Force and Position Control of Flexible-Link Robots

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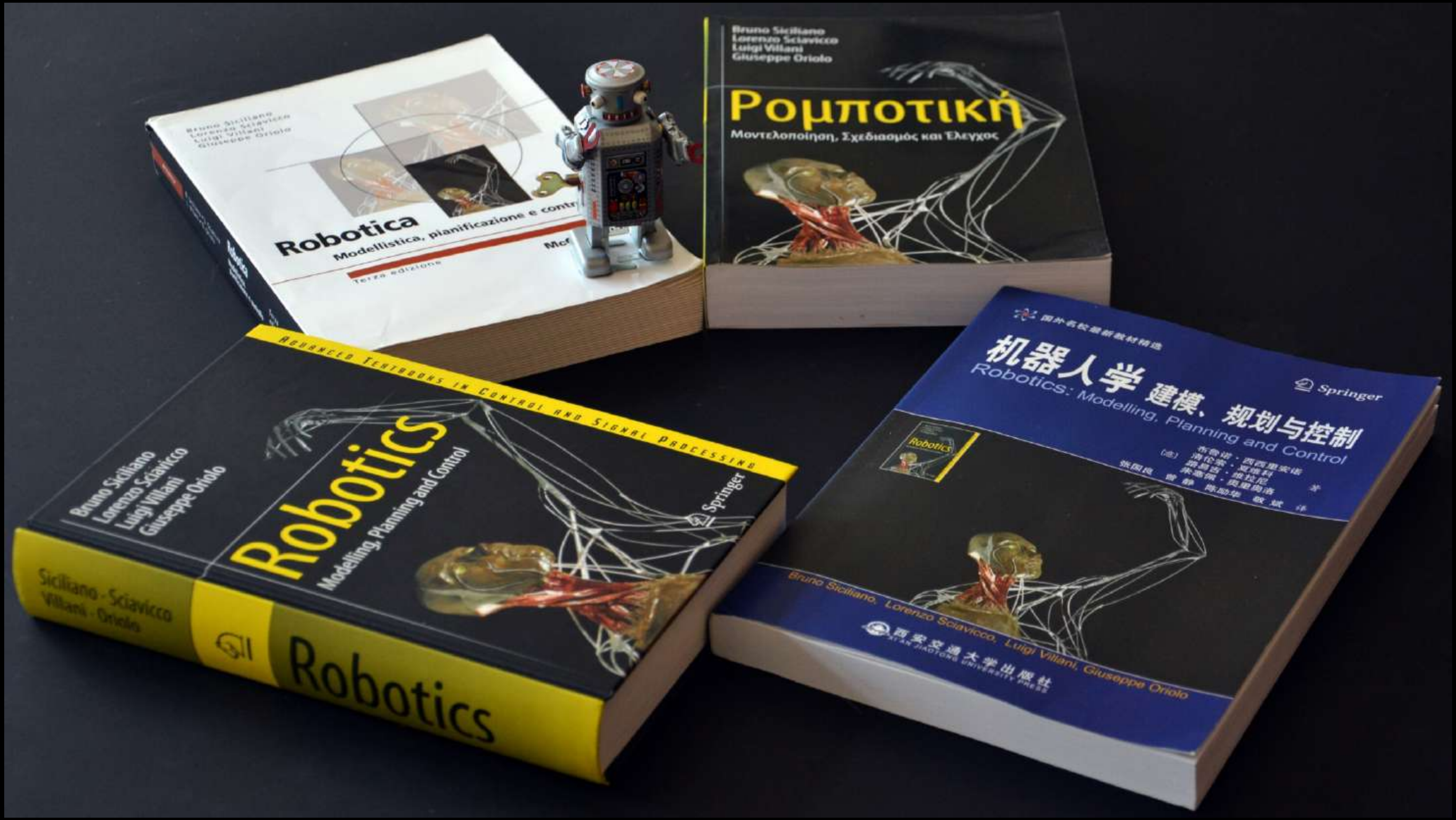
The Textbook



The Handbook

- Interaction control
- Modelling
  - Kinematics
  - Dynamics
  - Contact
- Indirect force and position regulation
  - CLIK algorithm
  - PD+ control
- Direct force and position control
  - Composite control
  - Force and position regulation
  - Force regulation and position tracking
- References

- In a wide number of applications, such as polishing, deburring, machining or assembling, it is necessary to control the interaction between the robot manipulator and the environment
  - The environment sets constraints on the geometric paths that can be followed by the end-effector
  - Purely motion control strategies for controlling the interaction will fail (rising contact force)
- The intrinsic compliance of a flexible-link robot may contribute to reduction in the value of the forces that can be generated when the interaction task is executed by a rigid robot
  - Using flexible robots to perform interaction tasks, some benefits may be gained, even though the distributed flexibility of the links makes the interaction control problem more complex than for rigid robots
- Interaction control strategies
  - Indirect force control (via motion control)
  - Direct force control (force feedback loop)

- The inherent difficulty of force control of flexible manipulators is due to problems similar to those arising in motion control
  - Kinematics and dynamics of the robot cannot be stated independent of the forces acting on the robot tip (end-point)
  - The additional deflections caused by contact forces must be suitably taken into account for the computation of inverse kinematics solutions
- Dynamics of a flexible manipulator in contact with the environment is very difficult to solve and simplifications must be made
  - If the assumed modes technique is adopted to model the flexible manipulator, the mode functions must satisfy the geometric boundary conditions, which are not altered by the contact with the environment, while the natural boundary conditions (i.e. those involving the balance of forces and moments at the ends of the links) are automatically taken into account by the Lagrange formulation of the dynamic model
- Another challenge is damping the vibrations that are naturally excited during the task execution
  - Adoption of singular perturbation theory when the link stiffness is large, and a two-time-scale model of the flexible manipulator can be derived

- Planar *n*-link flexible manipulators with revolute joints are considered
  - Links are subject to bending deformation in the plane of motion, i.e, torsional effects are neglected
- Position of a point along the deflected link

$$^{i}\mathbf{p}_i\,(x_i) = \left[\begin{array}{cc} x_i & w_i\,(x_i) \end{array}\right]^{\mathrm{T}}$$

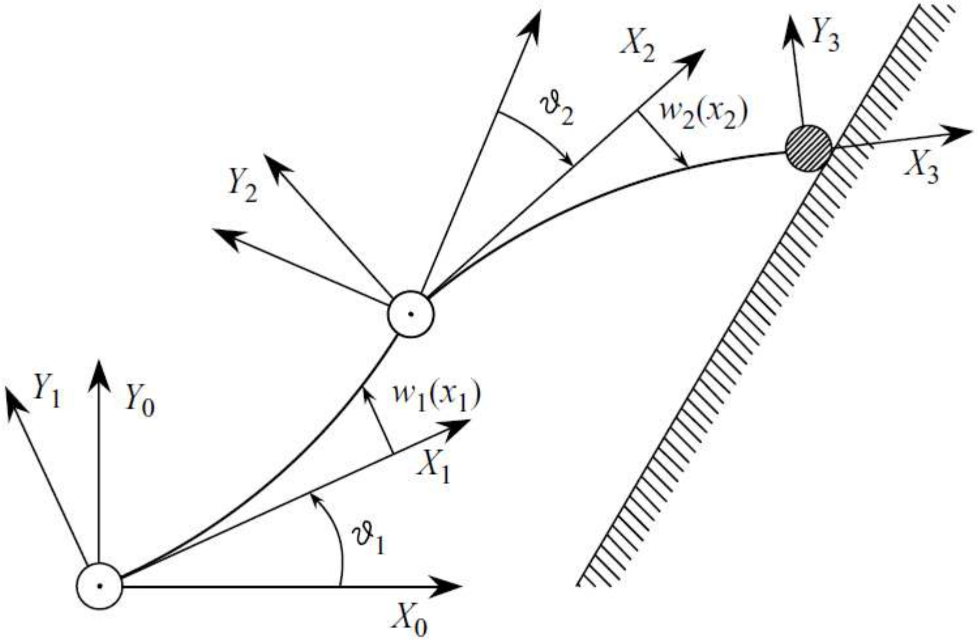
- Position of origin of frame (X_{i+1}, Y_{i+1})

$$^{i}\mathbf{r}_{i+1} \;=\; {}^{i}\mathbf{p}_i\,(L_i)$$

- Joint (rigid) rotation matrix $\mathbf{R}_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \\ \sin\theta_i & \cos\theta_i \end{bmatrix}$
- Link (flexible) rotation matrix $\mathbf{E}_i = \begin{bmatrix} 1 & -w'_{ie} \\ w'_{ie} & 1 \end{bmatrix}$

$$w'_{ie} = (\partial w_i / \partial x_i)\,|_{x_i = L_i}$$

$$\arctan w'_{ie} \simeq w'_{ie}$$



- Absolute position vectors

$$\mathbf{p}_i = \mathbf{r}_i + \mathbf{W}_i{}^i\mathbf{p}_i$$

$$\mathbf{r}_{i+1} = \mathbf{r}_i + \mathbf{W}_i{}^i\mathbf{r}_{i+1}$$

- Global transformation matrix from base frame to frame (X_{i+1}, Y_{i+1})

$$\mathbf{W}_i = \mathbf{W}_{i-1}\mathbf{E}_{i-1}\mathbf{R}_i = \hat{\mathbf{W}}_{i-1}\mathbf{R}_i \quad \hat{\mathbf{W}}_0 = \mathbf{I}$$

Link Flexibility

- Finite-dimensional model (of order m_i) of link flexibility
  - Euler–Bernoulli equation for flexible beams (separability in time and space of solutions)

$$(EI)_i \frac{\partial^4 w_i(x_i,t)}{\partial x_i^4} + \rho_i \frac{\partial^2 w_i(x_i,t)}{\partial t^2} = 0 \qquad i = 1, \dots, n$$

ρ_i uniform mass density

$(EI)_i$ constant flexural link rigidity

  - Link deflection (assumed modes)

$$w_i(x_i,t) = \sum_{j=1}^{m_i} \phi_{ij}(x_i)\,\delta_{ij}(t)$$

$\delta_{ij}(t)$ time-varying variables

$\phi_{ij}(x_i)$ spatial mode shapes

  - Mode shapes have to satisfy proper boundary conditions at the base (clamped) and at the end of each link (mass)

- Position of the manipulator end-point as a function of the (n × 1) joint variable vector and the (m × 1) deflection variable vector

$$\mathbf{p} = \mathbf{k}(\boldsymbol{\theta}, \boldsymbol{\delta}) \qquad m = \sum_{i=1}^{n} m_i$$

- Linear velocity of a point on the arm

$$\dot{\mathbf{p}}_i = \dot{\mathbf{r}}_i + \dot{\mathbf{W}}_i \mathbf{p}_i^i + \mathbf{W}_i \dot{\mathbf{p}}_i^i \qquad {}^i\dot{\mathbf{r}}_{i+1} = {}^i\dot{\mathbf{p}}_i(L_i)$$

$$\dot{\mathbf{W}}_i = \dot{\widehat{\mathbf{W}}}_{i-1} \mathbf{R}_i + \widehat{\mathbf{W}}_{i-1} \dot{\mathbf{R}}_i \qquad \dot{\mathbf{R}}_i = \mathbf{S}\mathbf{R}_i \dot{\theta}_i \qquad \mathbf{S} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$\dot{\widehat{\mathbf{W}}}_i = \dot{\mathbf{W}}_i \mathbf{E}_i + \mathbf{W}_i \dot{\mathbf{E}}_i \qquad \dot{\mathbf{E}}_i = \mathbf{S}\dot{w}'_{ie}$$

- Tip velocity as a function of joint velocity and link velocity

$$\dot{\mathbf{p}} = \mathbf{J}_\theta(\boldsymbol{\theta}, \boldsymbol{\delta})\,\dot{\boldsymbol{\theta}} + \mathbf{J}_\delta(\boldsymbol{\theta}, \boldsymbol{\delta})\,\dot{\boldsymbol{\delta}}$$

$$\mathbf{J}_\theta = \partial \mathbf{k}/\partial \boldsymbol{\theta}$$ rigid-body Jacobian

$$\mathbf{J}_\delta = \partial \mathbf{k}/\partial \boldsymbol{\delta}$$ flexible-body Jacobian

- Manipulator in contact with the environment → In view of virtual work principle (kinetostatics duality)
  - Vector $\mathbf{f}$ of forces exerted by the manipulator on the environment performing work on position variables

⇩

  - Vector $\mathbf{J}_\theta^T \mathbf{f}$ of joint torques performing work on joint variables
  - Vector $\mathbf{J}_\delta^T \mathbf{f}$ of elastic reaction forces performing work on link variables

- Finite-dimensional Lagrangian dynamic model of planar manipulator in contact with the environment

$$\mathbf{M}_{\theta\theta}(\theta,\delta)\ddot{\theta} + \mathbf{M}_{\theta\delta}(\theta,\delta)\ddot{\delta} + \mathbf{c}_{\theta}(\theta,\delta,\dot{\theta},\dot{\delta}) + \mathbf{g}_{\theta}(\theta,\delta) = \tau - \mathbf{J}_{\vartheta}^{T}(\vartheta,\delta)\mathbf{f}$$

$$\mathbf{M}_{\theta\delta}^{T}(\theta,\delta)\ddot{\theta} + \mathbf{M}_{\delta\delta}(\theta,\delta)\ddot{\delta} + \mathbf{c}_{\delta}(\theta,\delta,\dot{\theta},\dot{\delta}) + \mathbf{g}_{\delta}(\theta,\delta) + \mathbf{D}\dot{\delta} + \mathbf{K}\delta = -\mathbf{J}_{\delta}^{T}(\vartheta,\delta)\mathbf{f}$$

  - Matrix blocks of positive definite symmetric inertia matrix $\mathbf{M}_{\theta\theta}$ $\mathbf{M}_{\theta\delta}$ $\mathbf{M}_{\delta\delta}$
  - Vectors of Coriolis and centrifugal forces $\mathbf{c}_{\theta}$ $\mathbf{c}_{\delta}$
  - Vector of gravitational forces $\mathbf{g}_{\theta}$ $\mathbf{g}_{\delta}$
  - Diagonal and positive definite link stiffness matrix $\mathbf{K}$
  - Diagonal and positive semi-definite link damping matrix $\mathbf{D}$
  - Vector of input joint torques τ

Contact Force

- To analyse the performance of the position and force control algorithms, a model of the contact force is required
- Frictionless and planar elastic surface, which is locally a good approximation to surfaces of regular curvature (assuming contact is not lost)

$$\mathbf{f} = k_{\mathrm{e}} \mathbf{n} \mathbf{n}^T (\mathbf{p} - \mathbf{p}_{\mathrm{e}}) = k_{\mathrm{e}} \mathbf{n} \mathbf{n}^T (\mathbf{k}(\boldsymbol{\theta}, \boldsymbol{\delta}) - \mathbf{p}_{\mathrm{e}})$$

k_{e} surface stiffness

$\mathbf{p}_{\mathrm{e}}$ undeformed (constant) position of the surface

$\mathbf{n}$ (constant) unit vector of the direction normal to the surface

Indirect Force and Position Regulation

- The interaction of a flexible-link robot with a compliant environment can be managed by controlling both the contact force and the end-point position
- Control objective (without requiring direct measurement of the contact force)
  - Desired force along the normal to the surface $f_d \mathbf{n}$
  - Desired position on the contact plane $\mathbf{p}_d$

$\Downarrow$

- The desired force can be achieved only if the component normal to the plane of the desired position is chosen as

$$p_{dn} = \mathbf{n}^{\mathrm{T}} \mathbf{p}_d = k_e^{-1} f_d + p_{en}$$

p_{en} undeformed position of the surface

Two-Stage Control Strategy

- The first stage is in charge of solving the inverse kinematics problem to compute the desired vectors of joint variables and deflection variables that place the end-point of the flexible arm at a desired position
- In the second stage, which constitutes a joint regulator, the computed joint and link variables are used as set-points

- From the link dynamic equations in a static situation the deflections satisfy the equation

$$\mathbf{g}_\delta(\boldsymbol{\theta}, \boldsymbol{\delta}) + \mathbf{K}\delta = -\mathbf{J}_\delta^T(\boldsymbol{\theta}, \boldsymbol{\delta})\,\mathbf{f}$$

  - According to small deflection approximation $\mathbf{g}_\delta$ $\mathbf{J}_\delta$ $\mathbf{p}$ are a function of $\boldsymbol{\theta}$

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$$\boldsymbol{\delta} = -\mathbf{K}^{-1}\left(k_e \mathbf{j}_{\delta n}(\boldsymbol{\theta})\,(p_n(\boldsymbol{\theta}) - p_{en}) + \mathbf{g}_\delta(\boldsymbol{\theta})\right)$$

$$\mathbf{j}_{\delta n}(\boldsymbol{\theta}) = \mathbf{J}_\delta^T \mathbf{n}$$

$$p_n = \mathbf{n}^T \mathbf{p}$$

$$p_{en} = \mathbf{n}^T \mathbf{p}_e$$

- Differentiating equation of deflection variables

$$\dot{\boldsymbol{\delta}} = \mathbf{J}_{fg}(\boldsymbol{\theta})\,\dot{\boldsymbol{\theta}}$$

$$\mathbf{J}_{fg} = -\mathbf{K}^{-1}\left(k_e\mathbf{J}_f(\boldsymbol{\theta}) + \mathbf{J}_g(\boldsymbol{\theta})\right)$$

$$\mathbf{J}_f = \frac{\partial \mathbf{j}_{\delta n}}{\partial \boldsymbol{\theta}}(p_n - p_{en}) + \mathbf{j}_{\delta n}\frac{\partial p_n}{\partial \boldsymbol{\theta}}$$

$$\mathbf{J}_g = \partial \mathbf{g}_{\delta}/\partial \boldsymbol{\theta}$$

$$\Downarrow$$

$$\dot{\mathbf{p}} = \mathbf{J}_p(\boldsymbol{\theta}, \boldsymbol{\delta})\,\dot{\boldsymbol{\theta}} \qquad \mathbf{J}_p = \mathbf{J}_\theta + \mathbf{J}_\delta \mathbf{J}_{fg}$$

Overall Jacobian matrix relating joint to end-point velocity

- Rigid-body Jacobian modified with two terms accounting for the deflections induced by the contact force and gravity, respectively

CLIK Algorithm

- Formal analogy with the differential kinematics equation for a rigid arm
- Closed-Loop Inverse Kinematics (CLIK) algorithm – Jacobian transpose

$$\dot{\boldsymbol{\theta}} = \mathbf{J}_p^T(\boldsymbol{\theta}, \boldsymbol{\delta})\,\mathbf{K}_p(\mathbf{p}_d - \mathbf{p})$$

  - Using a Lyapunov argument, it can be shown that, as long as the vector $\mathbf{K}_p(\mathbf{p}_d - \mathbf{p})$ is outside the null space of $\mathbf{J}_p^T$, the end-point position error $\mathbf{p}_\mathrm{d} - \mathbf{p}$ asymptotically tends to zero

- Regulating the joint and deflection variables to the values $\boldsymbol{\theta}_d$ and $\boldsymbol{\delta}_d$ computed in the first stage

$$\tau = \mathbf{K}_1 (\boldsymbol{\theta}_d - \boldsymbol{\theta}) - \mathbf{K}_2\dot{\boldsymbol{\theta}} + \mathbf{g}_\theta(\boldsymbol{\theta}_d, \boldsymbol{\delta}_d) + \mathbf{J}_\theta^T(\boldsymbol{\theta}_d, \boldsymbol{\delta}_d) f_d \mathbf{n}$$

$\mathbf{K}_1$ $\mathbf{K}_2$ suitable positive definite matrix gains

$\mathbf{g}_\theta(\boldsymbol{\theta}_d, \boldsymbol{\delta}_d)$ $\mathbf{J}_\theta^T(\boldsymbol{\theta}_d, \boldsymbol{\delta}_d) f_d \mathbf{n}$ required to compensate for the gravity torque and contact force respectively

  - Asymptotic convergence to the corresponding set points

    $\mathbf{p} \rightarrow \mathbf{p}_d$ $\mathbf{f} \rightarrow f_d \mathbf{n}$

- PD+ regulator ensures asymptotic stability only in the presence of significant damping
  - When passive damping is too low, active vibration damping can be achieved by using full state-feedback
- The overall performance in terms of end-point position and force errors strongly depends on the accuracy of the static model of the flexible arm, as well as on the accuracy of the available estimates of the stiffness and position of the contact surface

- Planar two-link flexible robot with 0.1 kg payload at the end-point

$$\boldsymbol{\theta} = \begin{bmatrix} \theta_1 & \theta_2 \end{bmatrix}^T$$

- Expansion with two clamped-mass assumed modes for each link

$$\boldsymbol{\delta} = \begin{bmatrix} \delta_{11} & \delta_{12} & \delta_{21} & \delta_{22} \end{bmatrix}^T$$

  - Resulting natural frequencies of vibration

$$f_{11} = 1.40\,\text{Hz} \quad f_{12} = 5.10\,\text{Hz} \quad f_{21} = 536.09\,\text{Hz} \quad f_{22} = 20792.09\,\text{Hz}$$

- Stiffness matrix

$$\mathbf{K} = \begin{bmatrix} 38.79 & 513.37 \\ 536.09 & 20792.09 \end{bmatrix}$$

Parameter (unit)	Link 1	Link 2
Density (kg/m)	1	1
Length (m)	0.5	0.5
Centre of mass (m)	0.25	0.25
Mass (kg)	0.5	0.5
Hub mass (kg)	1	1

- The contact surface is a vertical plane $\mathbf{n} = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$
  - Point of the undeformed plane $\mathbf{p}_e = \begin{bmatrix} 0.55 & 0 \end{bmatrix}^T$ m
  - Contact stiffness $k_e = 50\mathrm{N/m}$
- CLIK algorithm
  - Matrix gains $\mathbf{K}_p = \mathrm{diag}\left\{ 500 \quad 500 \right\}$
  - Euler integration rule at 1 ms

$$\boldsymbol{\theta}_d(t_{k+1}) = \boldsymbol{\theta}_d(t_k) + T_c \mathbf{J}_p^T(\boldsymbol{\theta}_d(t_k), \boldsymbol{\delta}_d(t_k)) \mathbf{K}_p(\mathbf{p}_d(t_k) - \mathbf{p}(t_k))$$

$$\boldsymbol{\delta}_d(t_{k+1}) = -\mathbf{K}^{-1}(k_e \mathbf{j}_{\delta n}(\boldsymbol{\theta}_d(t_k))(p_n(\boldsymbol{\theta}(t_k)) - p_{en}) + \mathbf{g}_\delta(\boldsymbol{\theta}_d(t_k)))$$

- PD+ control
  - Matrix gains $\mathbf{K}_1 = \mathrm{diag}\left\{ 25 \quad 25 \right\}$ $\mathbf{K}_2 = \mathrm{diag}\left\{ 3 \quad 3 \right\}$

Contact Task

- The arm is initially placed with the tip in contact with the undeformed plane in the position

  $$\mathbf{p}(0) = [0.55 \quad -0.55]^{\mathrm{T}} \ \mathrm{m}$$

  with null contact force
- The corresponding generalized coordinates of the arm (computed by using the CLIK algorithm) are

  $$\boldsymbol{\theta} = \begin{bmatrix} -1.396 & 1.462 \end{bmatrix}^T \mathrm{rad} \quad \boldsymbol{\delta} = \begin{bmatrix} -0.106 & 0.001 & -0.009 & -0.0001 \end{bmatrix}^T \mathrm{m}$$

- It is desired to reach the end-point position

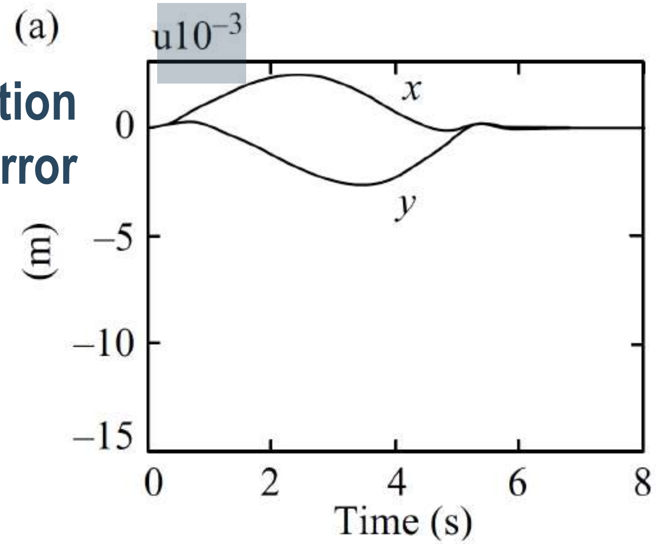
  $$\mathbf{p}_d = \begin{bmatrix} 0.55 & -0.50 \end{bmatrix}^T \mathrm{m}$$

  and a fifth-order polynomial trajectory with null initial and final velocity and acceleration is imposed from the initial to the final position with a duration of 5 s

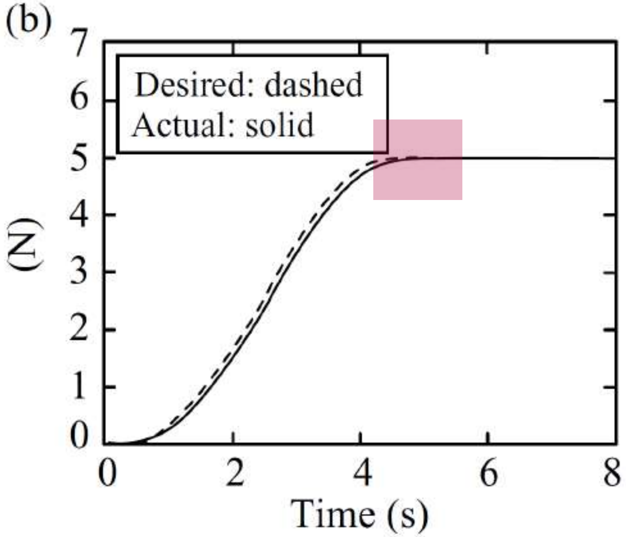
1st Case Study

$$\mathbf{f}_d = \begin{bmatrix} 5 & 0 \end{bmatrix}^T \text{N}$$

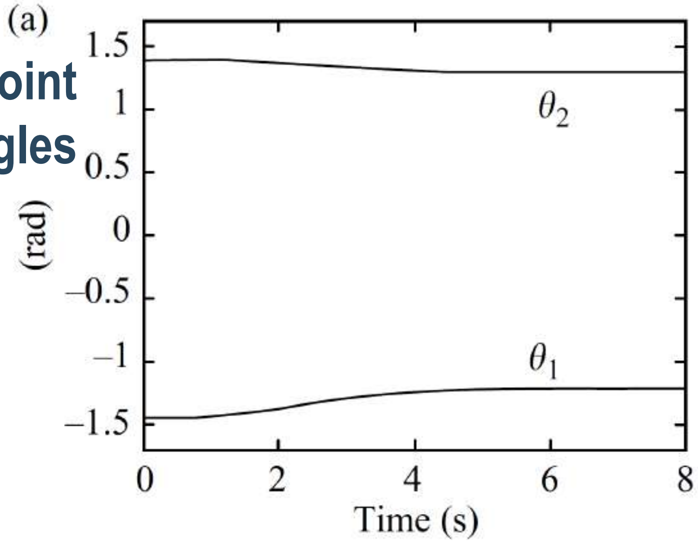
position error



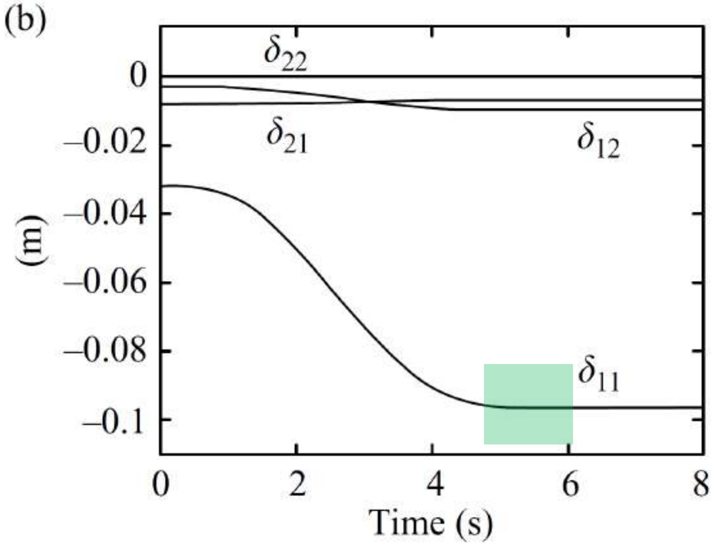
contact force



joint angles

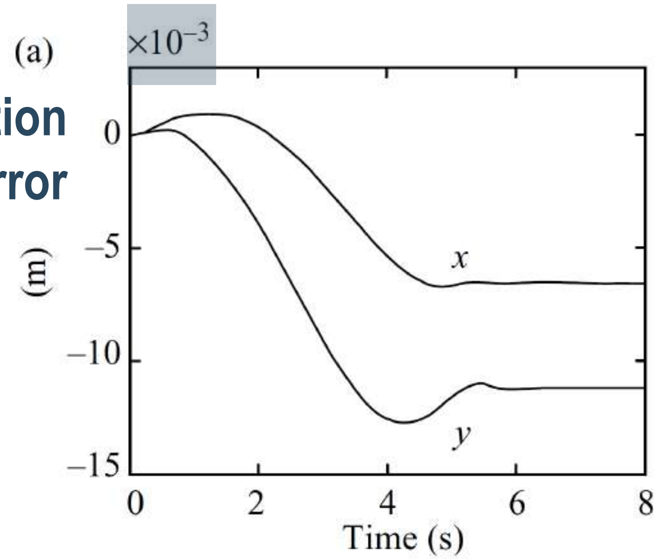


link deflections

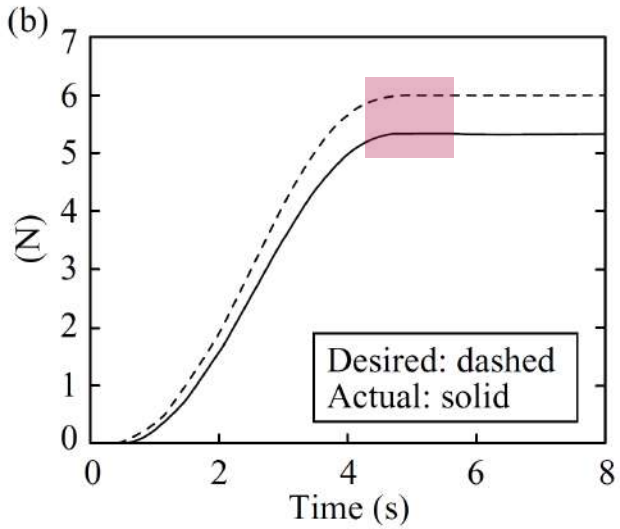


2^{nd} Case Study

position error



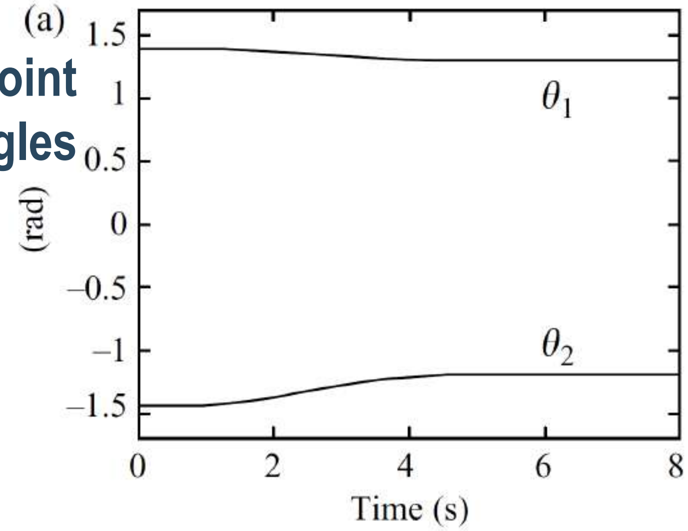
contact force



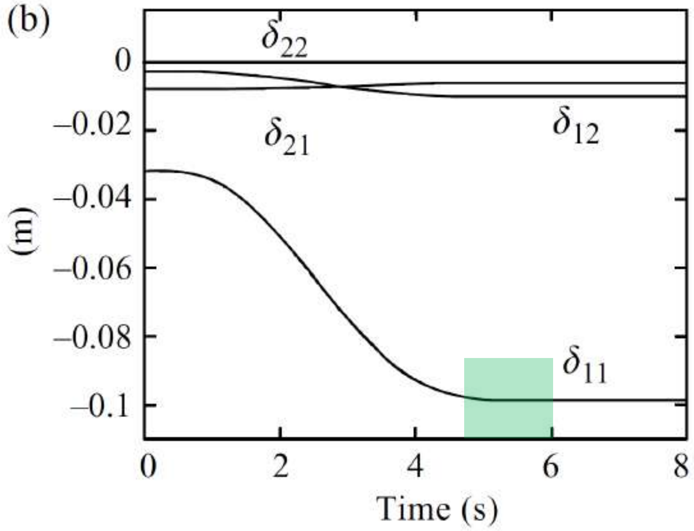
Contact stiffness is increased to 60 N/m

$$\mathbf{f}_d = \begin{bmatrix} 6 & 0 \end{bmatrix}^T \mathrm{N}$$

joint angles



link deflections



Direct Force and Position Control

- If a force sensor is available at the end-point of the robot, it is possible to achieve direct force control without requiring an exact estimate of the stiffness and of the position of the environment at rest
- If the dynamics related to link flexibility are suitably taken into account, tracking of a time-varying desired position can be achieved as well as regulation to a constant force

Composite Control Strategy

- When link stiffness is large, it is reasonable to expect that the dynamics related to link flexibility are much faster than the dynamics associated with rigid motion of the robot so that the system naturally exhibits a two-time-scale dynamic behaviour in terms of rigid and flexible variables
- The system can be decomposed into slow and fast subsystems by using singular perturbation theory; this leads to a composite control strategy for the full system based on separate control designs for the two reduced-order subsystems
- Assuming that full-state measurement is available and that a force sensor is mounted at the end-point of the robot, the joint torques can be conveniently chosen as

$$\boldsymbol{\tau} = \mathbf{g}_\theta(\boldsymbol{\theta}, \boldsymbol{\delta}) + \mathbf{J}_\theta^T(\boldsymbol{\theta}_d, \boldsymbol{\delta}_d)\,\mathbf{f} + \mathbf{u}$$

  to cancel out the effects of the static torques acting on the rigid part of the manipulator dynamics
  - The vector $\mathbf{u}$ is the new control input to be designed on the basis of the singular perturbation approach

Singularly Perturbed Model

- The time-scale separation between the slow and fast dynamics can be determined by defining the singular perturbation parameter $\varepsilon = 1/\sqrt{k_m}$

k_m smallest coefficient of diagonal stiffness matrix

- Elastic force $\mathbf{z} = \mathbf{K}\boldsymbol{\delta} = \frac{1}{\varepsilon^2}\hat{\mathbf{K}}\boldsymbol{\delta}$ $\quad \mathbf{K} = k_m\hat{\mathbf{K}}$

$$\mathbf{H} = \mathbf{M}^{-1} = \begin{bmatrix} \mathbf{H}_{\theta\theta} & \mathbf{H}_{\theta\delta} \\ \mathbf{H}_{\theta\delta}^T & \mathbf{H}_{\delta\delta} \end{bmatrix}$$

$$\ddot{\boldsymbol{\theta}} = \mathbf{H}_{\theta\theta}^T\left(\boldsymbol{\theta}, \varepsilon^2\mathbf{z}\right)\left(\mathbf{u} - \mathbf{c}_\theta\left(\boldsymbol{\theta}, \varepsilon^2\mathbf{z}, \dot{\boldsymbol{\theta}}, \varepsilon^2\dot{\mathbf{z}}\right)\right) - \mathbf{H}_{\theta\delta}\left(\boldsymbol{\theta}, \varepsilon^2\mathbf{z}\right)\Big[\mathbf{c}_\delta\left(\boldsymbol{\theta}, \varepsilon^2\mathbf{z}, \dot{\boldsymbol{\theta}}, \varepsilon^2\dot{\mathbf{z}}\right) + \mathbf{g}_\delta\left(\boldsymbol{\theta}, \varepsilon^2\mathbf{z}\right) + \varepsilon^2\mathbf{D}\hat{\mathbf{K}}^{-1}\dot{\mathbf{z}} + \mathbf{z} + \mathbf{J}_\delta^T\left(\boldsymbol{\theta}, \varepsilon^2\mathbf{z}\right)\mathbf{f}\Big]$$

$$\varepsilon^2\mathbf{z} = \hat{\mathbf{K}}\mathbf{H}_{\theta\delta}^T\left(\boldsymbol{\theta}, \varepsilon^2\mathbf{z}\right)\left(\mathbf{u} - \mathbf{c}_\theta\left(\boldsymbol{\theta}, \varepsilon^2\mathbf{z}, \dot{\boldsymbol{\theta}}, \varepsilon^2\dot{\mathbf{z}}\right)\right) - \hat{\mathbf{K}}\mathbf{H}_{\delta\delta}\left(\boldsymbol{\theta}, \varepsilon^2\mathbf{z}\right)\Big[\mathbf{c}_\delta\left(\boldsymbol{\theta}, \varepsilon^2\mathbf{z}, \dot{\boldsymbol{\theta}}, \varepsilon^2\dot{\mathbf{z}}\right) + \mathbf{g}_\delta\left(\boldsymbol{\theta}, \varepsilon^2\mathbf{z}\right) + \varepsilon^2\mathbf{D}\hat{\mathbf{K}}^{-1}\dot{\mathbf{z}} + \mathbf{z} + \mathbf{J}_\delta^T\left(\boldsymbol{\theta}, \varepsilon^2\mathbf{z}\right)\mathbf{f}\Big]$$

- Setting $\varepsilon = 0$ and solving for $\mathbf{z}$ gives

$$\mathbf{z}_s = \overline{\mathbf{H}}_{\delta\delta}^{-1}(\boldsymbol{\theta}_s)\,\overline{\mathbf{H}}_{\theta\delta}^{T}(\boldsymbol{\theta}_s)\left(\mathbf{u}_s - \overline{\mathbf{c}}_\theta(\boldsymbol{\theta}_s, \dot{\boldsymbol{\theta}}_s)\right) - \overline{\mathbf{c}}_\delta(\boldsymbol{\theta}_s, \dot{\boldsymbol{\theta}}_s) - \overline{\mathbf{g}}_\delta(\boldsymbol{\theta}_s) - \overline{\mathbf{J}}_\delta^{T}(\boldsymbol{\theta}_s)\,\mathbf{f}_s$$

$$\Downarrow$$

$$\ddot{\boldsymbol{\theta}}_s = \overline{\mathbf{M}}_{\theta\theta}^{-1}(\boldsymbol{\theta}_s)\left(\mathbf{u}_s - \overline{\mathbf{c}}_\theta(\boldsymbol{\theta}_s, \dot{\boldsymbol{\theta}}_s)\right)$$

$$\overline{\mathbf{M}}_{\theta\theta}^{-1}(\boldsymbol{\theta}_s) = \left(\overline{\mathbf{M}}_{\theta\theta}(\boldsymbol{\theta}_s) - \overline{\mathbf{M}}_{\theta\delta}(\boldsymbol{\theta}_s)\,\overline{\mathbf{M}}_{\delta\delta}^{-1}(\boldsymbol{\theta}_s)\,\overline{\mathbf{M}}_{\theta\delta}^{T}(\boldsymbol{\theta}_s)\right)$$

$\overline{\mathbf{M}}_{\theta\theta}(\boldsymbol{\theta}_s)$ inertia matrix of the equivalent rigid robot

$\overline{\mathbf{c}}_\theta(\boldsymbol{\theta}_s, \dot{\boldsymbol{\theta}}_s)$ vector of the corresponding Coriolis and centrifugal torques

- Setting $t_f = t/\varepsilon$, treating the slow variables as constants in the fast time-scale and introducing the fast variables $\mathbf{z}_f = \mathbf{z} - \mathbf{z}_s$

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$$\frac{\mathrm{d}^2\mathbf{z}_f}{\mathrm{d}t_f^2} = -\hat{\mathbf{K}}\overline{\mathbf{H}}_{\delta\delta}(\boldsymbol{\theta}_s)\,\mathbf{z}_f + \hat{\mathbf{K}}\mathbf{H}_{\theta\delta}^T(\boldsymbol{\theta}_s)\,\mathbf{u}_f \qquad \mathbf{u}_f = \mathbf{u} - \mathbf{u}_s$$ fast control

- On the basis of the above two-time-scale model

$$\mathbf{u} = \mathbf{u}_s\left(\boldsymbol{\theta}_s, \dot{\boldsymbol{\theta}}_s\right) + \mathbf{u}_f\left(\mathbf{z}_f, \mathrm{d}\mathbf{z}_f/\mathrm{d}t_f\right) \qquad \mathbf{u}_f\left(\mathbf{0},\mathbf{0}\right) = \mathbf{0}$$

- End-point acceleration

$$\ddot{\mathbf{p}} = \mathbf{J}_\theta\left(\boldsymbol{\theta},\boldsymbol{\delta}\right)\ddot{\boldsymbol{\theta}} + \mathbf{J}_\delta\left(\boldsymbol{\theta},\boldsymbol{\delta}\right)\ddot{\boldsymbol{\delta}} + \mathbf{h}\left(\boldsymbol{\theta},\boldsymbol{\delta},\dot{\boldsymbol{\theta}},\dot{\boldsymbol{\delta}}\right) \qquad \mathbf{h} = \dot{\mathbf{J}}_\theta\dot{\boldsymbol{\theta}} + \dot{\mathbf{J}}_\delta\dot{\boldsymbol{\delta}}$$

$$\Downarrow$$

$$\ddot{\mathbf{p}}_s = \overline{\mathbf{J}}_\theta\left(\boldsymbol{\theta}_s\right)\overline{\mathbf{M}}_{\theta\theta}^{-1}\left(\boldsymbol{\theta}_s\right)\left(\mathbf{u}_s - \overline{\mathbf{c}}_\theta\left(\boldsymbol{\theta}_s,\dot{\boldsymbol{\theta}}_s\right)\right) + \overline{\mathbf{h}}\left(\boldsymbol{\theta}_s,\dot{\boldsymbol{\theta}}_s\right)$$

- The slow dynamic model enjoys the same notable properties of the rigid robot dynamic model, hence the control strategies used for rigid manipulators can be adopted
- The fast subsystem is a marginally stable linear slowly time-varying system that can be stabilised to the equilibrium manifold $\dot{\mathbf{z}}_f = \mathbf{0}\ (\dot{\mathbf{z}} = \mathbf{0})\ \mathbf{z}_f = \mathbf{0}\ (\mathbf{z} = \mathbf{z}_s)$

$$\mathbf{u}_f = \mathbf{K}_1\dot{\mathbf{z}}_f + \mathbf{K}_2\mathbf{z}_f$$

Force and Position Regulation

- Simultaneous regulation of the contact force and position to constant set points
- Parallel control approach, especially effective in the case of inaccurate contact modelling
  - Force control loop working in parallel to a position control loop along each task space direction
  - The logical conflict between the two loops is managed by imposing dominance of force control over position control, i.e., force regulation is always guaranteed at the expense of a position error along the constrained directions
- Parallel regulator for the slow subsystem

$$\mathbf{u}_s = \bar{\mathbf{J}}_\theta^T(\boldsymbol{\theta}_s)\, k_P (\mathbf{p}_r - \mathbf{p}_s) - k_D \dot{\boldsymbol{\theta}}_s$$

$$\mathbf{p}_r = \mathbf{p}_d + k_P^{-1}\left(k_F(\mathbf{f}_d - \mathbf{f}_s) + k_I \int_0^t (\mathbf{f}_d - \mathbf{f}_s)\, d\tau\right)$$

Equilibrium Under Parallel Control

- Model of compliant environment with contact force normal to the plane in the slow time-scale

$$\mathbf{f}_s = k_e \mathbf{n}\mathbf{n}^T (\mathbf{p}_s - \mathbf{p}_o)$$

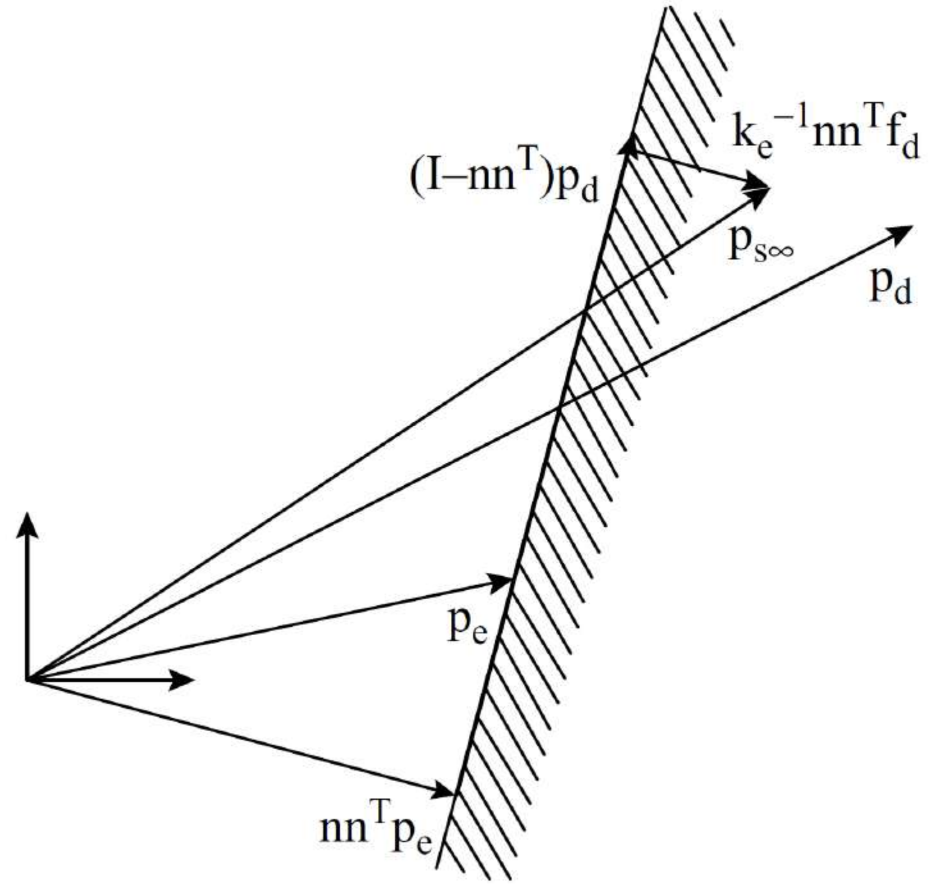
  - A null force error can be obtained only if the desired force is aligned with the normal
  - A null position error can be obtained only on the contact plane while the component of the position along the normal has to accommodate the force requirement

- The closed-loop system has an exponentially stable equilibrium

$$\mathbf{p}_{s\infty} = \left(\mathbf{I} - \mathbf{n}\mathbf{n}^T\right)\mathbf{p}_d + \mathbf{n}\mathbf{n}^T\left(k_e^{-1}\mathbf{f}_d + \mathbf{p}_o\right)$$

$$\mathbf{f}_{s\infty} = k_e \mathbf{n}\mathbf{n}^T (\mathbf{p}_{s\infty} - \mathbf{p}_o) = \mathbf{f}_d$$

- If $\mathbf{f}_d$ is not aligned with $\mathbf{n}$, then a drift motion of the end-point occurs along the plane; better set $\mathbf{f}_d = \mathbf{0}$



Force Regulation and Position Tracking

- If tracking of a time-varying position on the contact plane is desired (with an order ε approximation), an inverse dynamics parallel control scheme can be adopted for the slow subsystem (non-redundant robot)

$$\mathbf{u}_s = \overline{\mathbf{B}}_{\theta\theta}(\boldsymbol{\theta}_s)\,\overline{\mathbf{J}}_\theta^{-1}(\boldsymbol{\theta}_s)\left(\mathbf{a}_s - \overline{\mathbf{h}}\left(\boldsymbol{\theta}_s, \dot{\boldsymbol{\theta}}_s\right)\right) + \overline{\mathbf{c}}_\theta\left(\boldsymbol{\theta}_s, \dot{\boldsymbol{\theta}}_s\right)$$

$$\Downarrow$$

$$\ddot{\mathbf{p}}_s = \mathbf{a}_s$$

$$\mathbf{a}_s = \ddot{\mathbf{p}}_r + k_D(\dot{\mathbf{p}}_r - \dot{\mathbf{p}}_s) + k_P(\mathbf{p}_r - \mathbf{p}_s)$$

$$\mathbf{p}_r = \mathbf{p}_d + \mathbf{p}_C$$

$\mathbf{p}_C$ solution of $k_A\ddot{\mathbf{p}}_C + k_V\dot{\mathbf{p}}_C = \mathbf{f}_d - \mathbf{f}_s$

- Regulation of contact force to the desired set point and tracking of time-varying component of the desired position on the contact plane

- Planar two-link flexible robot placed in the same initial position with the end-point in contact with the plane and null contact force
- It is desired to reach the end-point position

  $$\mathbf{p}_d = \begin{bmatrix} 0.55 & -0.35 \end{bmatrix}^T \text{ m}$$

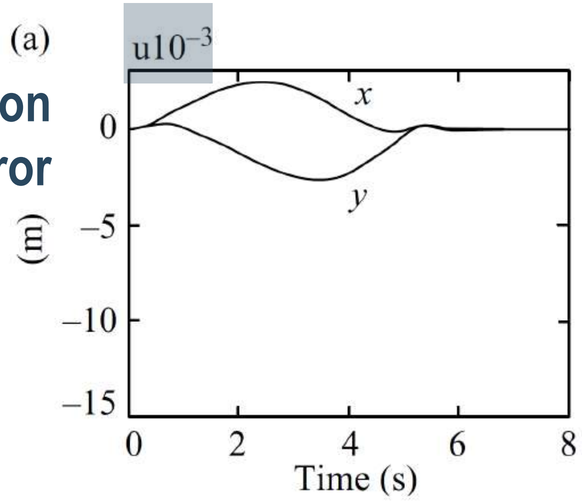
  and a fifth-order polynomial trajectory with null initial and final velocity and acceleration is imposed from the initial to the final position with a duration of 5 s
- The desired force is taken from zero to the desired value

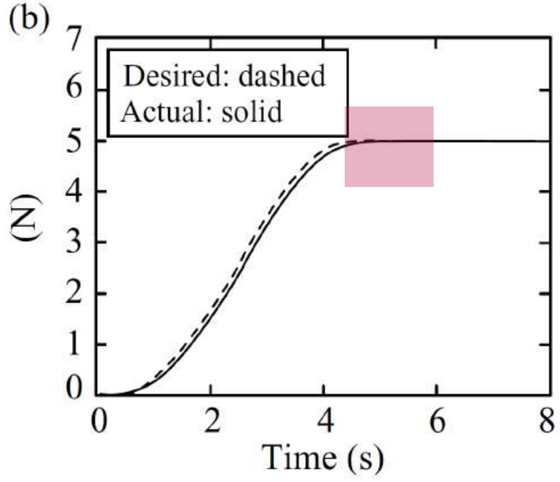
  $$\mathbf{f}_d = \begin{bmatrix} 5 & 0 \end{bmatrix}^T \text{ N}$$

  according to a fifth-order polynomial trajectory with null initial and final derivatives and duration of 1 s
- Fast control with $\varepsilon = 0.1606$ solving LQ problem

$$\mathbf{K}_1 = \begin{bmatrix} -0.0372 & -0.0204 & -0.0375 & 0.1495 \\ 0.0573 & 0.0903 & 0.0080 & -0.7856 \end{bmatrix} \quad \mathbf{K}_2 = \begin{bmatrix} -0.1033 & -0.0132 & -0.0059 & -0.0053 \\ -0.0882 & 0.0327 & -0.0537 & -0.0217 \end{bmatrix}$$

position error





contact force

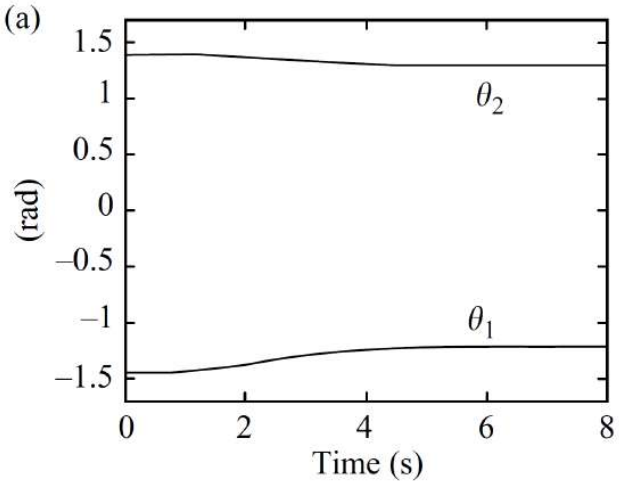
Parallel regulator

$k_P = 100$

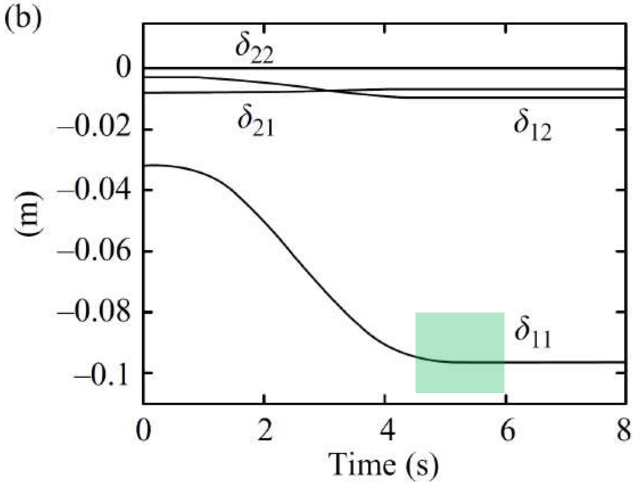
$k_D = 4$

$k_F = 100$

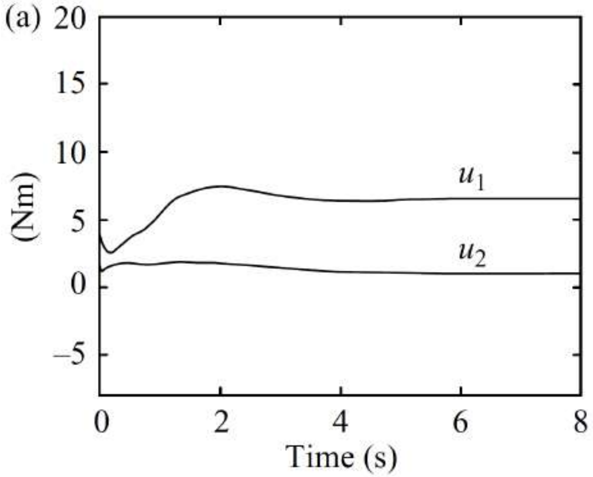
$k_I = 500$



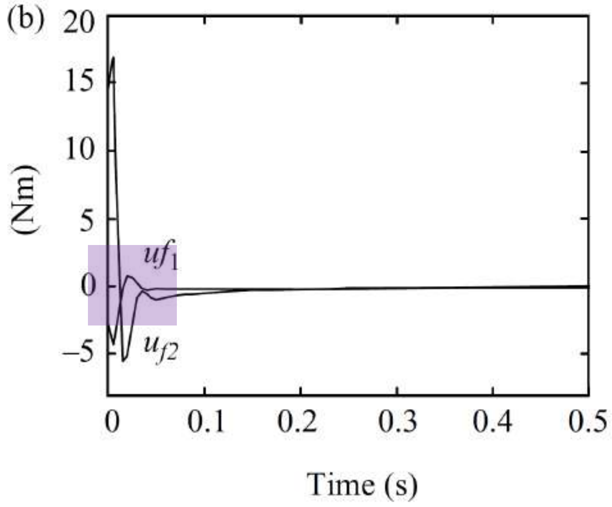
joint angles



link deflections

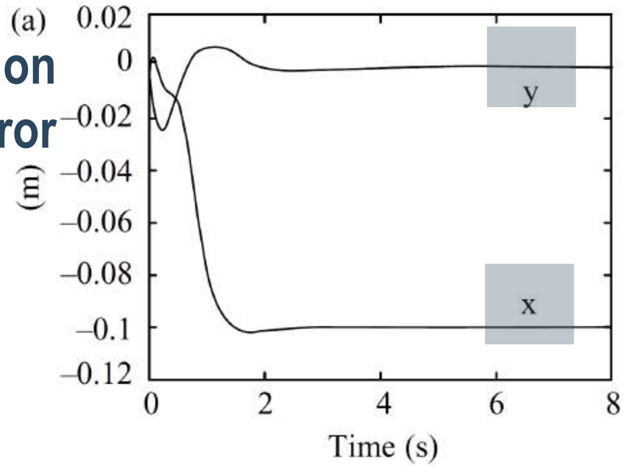


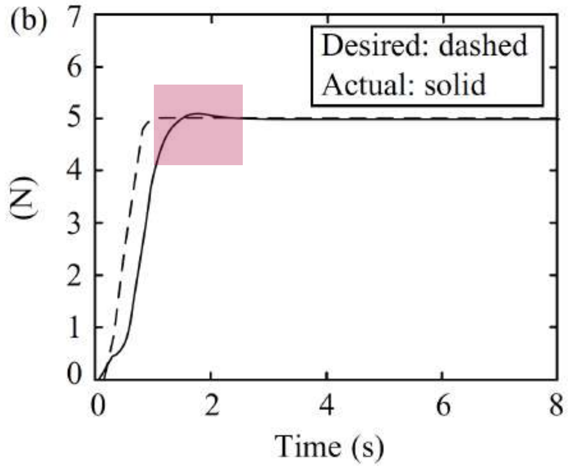
joint torques



fast control

position error

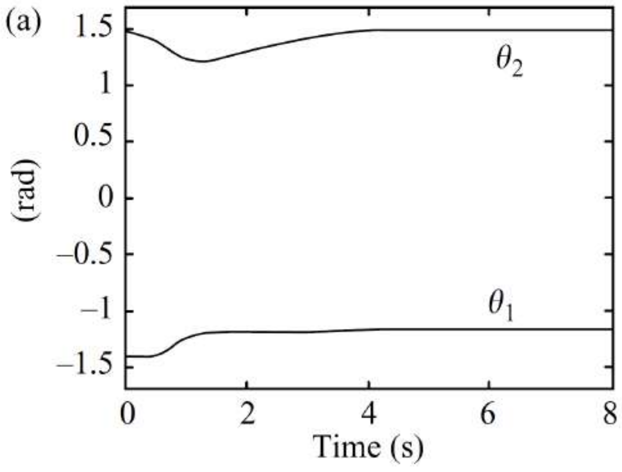




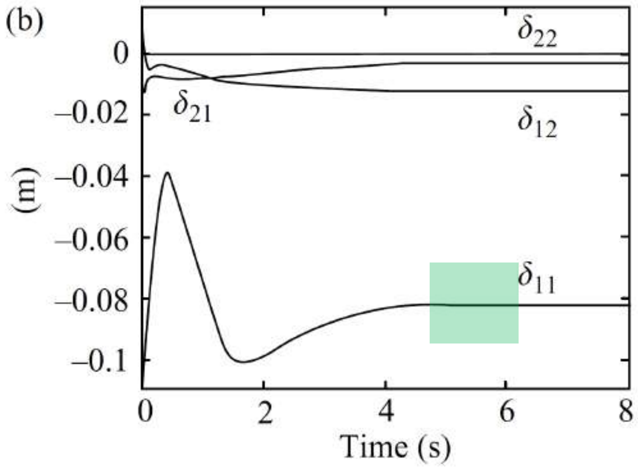
contact force

Inverse dynamics parallel control

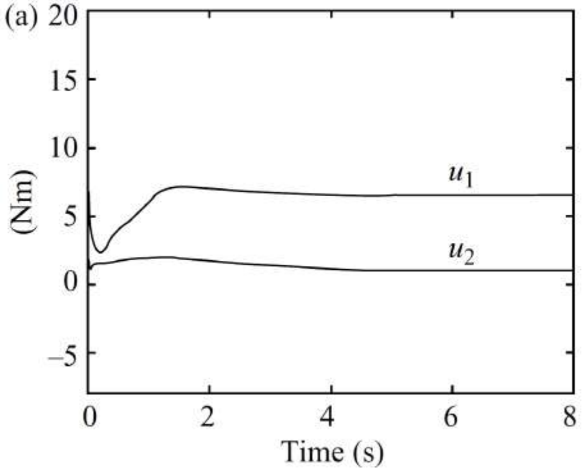
$$k_P = 100$$
$$k_D = 22$$
$$k_A = 0.7813$$
$$k_V = 13.75$$



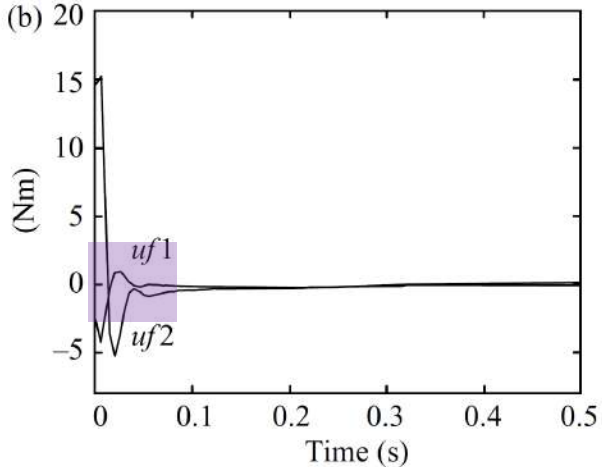
joint angles



link deflections



joint torques



fast control

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Thank You ☺