Vehicle motion control

SIDRA Summer School,
Bertinoro 2023

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CHALMERS

UNIMORE

Dipartimento di Ingegneria "Enzo Ferrari",
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Modena

Lecture objectives

- Formulating and solving a vehicle motion control problem

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  3. Control design

State-feedback control design

Recall that

$$\begin{bmatrix} \dot{e}_1 \\ \ddot{e}_1 \\ \dot{e}_2 \\ \ddot{e}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{2C_{\alpha f}+2C_{\alpha r}}{mV_x} & \frac{2C_{\alpha f}+2C_{\alpha r}}{m} & \frac{-2C_{\alpha f}l_f-2C_{\alpha r}l_r}{mV_x} \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{2C_{\alpha f}l_f-2C_{\alpha r}l_r}{I_zV_x} & \frac{2C_{\alpha f}l_f-2C_{\alpha r}l_r}{I_z} & -\frac{2C_{\alpha f}l_f^2+2C_{\alpha r}l_r^2}{I_zV_x} \end{bmatrix} \begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \dot{e}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{2C_{\alpha f}}{m} \\ 0 \\ \frac{2l_fC_{\alpha f}}{I_z} \end{bmatrix} \delta + \begin{bmatrix} 0 \\ -\frac{2C_{\alpha f}l_f-2C_{\alpha r}l_r}{mV_x} - V_x \\ 0 \\ -\frac{2C_{\alpha f}l_f^2+2C_{\alpha r}l_r^2}{I_zV_x} \end{bmatrix} \dot{\psi}_{des}$$

By setting $x = \begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \dot{e}_2 \end{bmatrix}$, $u = \delta$, $d = \dot{\psi}_{des}$ the model can be compactly rewritten as

$$\dot{x} = A(V_x)x + B(V_x)u + B_d(V_x)d.$$

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$$\delta = -Kx = -k_1 e_1 - k_2 e_2 = k_3 e_3 - k_4 e_4.$$

If the system is controllable, then the eigenvalues of the closed-loop matrix $A - BK$ can be arbitrarily placed (`K=place(A,B,p)`).

Exercise. Is the stability of $A - BK$ enough to achieve zero steady-state tracking error?

State-feedback lateral control design. Exercise

Consider the road-aligned model with the following parameters

$$m = 1537,\ I_z = 2873,\ l_f = 1.1,\ l_r 1.58,\ C_{\alpha f} = 8 \cdot 10^4,\ C_{\alpha r} = 8 \cdot 10^4.$$

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to have a (*dominant*) second order type of response.

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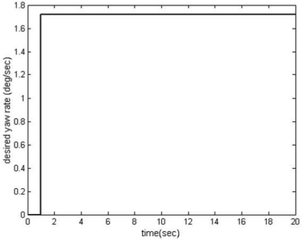

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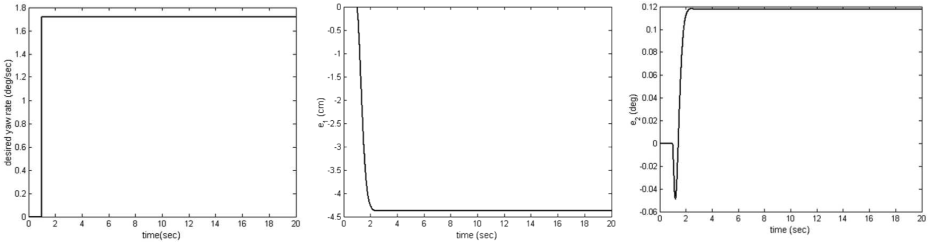

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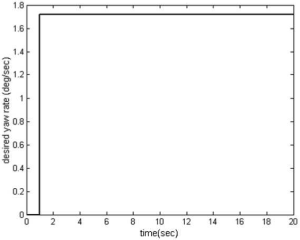

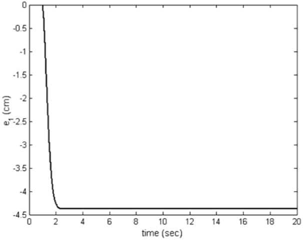

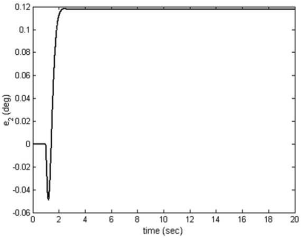

Why aren't e_1 and e_2 not converging to 0?

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At steady-state $(A - BK)x_{ss} + B\delta_{ff} + B_d\frac{V_{des}}{R} = 0$. By solving for x_{ss}

$$x_{ss} = \begin{bmatrix} \dfrac{\delta_{ff}}{k_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -\dfrac{1}{k_1}\dfrac{mV_x^2}{R(l_f + l_r)}\left[\dfrac{l_r}{2C_{\alpha f}} - \dfrac{l_f}{2C_{\alpha r}} + \dfrac{l_f}{2C_{\alpha r}}k_3\right] - \dfrac{1}{k_1 R}(l_f + l_r - l_r k_3) \\ 0 \\ \dfrac{1}{2RC_{\alpha r}(l_f + l_r)}\left[-2C_{\alpha r}l_f l_r - 2C_{\alpha r}l_r^2 + l_f m V_x^2\right] \\ 0 \end{bmatrix}$$

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While e_1 can be made zero, e_2 cannot.

Feedforward control and orientation error

$e_1 = 0$ is achieved with

$$\delta_{ff} = \frac{mV_x^2}{RL}\left[\frac{l_r}{2C_{\alpha f}} - \frac{l_f}{2C_{\alpha r}} + \frac{l_f}{2C_{\alpha r}}k_3\right] + \frac{L}{R} - \frac{l_r}{R}k_3,$$

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Observe that, when $e_1 = 0$, e_2 can be made 0 as well at the speed

$$V_x = \sqrt{\frac{l_r}{ml_f}2C_{\alpha r}(l_f + l_r)}.$$

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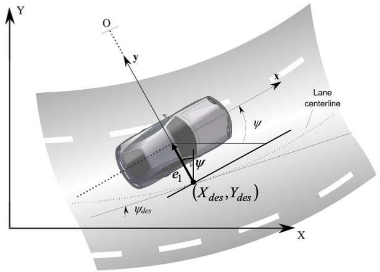
that is, the orientation of the vehicle velocity vector in the global frame is the same as the path to follow.



Effects of decoupled longitudinal and lateral control

Although the lateral and the longitudinal dynamics are coupled,

$$\dot{x} = A(V_x)x + B(V_x)u + B_d(V_x)d, \; x = \begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \dot{e}_2 \end{bmatrix}, \; u = \delta, \; d = \dot{\psi}_{des},$$

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This leads to the CL matrix $A(V_x) - B(V_x)K$, which must be stable (eigenvalues with negative real part in CT, within the unitary circle in DT) *regardless the speed* V_x.

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Let $V_x \in [V_{min},\ V_{max}]$ and

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Wide speed intervals can be partitioned into smaller intervals and state-feedback gains designed (*speed-dependent gain-scheduling*).

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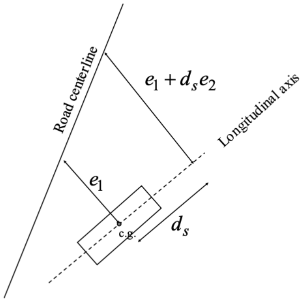

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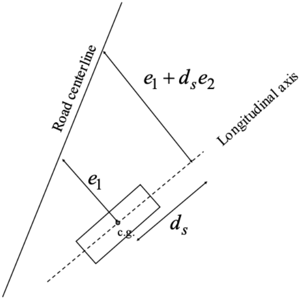
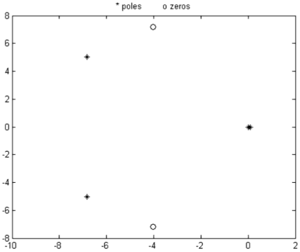
$$y = e_1 + d_s e_2.$$





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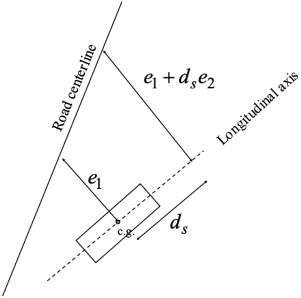
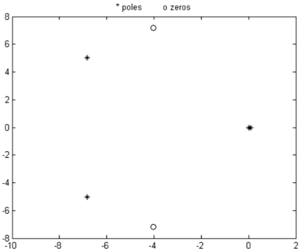
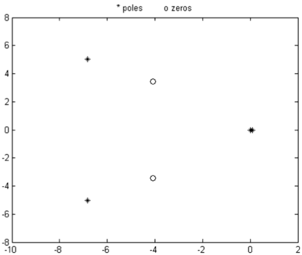
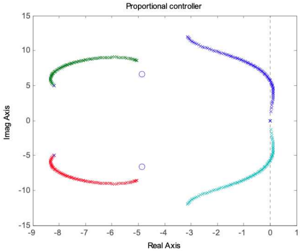


If a proportional controller $\delta = -Ky$ is used







Longitudinal (speed) control

Assumption. A low level powertrain control system exists delivering a requested vehicle longitudinal acceleration a_{des} according to (first order dynamics)

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A reasonable value for the time constant is $\tau = 0.5\ s$.

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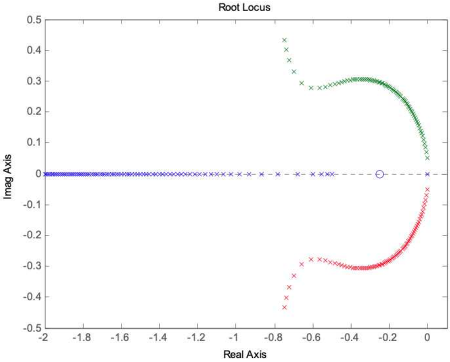

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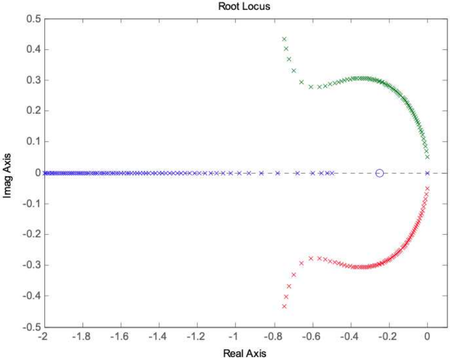
- Two eigenvalues on the origin,



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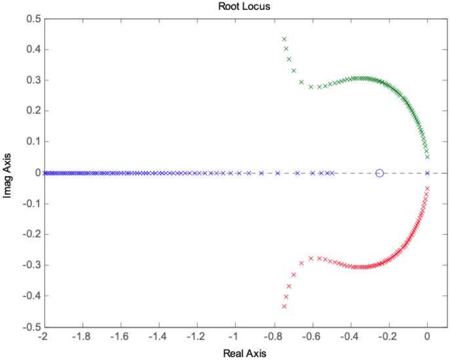

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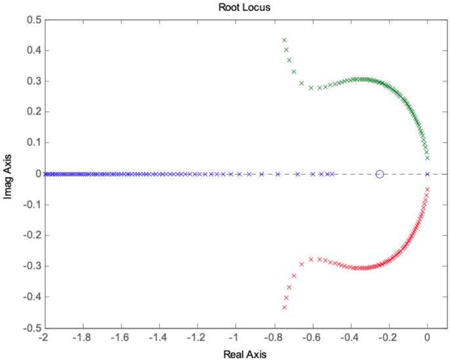

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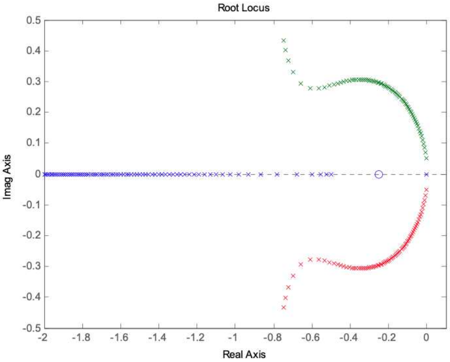

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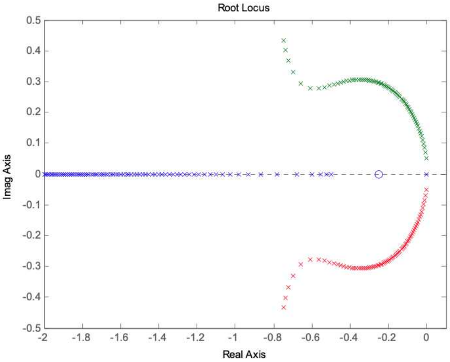

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- there's a value of k_P where max damping is attained, the CL system becomes less damped,
- use sisotool, rlocus in Matlab to place the eigenvalues of the CL matrix as desired.