# Model Based Control of Soft Robots

Project Assignment, SIDRA 2021

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Consider the soft planar segment in figure. This simple soft robot is approximated to bend with a curvature $\theta \in \mathbb{R}$, which is constant in space but variable in time (CC segment). Its total length is also *not* constant in time. Instead, it uniformly elongates and its length $L_0 + \delta L(t)$, with $L_0 \in \mathbb{R}$ being the rest length. The local coordinate $s \in [0, 1]$ is defined such that the arc[1] going from $\{S_0\}$ to $\{S_s\}$ subtends an angle $s\theta$.

The rest length is $L_0 = 1$m. Gravity acceleration points orthogonal to the plane of bending, which means that $U_{\mathrm{G}} = 0$. The elastic potential is

$$U_{\mathrm{K}} = \int_0^1 \frac{1}{2} k_1(s)\theta^2 \mathrm{d}s + \frac{1}{2} k_2 \delta L^2. \tag{1}$$

The segment is thin and its characteristics homogeneous, i.e.

- Mass: $m(s) \equiv 1$Kg
- Inertia: $j(s) \equiv 0$
- Stiffness: $k_1(s) \equiv 1$Nm/rad

For the sake of simplicity, the segment is assumed actuated via a pure torque $\tau$, applied at the tip.

Your tasks are the following:

(i) Derive the dynamic equations of this 2 DoF system, in the form

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + K(q) + \begin{bmatrix} 0.25 & 0 \\ 0 & 0.5 \end{bmatrix} \dot{q} = A\tau,$$

with $q = (\theta, \delta L) \in \mathbb{R}^2$, $M, C \in \mathbb{R}^{2\times 2}$, $A \in \mathbb{R}^{2\times 1}$, and $K \in \mathbb{R}^2$, and where for simplicity we have have provided already the damping matrix $D$.

**Tip:** Note that this task is quite similar to the derivation of the CC segment dynamics we have done in class. So, same steps should be followed.

[1] Which is the length of this arc?

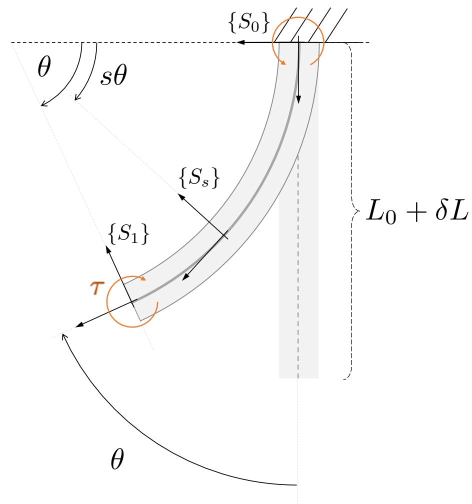


Figure 1: A constant curvature segment with variable length, actuated via a pure torque at the tip.

The main difference is that the total length is $L_0 + \delta L$ rather than $L_0$, and that $\delta L$ is a Lagrangian variable. So, start with the forward kinematics $h(s, (\theta, \delta L)) = x(s, t)$, then Jacobians, then ...

(ii) Simulate[2] the evolution of said system when starting from the initial condition $(\theta(0), \delta L(0)) = (\pi/4, 0)$, $(\dot{\theta}(0), \dot{\delta L}(0)) = (0, 0)$, for $\tau \equiv 0$ and $k_2 \in \{0.1, 1, 10\}$N/m.

**Tip:** many expressions are numerically ill-conditioned for $\theta \simeq 0$. The simplest fix to this is to introduce something like the following lines of code at the beginning of each function

```
if abs(theta) < 1e-3
    theta = 1e-3;
end
```

(iii) Which equilibria can be obtained by varying $\tau \in \mathbb{R}$?

(iv) Propose a control action $\tau$ such that $\theta$ converges to any given $\bar{\theta}$, no matter the initial condition.

Simulate the closed loop evolution for $\bar{\theta} = \pi/2$ when starting from $(\theta(0), \delta L(0)) = (0, 0)$, $(\dot{\theta}(0), \dot{\delta L}(0)) = (0, 0)$, with $k_2 = 1$N/m.

[2]The use of MatLab/Simulink is preferred but not mandatory. One simulation is required for each of the three values of stiffness.

(v) Can the controller $\tau$ be devised in such a way that there is no overshoot in $\theta$ for the same initial condition and desired reference as in point (iv)? Motivate your answer, possibly including simulations.

The results should be submitted as a single zip file, named as your surname, and containing

- A report in pdf format, where the steps/reasoning necessary to get to the results are summarized **succinctly**. Please divide it in five sections, one for each question. Plots of $q$ in time must be included whenever a simulation is required to answer the question
- Source codes with comments (enough to navigate the code, do not overdo it)

You can ask questions about this assignment via email to c.dellasantina@tudelft.nl. I may write/call you myself to ask a few questions about your assignment.