SIDRA —Summer School 2024

Data-Driven Control Design course – Homework assignments

Assignment #1 (Two-cart and spring system)

Consider a sampled-data version of a two-cart and spring system. The dynamics of the system is given by the difference equation

$$x(t+1) = Ax(t) + Bu(t) + d(t)$$

with

$$A = \begin{bmatrix} 0.5780 & 0.8492 & 0.4220 & 0.1508 \\ -0.6985 & 0.5780 & 0.6985 & 0.4220 \\ 0.4220 & 0.1508 & 0.5780 & 0.8492 \\ 0.6985 & 0.4220 & -0.6985 & 0.5780 \end{bmatrix}, \quad B = \begin{bmatrix} 0.4610 \\ 0.8492 \\ 0.0390 \\ 0.1508 \end{bmatrix} \tag{1}$$

where x is the state, u is the control input, and d in an unmeasured process disturbance. The four components of the state are position (variable x_1) and velocity (x_2) of the first cart, and position (x_3) and velocity (x_4) of the second cart; u represent the force applied to the first cart.

Choose two of the following problems:

P1 State-feedback controller. Assume that the whole state of the system is available for measurements. Perform an experiment on the system with $\|u\| \leq 1$ and $\|d\| \leq 0.1$, and design a robust state-feedback controller as discussed in Lecture 2 (see also [R1]). Implement the control system in Matlab/Simulink and simulate the behavior of the closed-loop system. (Note: A zero-regulation problem is not realistic here because we cannot jointly regulate x_1 and x_3 to zero. Nonetheless, the design could be easily extended to an arbitrary nonzero equilibrium as we discussed in Lecture 3.)

P2 Linear quadratic regulation (LQR). Assume again that the whole state of the system is available for measurements. Perform an experiment on the system with $\|u\| \leq 1$ and $d \equiv 0$, and design the state-feedback controller that minimizes the following cost:

$$J = \sum_{k=1}^{\infty} \left(x(k)^\top Q_x x(k) + u(k)^\top R u(k) \right)$$

with weights $Q_x = I_4$ and $R = 1$. The resulting controller is known as the Linear Quadratic Regulator. We have not addressed this problem during class but the relevant material can be found in Lecture 1 and in reference [R2]. Implement the control system in Matlab/Simulink and simulate the behavior of the closed-loop system. (Note: As before, a zero-regulation problem is not realistic here because we cannot jointly regulate x_1 and x_3 to zero. Nonetheless, the design could be easily extended to an arbitrary nonzero equilibrium as we discussed in Lecture 3.)

P3 Output-feedback control. Assume now that the only variable which is available for measurements is $y = x_3$ (position of the second cart). Perform an experiment on the system with $\|u\| \leq 1$ and $d \equiv 0$, and design an output-feedback controller as we discussed in Lecture 1 (see also [R2]). Implement the control system in Matlab/Simulink and simulate the behavior of the closed-loop system. (Note: Unlike the previous cases, you will now obtain a dynamic controller.)

References

[R1] C. De Persis and P. Tesi, *Learning Controllers for Nonlinear Systems from Data*, Annual Reviews in Control, volume 56 (100915), 2023.

[R2] C. De Persis and P. Tesi, *Formulas for Data-driven Control: Stabilization, Optimality and Robustness*, IEEE Transactions on Automatic Control, volume 65, 2019.

Assignment #2 (One-link robot arm) Consider a robot arm system:

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\frac{K_c}{J_2}x_1 - \frac{F_2}{J_2}x_2 + \frac{K_c}{J_2 N_c}x_3 - \frac{mg\nu}{J_2}\cos x_1 \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= -\frac{K_c}{J_1 N_c}x_1 + \frac{K_c}{J_1 N_c^2}x_3 - \frac{F_1}{J_1}x_4 + \frac{1}{J_1}u
\end{aligned}$$

where x_1 and x_3 represent the angular positions of the link and of the actuator shaft, respectively, while u is the torque produced at the actuator axis. All the other quantities that appear in the above differential equations are unknown parameters that take on values $K_c = 0.4$, $F_2 = 0.15$, $J_2 = 0.2$, $N_c = 2$, $F_1 = 0.1$, $J_1 = 0.15$, $m = 0.4$, $g = 9.8$ and $\nu = 0.1$.

Reproduce both Example 1 and Example 3 in reference [R3]. The procedure discussed in Example 1 returns a state-feedback controller, while the procedure discussed in Example 3 returns a PI controller for set-point tracking. Implement both control systems in Matlab/Simulink and simulate the behavior of the closed-loop system. The relevant material is also available in Lecture 5.

References

[R3] Z. Hu, C. De Persis, and P. Tesi, *Enforcing Contraction via Data*, submitted. Preprint available at https://arxiv.org/abs/2401.07819.