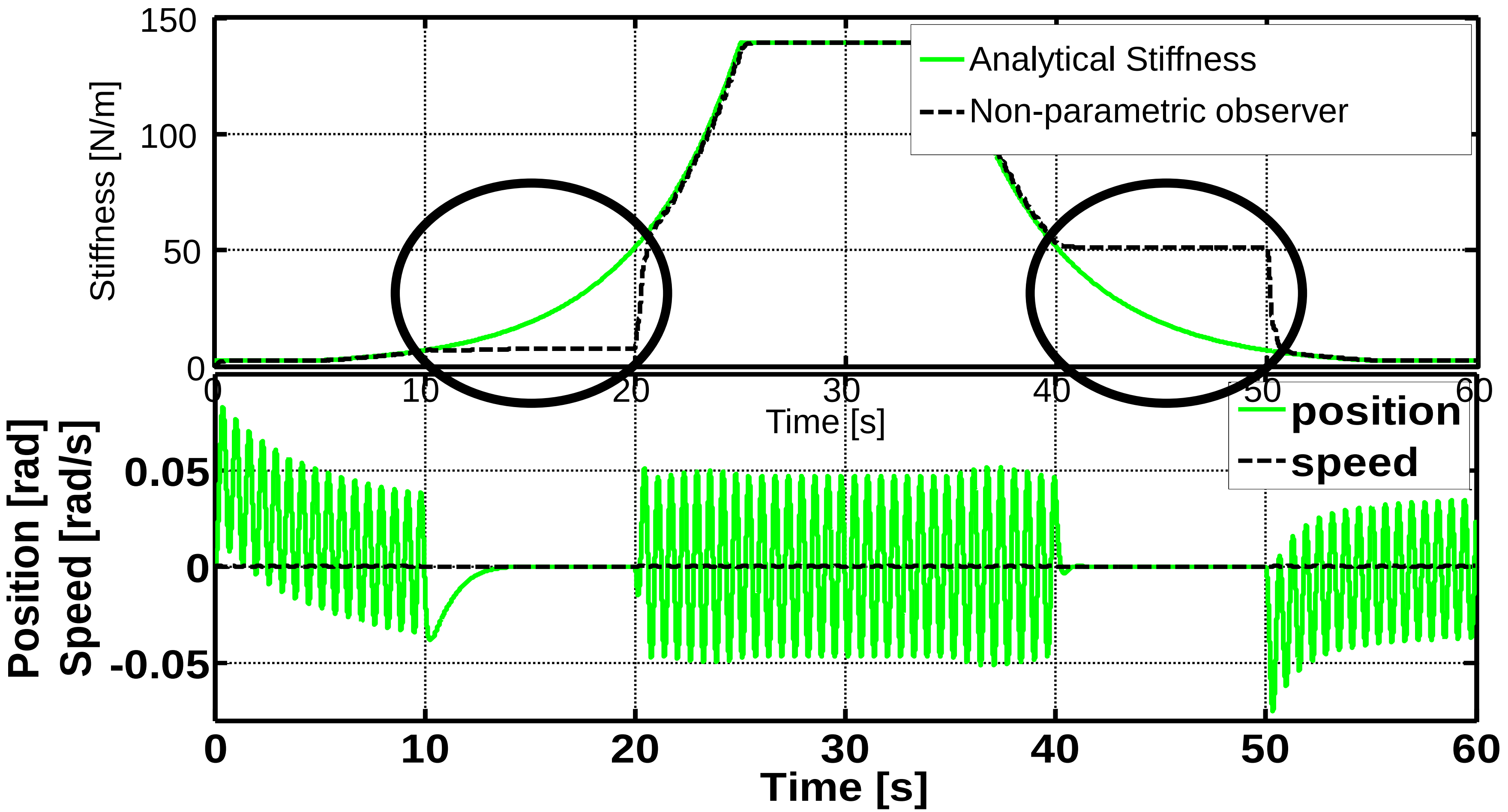
Explanation of the bound

$$|\tilde{\sigma}| > \frac{|\sigma_y|}{\alpha} + \left(|s_u| + \frac{|\sigma_u|}{\alpha}\right)\frac{\dot{u}}{\dot{y}}$$

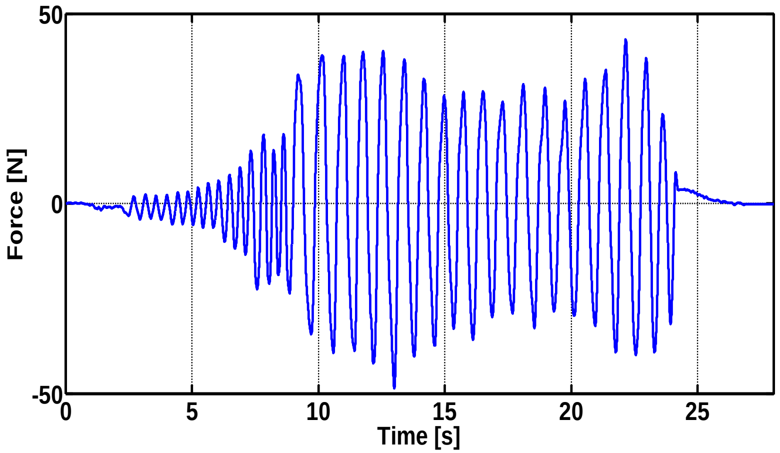


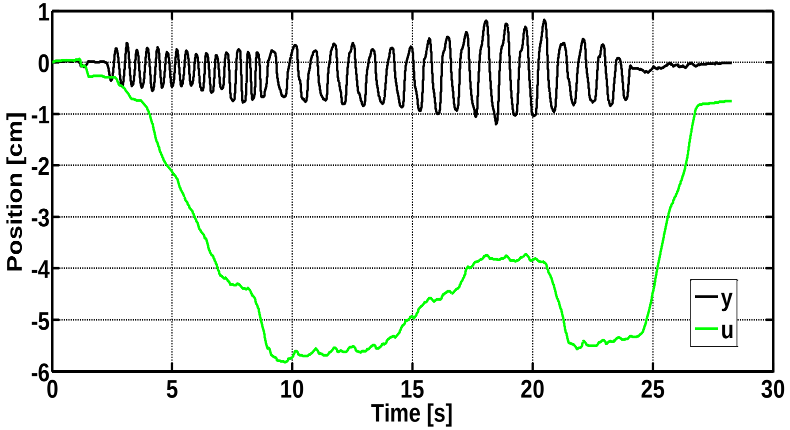
Explanation of the bound

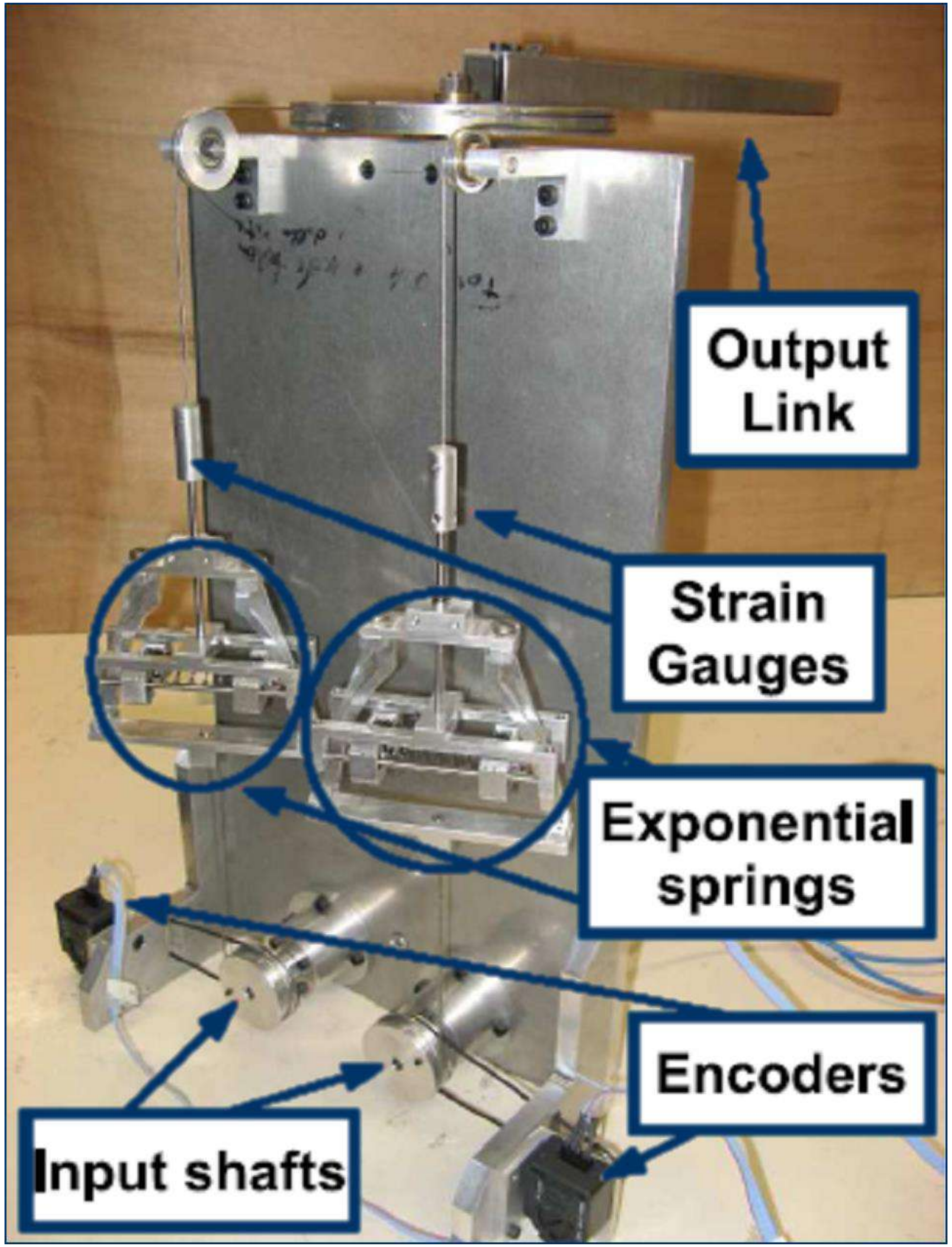
$$|\tilde{\sigma}| > \frac{|\sigma_y|}{\alpha} + \left(|s_u| + \frac{|\sigma_u|}{\alpha}\right)\frac{\dot{u}}{\dot{y}}$$



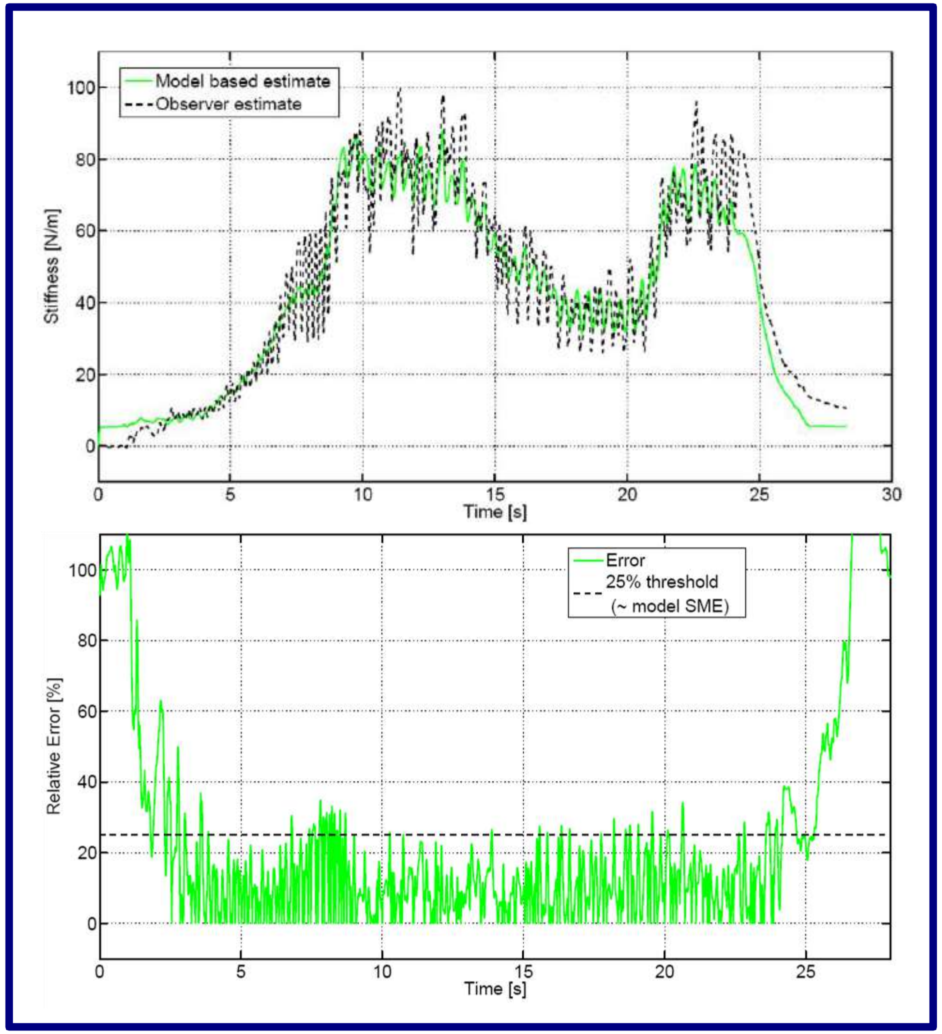
Experimental setup

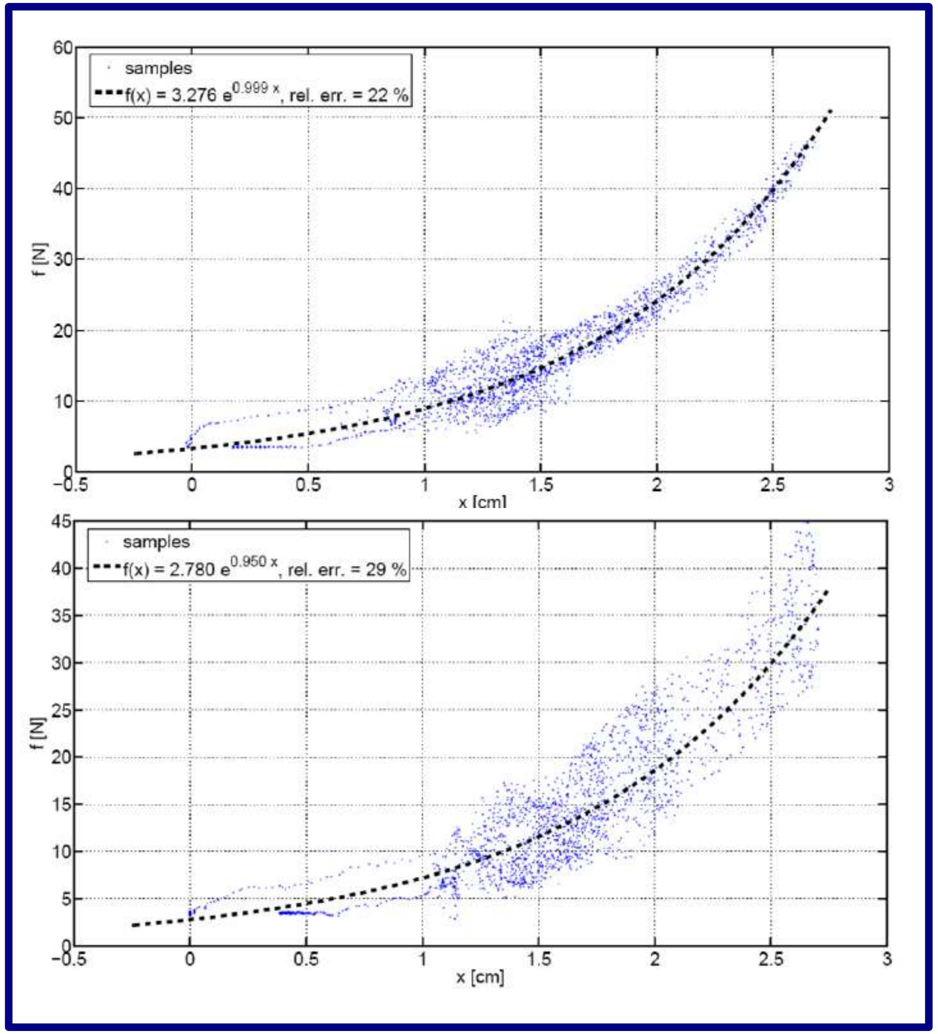






Experimental results





“A Non-Invasive, Real-Time Method for Measuring Variable Stiffness” G. Grioli, A. Bicchi Robotic Science and Systems 2010, Zaragoza, Spain.

- Still needed force or torque sensors

2011 IEEE International Conference on
Robotics and Automation
May 9-13, 2011 Shanghai, China

Residual-based Stiffness Estimation in Robots with Flexible Transmissions

Courtesy of

Fabrizio Flacco
Alessandro De Luca
Dipartimento di Informatica e Sistemistica

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2^{nd} Order Residual Based Estimator

Second order residual

$$r_\sigma = K_{\sigma,1}\left(p_\theta + D_\theta\theta - \int_0^t \left(\tau + \int_0^{t_1} r_\sigma\, dt_2\right) dt_1\right) - K_{\sigma,2}\int_0^t r_\sigma\, dt_1$$

It is easy to check that

$$\dot{r}_\sigma = K_{\sigma,1}\left(\dot{p}_\theta + D_\theta\dot{\theta} - \tau - \int_0^t r_\sigma\, dt_1\right) - K_{\sigma,2} r_\sigma$$

$$\ddot{r}_\sigma = K_{\sigma,1}\left(\sigma(\phi)\dot{\phi} - r_\sigma\right) - K_{\sigma,2}\,\dot{r}_\sigma$$

Second order filter of

$$\boxed{\dot{\tau}_e(\phi) = \frac{\partial \tau_e(\phi)}{\partial \phi}\,\dot{\phi} = \sigma(\phi)\dot{\phi}} \qquad \sigma(\phi) \simeq \widehat{\sigma}(\phi) = \frac{r_\sigma}{\dot{\phi}}$$

Black-Box Estimator of Stiffness

Proposed solution

$$\begin{aligned}\widehat{\sigma}(k+1) &= \widehat{\sigma}(k) + K_p\left(r_\sigma\dot{\phi} - \widehat{\sigma}(k)\dot{\phi}^2\right)\\ &\simeq \widehat{\sigma}(k) + K_p\dot{\phi}^2\left(\sigma(\phi) - \widehat{\sigma}(k)\right)\\ &= \left(1 - K_p\dot{\phi}^2\right)\widehat{\sigma}(k) + \sigma(\phi)K_p\dot{\phi}^2\end{aligned}$$

Stable if

$$\left|1 - K_p\dot{\phi}^2\right| < 1 \quad \Rightarrow \quad K_p\dot{\phi}^2 \in [0, 2)$$

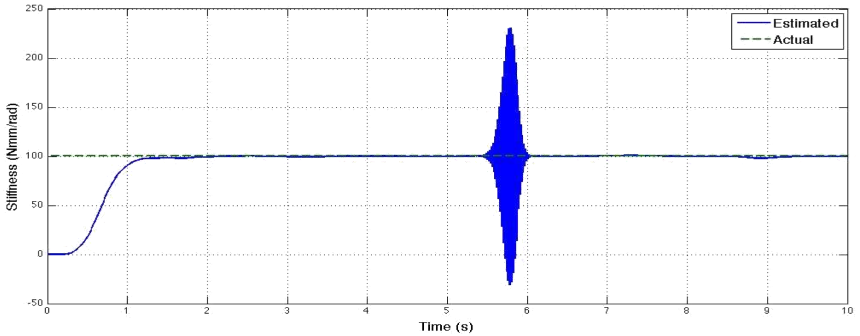
Stability recovery

$$\text{if } K_p\dot{\phi}^2 > K_{max} \quad \Rightarrow \quad K_p = \frac{K_{max}}{\dot{\phi}^2}$$

Black-Box Estimator of Stiffness

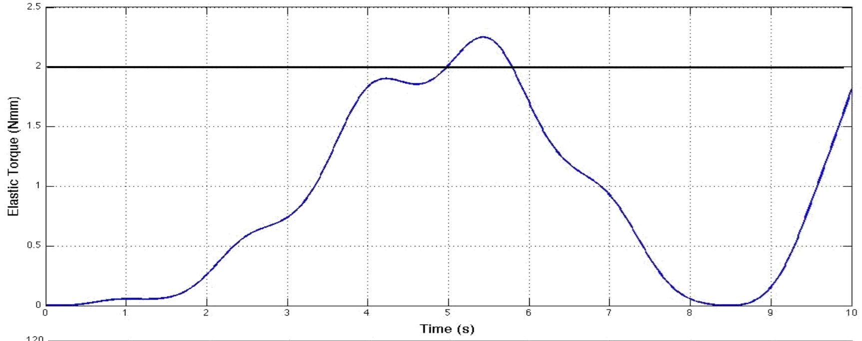
Constant stiffness

Estimated Stiffness
without stability recover

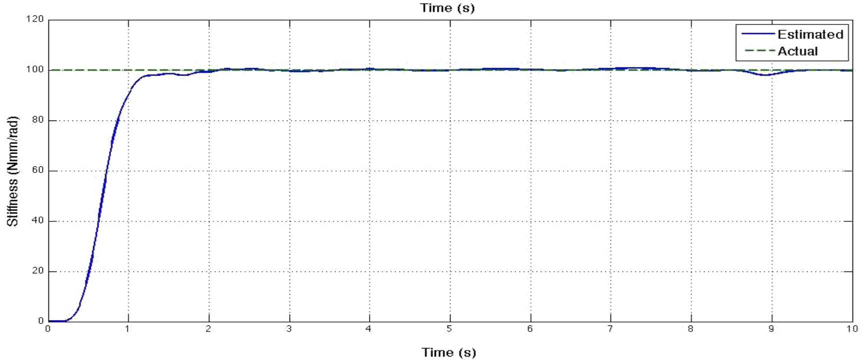


Proportional Factor

$$K_p\dot{\phi}^2$$

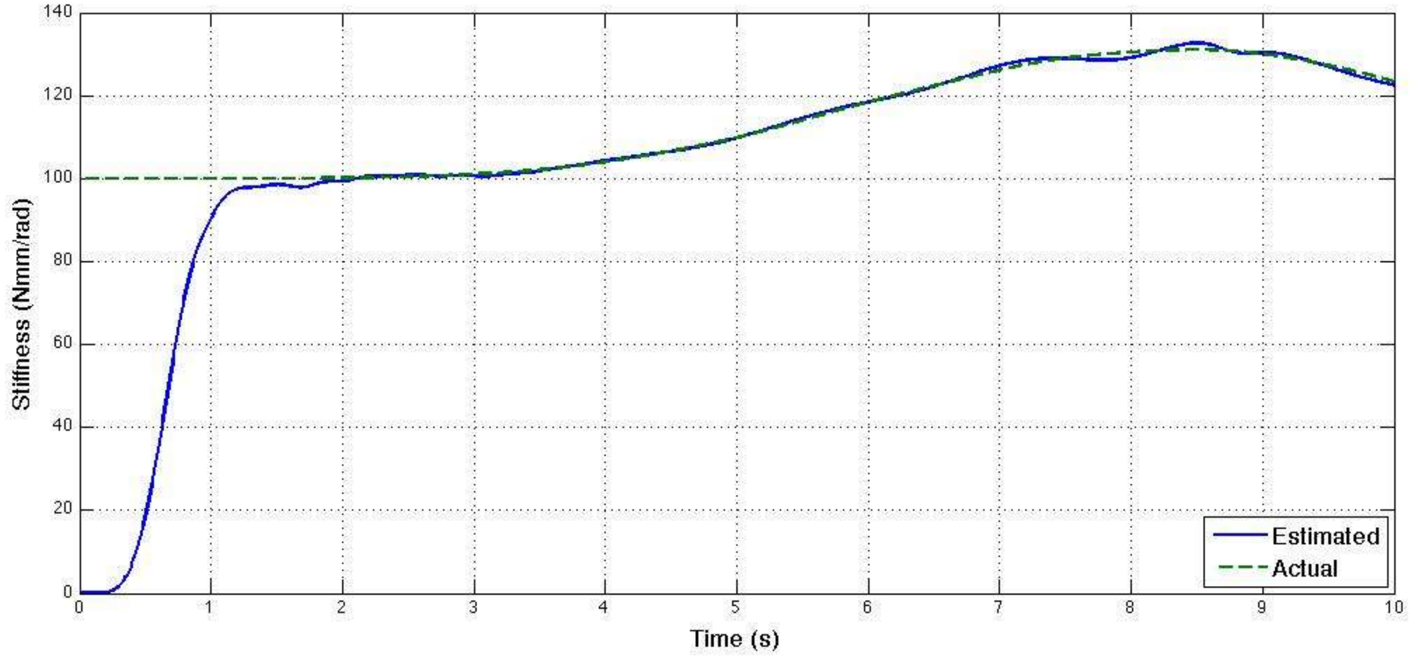


Estimated Stiffness with
stability recover



Black-Box Estimator of Stiffness

Nonlinear stiffness $\tau_e(\phi) = K_a\phi + K_b\phi^3$

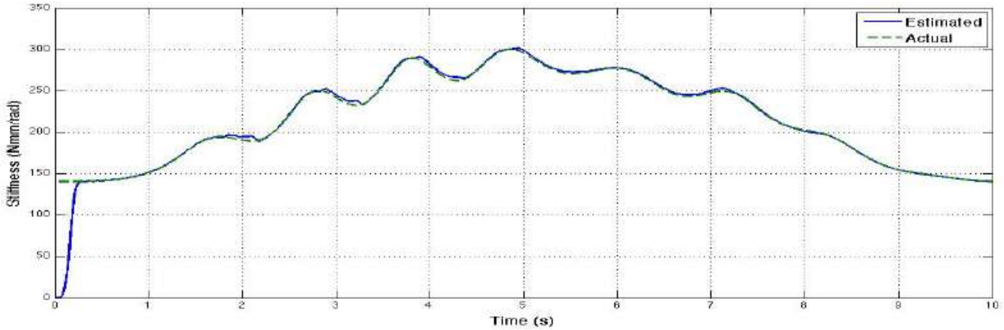


Starting without knowledge of the stiffness,
after a transient of about 1 sec,
the actual stiffness value is dynamically tracked

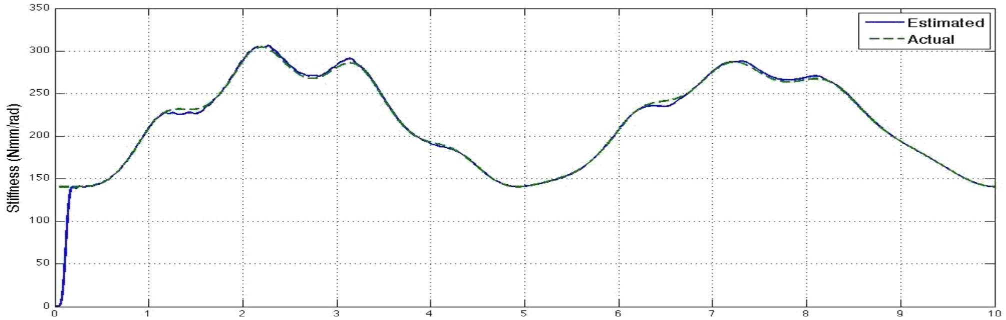
Black-Box Estimator of Stiffness

VSAII

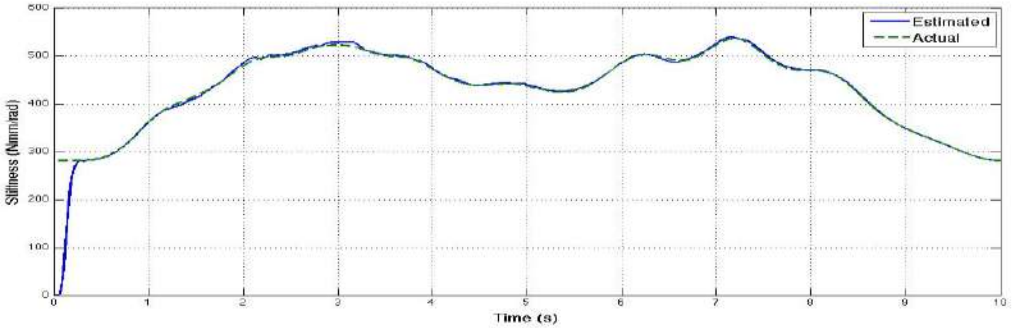
Estimated stiffness for the first motor



Estimated stiffness for the second motor



Estimated total joint stiffness



Other works

A real-time robust observer for an Agonistic-Antagonist Variable Stiffness Actuator

T. Ménard[1],

G. Grioli[2,3] and A. Bicchi[2,3]

[1]GREYC, University of Caen, Caen, France

[2]Advanced Robotics Dept., Istituto Italiano di Tecnologia, Genova, Italy

[3]Centro Interdipartimentale di Ricerca ``E. Piaggio'', University of Pisa, Italy

Our approach

1. Derive a relationship involving the stiffness
2. Transform this relationship into a relationship between integrals of the measured signals

[...]

3. Estimate the stiffness

ICRA 2020

An Input Observer-Based Stiffness Estimation Approach for Flexible Robot Joints

Courtesy of *Adriano Fagiolini, Maja Trumić, Kosta Jovanović*

UNIVERSITÀ DEGLI STUDI DI PALERMO

dipartimento di ingegneria unipa

UNIVERSITY OF BELGRADE

School of Electrical Engineering - ETF

Steps

Robot dynamics

$$I\, q^{(3)} + \beta q^{(2)} + mgl\, \dot{q} \cos q = \dot{\tau}_e$$

Flexibility torque reconstruction

$$\hat{\dot{\tau}}_{e,k} = \hat{u}_k + mgl\, \dot{q}_k \cos q_k$$

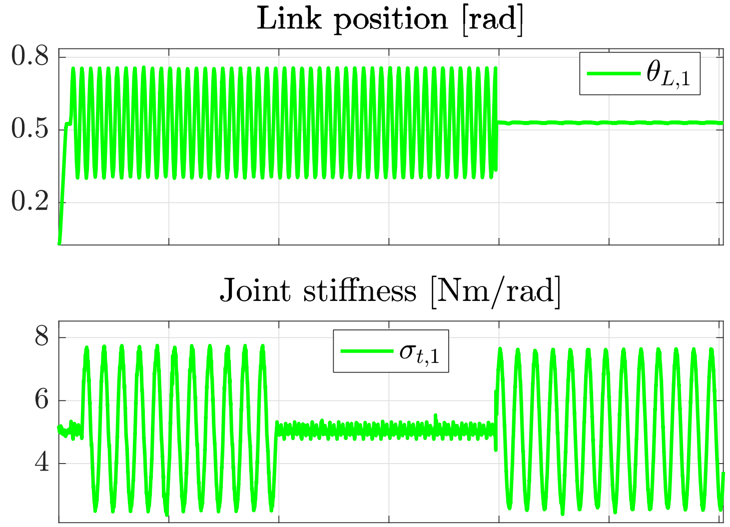
Observer

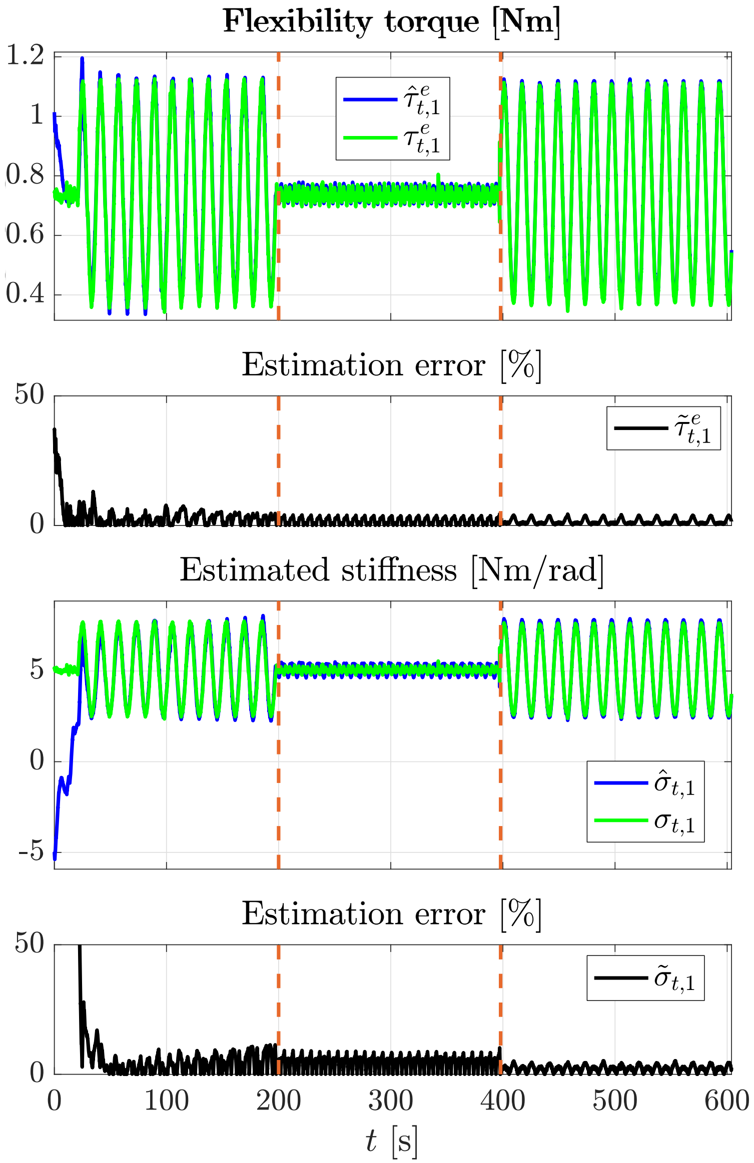
$$\hat{x}_{k+1} = E\, \hat{x}_k + FY_k$$
$$\hat{u}_k = G \begin{pmatrix} \hat{x}_{k+1} - A\hat{x}_k \\ y_k - C\, x_k \end{pmatrix}$$

Stiffness Estimation

$$\dot{\tau}_e \approx (\pi_{a,0} + \pi_{a,2}\phi_a^2 + \pi_{a,4}\phi_a^4)\dot{\phi}_a + (\pi_{b,0} + \pi_{b,2}\phi_b^2 + \pi_{b,4}\phi_b^4)\dot{\phi}_b$$
$$\sigma \approx \pi_{a,0} + \pi_{a,2}\phi_a^2 + \pi_{a,4}\phi_a^4 + \pi_{b,0} + \pi_{b,2}\phi_b^2 + \pi_{b,4}\phi_b^4$$

Experiments – 1-DoF





Torque Mean Square Error $\sim 10^{-5}$

Stiffness Mean Square Error $\sim 10^{-1}$

Torque Mean Square Error $\sim 10^{-6}$

Stiffness Mean Square Error $\sim 10^{-2}$

Conclusions

- When controlling a VSA closing the loop of the stiffness is a **problem**
- Approaches:
  - Characterization and identification
    - Datasheets
  - Online estimation
    - Parametric Estimators
    - Non-parametric Estimators
- There are still open problems

Open problems

- Extension to damping & inertia estimation

Serio, A., Grioli, G., Sardellitti, I., Tsagarakis, N. G., & Bicchi, A. (2011, May). A decoupled impedance observer for a variable stiffness robot. In *2011 IEEE international conference on robotics and automation* (pp. 5548-5553). IEEE.

- Closed loop stiffness identification and control

Flacco, F., & De Luca, A. (2011). Stiffness estimation and nonlinear control of robots with variable stiffness actuation. *IFAC Proceedings Volumes*, *44*(1), 6872-6879.

Trumić, M. B. (2021). *Stiffness estimation and adaptive control of soft robots* (Doctoral dissertation, Univerzitet u Beogradu-Elektrotehnički fakultet).

- What happens when there is interaction?

Bibliography

- Grioli, Giorgio, and Antonio Bicchi. **"A non-invasive, real-time method for measuring variable stiffness."** *Robotics Science and Systems VI* (2010): 90-96.
- Flacco, Fabrizio, and Alessandro De Luca. **"Stiffness estimation and nonlinear control of robots with variable stiffness actuation."** *IFAC Proceedings Volumes* 44.1 (2011): 6872-6879.
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Thank You!

Characterization and Estimation of the Stiffness (or Impedance) of a Robot

by Giorgio Grioli